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Volume IV. *July–August, 1896.* *Number 1.*

THE

PHYSICAL REVIEW.

ON THE MEASUREMENT OF THE EXPANSION OF METALS BY THE INTERFERENTIAL METHOD.

By Edward W. Morley and William A. Rogers.

Note by William A. Rogers.

DURING the summer of the year 1890 Professor Morley suggested to the writer the method, described in the paper herewith presented, for determining the coefficient of expansion of a bar of metal between the freezing and the boiling points of water in terms of a wave-length of light of known refrangibility.

After a full discussion of the form of apparatus best adapted for this purpose, it was arranged that I should construct a simplified form which should suffice for some preliminary experiments; that Professor Morley should come to Waterville during his summer vacation, and that we should jointly undertake the experiment.

The first series of observations was made in July, 1891. Both of the bars selected, one of steel and the other of bronze, were observed continuously for a period of twelve hours in melting ice. During this time the relative lengths of the two bars suffered no perceptible change. The ice which surrounded the steel bar was then removed, and the bar was kept in steam for several hours, the bronze bar remaining in melting ice. Under this condition the difference in the lengths of the two bars was also measured.

In this experiment, however, as was foreseen, it was found to be impossible to count the number of fringes of the sodium light employed, on account of the difficulty of obtaining sufficient freedom from the condensation of moisture upon the mirrors during the passage from low to high temperatures. The only thing which could be done under these circumstances was to measure the difference in length by means of a microscope attached to a plate upon which the steel bar was mounted, the graduated scale employed being disconnected from this plate.

This experiment confirmed our previous impression that all observations of this character must be made *in vacuo.*

It was therefore found necessary to reconstruct the refractometer. Profiting by the experience already gained, several modifications were made in the construction of the new apparatus. The bars compared were mounted in boxes of rolled brass. It was supposed that the air would be unable to enter the boxes through the pores of this metal. It was found, however, impossible to maintain a vacuum during the time required for a complete observation. After a fruitless trial of fourteen months it was decided to commence *de novo.*

The two brass boxes and the box for receiving the diagonal mirrors were cast in one piece, which weighed about 500 lbs. Every part of this casting was then covered on the outside with solder to the depth of one-eighth of an inch. The work was done with the ordinary soldering iron. The magnitude of this undertaking may be inferred when it is stated that the boxes with their connections are nearly 60 inches in length and 6 x 6 inches in cross-section. One hundred and fifty pounds of solder were required for this purpose. The construction of this refractometer was completed in 1894. Several months were occupied in testing the apparatus in every possible way. Early in June, 1895, I wrote to Professor Morley that, as far as I could see, everything was ready for the final trial. He therefore came to Waterville early in July for this purpose.

While the two experiments recorded in this paper are not in every respect satisfactory, they demonstrate with entire certainty the practical feasibility of the new method. In the first series of observations, a little oil was forced through the packing and dropped upon one of the mirrors when the temperature of 65° C.

was reached, thus compelling the abandonment of the experiment at this point.

In the second series, after the measures had been made at 65°, the vapor of sulphur extracted from the rubber packing tarnished the mirrors to such an extent that while the fringes could very easily be counted near the points of maximum visibility, it was found impossible to make the count at the minimum point beyond the second order. It was apparent, however, that there would be no difficulty in making the count at 100°, after freedom from the sulphur vapor should be secured.

In another respect the result of this experiment was a little disappointing. Throughout the observations the readings for temperature were made with the air thermometer; but at the very last moment it was found that, owing to a mechanical defect, which could not have been known until after the reduction of the observations, the readings taken could not be depended upon. It therefore became necessary to make a careful investigation of all the thermometers employed in the experiment. This discussion is given in the second part of this paper. The general features of the different forms of the apparatus employed in this investigation will be seen from the diagram and photographic views shown in the plate accompanying Part II.

SHANNON PHYSICAL LABORATORY, COLBY UNIVERSITY,
March 1, 1896.

PART I.

OPTICAL PRINCIPLES ON WHICH THE METHOD DEPENDS.

BY EDWARD W. MORLEY.

Two methods for determining the expansion of metallic bars are in common use. In the first method, micrometers are employed to measure, at different temperatures, the distance between two lines cut near the ends of the bars in question. If the bars compared are both exposed to the same temperatures, the measurements determine the difference between their expansions; but if one bar is kept at a constant temperature, while that of the other is varied, the measurements determine the absolute expansion of the latter.

In the second method, the changes in the length of a bar exposed successively to different temperatures are determined by interferential observations. This method gives, not absolute expansions, but the difference between the expansion of the bar and that of the support of the interferential apparatus, which can be previously determined by interferential observations of great accuracy. The method has been used only for bars of but a few millimeters in length.

The accuracy of which these methods are capable depends on the precision of measurements of length and of temperature. The International Bureau of Weights and Measures estimates that coefficients of expansion may be determined within one part in twelve hundred.

A method which was first suggested some eight years ago, and which seems likely to attain a somewhat greater accuracy, has now been put in practice, although with some of the imperfections to be expected in a first attempt. The results obtained are of such a character that it is now proper to describe the method more fully than has been done heretofore.

The new method resembles, in certain respects, each of the former methods. It resembles the micrometric method, for bars of any length may be used, and their lengths are compared while the temperature of one is varied, and that of the other is either varied or kept constant. It resembles Fizeau's method, for interferential phenomena are utilized. But the new process differs so materially from both of the older processes, that it cannot fairly be called a modification of either. Interferential observations replace the micrometric observations of the first process; but they differ both in method and in principle from those of Fizeau, for they are used, just like micrometric observations, to measure the difference of the length of two bars.

We have, in this interferential method of comparison, much greater accuracy in measuring small differences of length. We are also able, incidentally, to place the bars, during the comparison, in conditions especially favorable for accurate determination of their temperature. It is therefore fair to expect ultimately a decided gain in accuracy of measurements of expansion.

To explain the principle of this method, let *ab*, Fig. 1, represent a plane mirror, whose thickness we may for a moment imagine to be negligible. Suppose this mirror to be a film of silver of such thickness that thirty or forty per centum of the light falling on it is transmitted, and the remainder reflected. Let *c* and *d* be the axes of two metal bars ending in points; let the bars and the mirror be so adjusted that the image of *c*, seen by the eye placed at *k*, is parallel with *d*, and near it. Let *e*, *f*, *g*, *h*, be four plane mirrors, each held by a spring against one of the points which terminate the bars, and also against the points of two adjusting screws, by means of which we make the four mirrors perpendicular to the axes of the bars. Let means be provided for moving the bar *d*, with its mirrors, in the direction of its length. For the present, imagine the thickness of the mirrors *e* and *g* to be negligible.

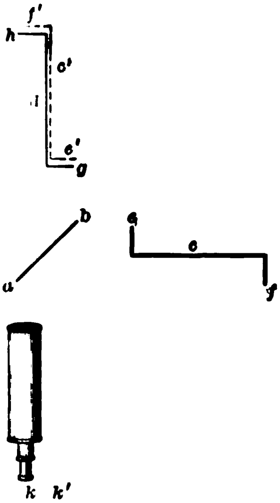


Fig. 1.

If now a source of light is placed to the left of *ab*, part of a beam falling on *ab* will be transmitted to *e* and *f*, most of this fraction will be there reflected back to *ab*, and in part to the eye at *k*. If we suppose, for example, that *ab* reflects seventy per centum of the light falling on it, and suppose no loss to occur in the reflections at *e*, *f*, *g*, *h*, and none by absorption in *ab*, twenty-one per centum of the light falling on *ab* will reach the eye at *k* by the route just mentioned. On the other hand, seventy per centum of the light incident on *ab* will be reflected to the mirrors *g* and *h*, and twenty-one per centum will be transmitted through *ab* to *k*. The transmitted and reflected rays will therefore be of equal intensity when they reach the eye, and will be in condition to produce interference phenomena of maximum distinctness.

It is obvious that the effect of the thinly silvered mirror *ab* is to superpose, optically, the system *ecf* upon the system *gdh*. To save circumlocution, let us represent the *apparent* position of *ecf* by the accented letters $e'c'f'$.

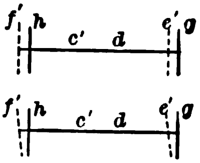
If the source of light is monochromatic, and if the distances of *g* and *h* from the eye at *k* do not differ too much from those of e' and f', interference fringes will be seen by the eye at *k*. When f' and *h* are not equidistant from *k*, these fringes will have their maximum distinctness if f' and *h* are very nearly parallel; the interference fringes will then be seen as circles when the observing telescope is adjusted for parallel rays. The same is true for e' and *g*. Suppose that such parallelism of f' and *h*, as well as of e' and *g*, is obtained and is preserved while we slowly move the system *gdh* so as to bring *h* nearer to f'. Each of the circular fringes seen at *k* will enlarge and disappear, while new fringes will appear at the center of the system of circular fringes. As f' and *h* approach, the number of circles which can be seen at once diminishes; at last, the field of view contains but a fraction of the diameter of the two inner fringes, and the continued approach of f' and *h* produces simple alternations of light and darkness. If, at the right time, white light is substituted for monochromatic, alternations of color will begin to be seen, till f' and *h* coincide, when we shall perceive the darkness which is produced by two equal beams of white light whose paths have been optically equivalent while one of them has suffered the change of phase which is due to an internal reflection.

Now, when the planes f' and *h* coincide, it is obvious that the ends of the axes of c' and *d* are in the same plane perpendicular to these axes. At this moment let us restore the monochromatic light, move the telescope to the position k', and observe the interference phenomena at the surfaces e' and *g*. It is obvious that, if e' and *g* are very nearly parallel, their distance is equal to the difference in the length of the two bars. We can move *d* in the direction of its length till e' and *g* coincide, and we can determine when they coincide by the appearance of the central black fringe in white light. Further, if we can determine how far *d* has to be moved to produce alternate coincidence of f' with *h*, and of e'

with g, we shall have a measure of the difference of length of c' and d.

If this difference is not more than one or two tenths of a millimeter, a convenient method is to count the number of half-wave-lengths contained in it, and to measure the residual fraction. For such small differences, the new method depends on such direct counting, but for greater differences, another principle is to be used.

The mirrors e' and g, f' and h, have been supposed to be rigorously parallel, because, in this case, the interference phenomena have the maximum of distinctness, and because the geometrical relations are simple. But another adjustment of the mirrors is commonly more convenient in practice. For this second adjustment of the mirrors, it is necessary that the axes c' and d should very nearly coincide. If such coincidence is secured, and if we are looking at $e'c'f'$ and gdh from one side, we should see them as in Fig. 2, supposing that $e'c'f'$ were a real object and not an image. Now let e' and f', still touching the points of c', be rotated a minute of arc, or less, about an axis in the plane of Fig. 1, and perpendicular to the plane of Fig. 2, so as to take the position shown in Fig. 3. The interference fringes will now appear as lines; these are straight, and of maximum distinctness when e' and f' are equidistant from the eye; they are also sufficiently distinct with the greatest difference of distance which is required in measuring the expansion of bars not more than one or two meters long.



Figs. 2 and 3.

With this adjustment, and with monochromatic light, we see the interference fringes when the telescope is adjusted for distinct vision of the surfaces of the mirrors e', g, or f', h, respectively. Suppose we are observing f', h, Fig. 1; suppose also that the upper half of f' was tilted backwards; if we slowly bring h nearer to f', the interference fringes, which appear as horizontal lines, will actually move upwards, and will seem to move downwards as seen in an inverting telescope. When the lower edges of f' and h are within about ten half-wave-lengths, colored fringes will begin to be seen in white light; when these lower edges coincide, the central black fringe will be there seen in white light. As the motion of *d* continues, this central fringe will move upwards (apparently downwards); when it passes through that point of the mirror *f* which rests on the point of the bar *c'*, the end of *c'* coincides with the plane *h*, and with the end of the bar *d*. Let us now move the telescope to the position *k'*, and obtain distinct vision of the surfaces *e'* and *g*; then, if the distance by which *d* must be moved to make the remaining ends of *c'* and *d* coincide is a whole number of half-wave-lengths of the monochromatic light used, a black fringe will pass through that point of the mirror *e'* which rests upon the point of the bar *c'*. But if this distance is a whole number, together with some fraction, the magnitude of this fraction is determined by micrometric or other measurement; after which, the whole number is observed by slowly moving *d*, and counting the number of fringes which pass a mark ruled on the surface of the mirror. We shall have thus determined the difference of length of *c* and *d*, as expressed in half-wave-lengths of the monochromatic light used.

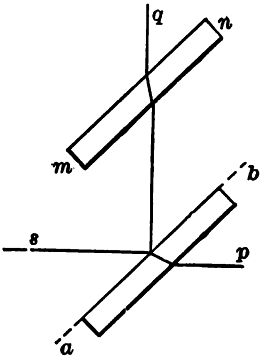


Fig. 4.

The thickness of the mirrors has been neglected in this explanation, and must now be considered. The mirror *ab*, Fig. 1, consists of a thin layer of silver deposited on the surface of a plane parallel plate of optical glass, *ab*, Fig. 4. A second plate, *mn*, of equal thickness, cut from the same disk, is set parallel to *ab*. The rays *sp* and *sq* pass through the same thickness of glass; if they also pass through the same thickness of air, at the same temperature, they will have equivalent optical paths.

The mirrors *f* and *h*, Fig. 1, have their reflecting surfaces supported on the points which determine the effective lengths of the bars. Their thickness, therefore, has no effect. But, with the constructions so far used in actual experiment, the mirrors *e* and *g* are supported on one surface while they reflect light from the other. Now these mirrors are plane-parallel plates of glass; if their temperatures are the same, and their thickness is the same, their effects will cancel each other. If their temperatures are the same, but their thicknesses different, their effects may be made to cancel by reversal. But if their temperatures are different, the coefficient of expansion of one of them must be known.

Two constructions are possible in which the expansion of the glass of the mirror need not be known. In one of these, both mirrors on the bar whose temperature is varied are supported by the reflecting surface. But this involves the cutting of a notch in the edge of the bar, and so is limited in application.

We have tried various methods for supporting the mirrors perpendicular to the axes of the bars, and have others in mind. These vary according to the purpose of the experiment. In one form, convenient when many measurements have to be made on the same pair of bars, the mirrors *b* and *c*, Fig. 5, rest on points on the bar *aa*, and also upon screws *de*, which are carried on pieces clamped to the bar. One such piece is shown in Fig. 6. Springs which press the mirror against the supporting points are also attached to the same piece; so that the mirrors, the adjusting screws, and the springs are somewhat permanently associated, and may be handled or adjusted as a whole. It is then convenient to support the bar by means of a hole, a groove, and a plane, resting on three points; in which case, the bar and its mirrors may be removed from the interferential comparer and again replaced without needing readjustment.

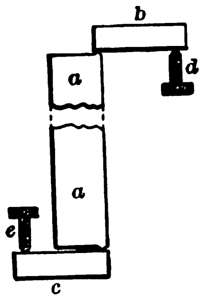


Fig. 5.

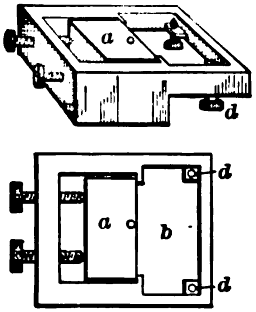
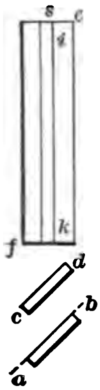


Fig. 6.

When the mirrors are supported in this way, they are adjusted so as to be perpendicular to the axis of the bar by the use of a collimating telescope, in a way which is perhaps too simple to need description. The two bars and their mirrors are then placed in the interferential comparer, the mirrors of the bar which is to be kept at constant temperature are left undisturbed throughout the experiment, while the mirrors on the other bar are given the slight inclination needed to make the direction and width of the interference fringes convenient for observation.

When the mirrors are supported and adjusted as now described, the subsequent adjustment of the bars for coincidence of the axes *c'* and *d*, Fig. 1, is somewhat troublesome. If experiments are to be made on a number of different bars in succession, it may be well to facilitate this adjustment of the axes in coincidence. In the method used for this purpose, the bars are first put in position in the interferential comparer, so that the lines joining the points on which the mirrors are to be supported, are made to coincide; after which the mirrors are brought up to the ends of the bars, and made perpendicular to them.



For this purpose, the metal base of the apparatus has two grooves, *ik*, *em*, Fig. 7. In these grooves are fitted four pieces *oo*, Fig. 8, each having a round aperture with cross-lines *p*. These pieces and the apparatus are adjusted so that, if two pieces are placed at *i* and *e*, Fig. 7, with lights at *r* and *s*, the two sets of cross-wires coincide, one seen direct, and one reflected at the surface *ab*. Two other pieces set at *k* and *m* are, in like manner, made to coincide. These four pieces are marked so as always to be used in the same positions.

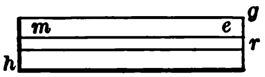


Fig. 7.

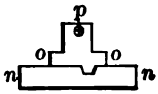


Fig. 8.

With this preparation, we can readily put two bars into the required position. We replace the pieces at *i* and *k*, and then adjust the bar with the points at its ends in the position shown by the cross-wires at *i* and *k*; we do the same with the other bar and the cross-wires at *m* and *e*, when the axes *c'* and *d*, Fig. 1, will coincide.

The mirrors may now be applied to the ends of the bars. The mirrors are held as shown in elevation by Fig. 9, and in plan by Fig. 10. The mirror *a* is contained in a brass frame *cc*. On this frame are two pivots, *ee*, which turn in holes made in the ends of the springs *dd*, which press the mirror against *b*, the point of the bar, whenever the mirror and its supports are brought sufficiently near the bar. Other springs, *ff*, press the frame against adjusting screws *gg*.

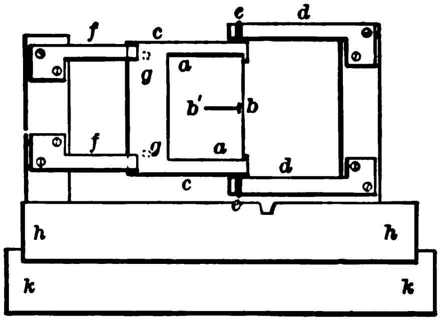


Fig. 9.

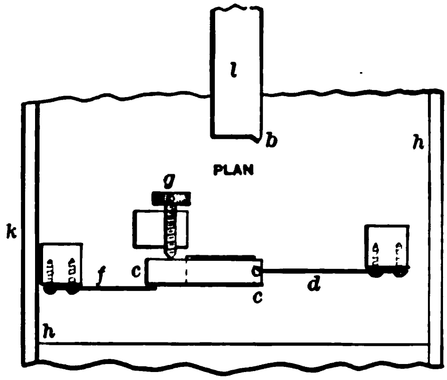


Fig. 10.

During the adjustment of the bars to the proper position with respect to each other, the mirrors are slipped back twenty or thirty millimeters from the ends of the bar, so that the pieces *o*, Fig. 8, can be put at *i*, *k*, *e*, *m*, Fig. 7, at the ends of the bar. When the bar has been adjusted to the position shown by the cross-wires, and secured there, these pieces are removed, and the mirrors and their supports are moved up to the ends of the bar by sliding the base *h*, Figs. 9 and 10, in the ways which carry it. When they have been clamped, the screws *g* are used to make the mirrors very nearly perpendicular to the axes of the bars by means of a collimating telescope which has been previously adjusted to the required position.

On the mirrors, lines *bb'*, Fig. 9, may be ruled, so as to mark the points of contact of the mirrors and the points of the bars. When the central black interference band in white light is brought to coincide with *bb'*, the end of one bar coincides with the image of the other bar within limits which it is easy to make less than the thirty-thousandth of a millimeter.

Neither of these methods of supporting the mirrors is the best; but it will be best to explain some other points before describing the method which will probably be found most accurate as well as convenient when we shall have had time to prepare suitable apparatus.

If the difference of length of two bars is determined in this apparatus, when both bars have the same temperature, the fact that the two rays of light pass through air introduces no error. But when the two bars have different temperatures, the optical path of the ray near the cold bar is made too great by the greater refractive index of the colder air. For this error, a suitable numerical correction could be applied. But, with any convenient arrangement of the apparatus, the presence of air produces another difficulty, which is more serious. Irregular currents are set up by the difference of density of the hot and the cold air, which interfere with the optical observations as well as with the accurate determination of the temperatures of the bars. It is therefore necessary to inclose the whole optical apparatus in a suitable chamber from which the air can be removed.

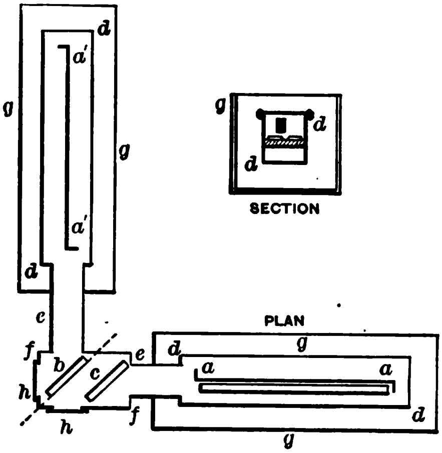


Fig. 11.

The arrangement used in the experiments which we have made is shown in Fig. 11. *aa*, $a'a'$, are the bars and their mirrors; *b*, *c*, are the diagonal mirror and its compensation. The bars are contained in two brass boxes *dd*, *dd*, one of which is also shown in section. These are placed in larger iron boxes *gg*, designed to hold water, ice, or steam, for maintaining the desired temperature. Tubes *ee* connect *dd*, *dd*, to the box *ff*, which contains the diagonal mirror and its compensation. *hh* are two openings covered with glass; through one light is admitted to the apparatus, and the observing telescope is placed before the other.

A screw, not shown in the drawings, provides the means of moving the bar *a′a′* in the direction of its length. The screws for adjusting the mirrors perpendicular to the axes of the corresponding bars are connected to handles which pass through stuffing boxes in the air-tight case.

It is obvious that, if we desire to measure expansions more accurately than to one part in one thousand, we must be prepared to measure differences of temperature with corresponding accuracy. Now, in our present apparatus, the bar *a′a′* is to be kept at a constant temperature by means of ice. We have accordingly thought it sufficient, at least in preliminary experiments, simply to verify the constancy of its temperature by means of delicate mercurial thermometers. These thermometers, each graduated in fiftieths of a degree, are introduced through the cover of the box; their bulbs rest on the bar *a′a′*, while their scales are in position for convenient observation.

The bar *aa* is that whose expansion is to be measured; its temperature must therefore be determined with accuracy. In our present apparatus, a square brass tube, about a meter long and twenty-five millimeters in diameter, is used as the bulb of an air-thermometer, and is placed in contact with the bar *aa*, to which it may well be attached by tying with wire. A capillary brass tube connects this thermometric reservoir with a suitable apparatus for measuring the pressure of the inclosed hydrogen.

When we desire to measure the difference of length of the two bars at some temperature which can be maintained constant by some physical process, like that of melting ice, we can make sure that the bars have actually reached a constant temperature, by observing whether the interference phenomena have become stationary. When the thermometers have also become stationary in their indications, we know that the thermometers indicate the temperature of the bars, for the bars, the thermometers, and their envelope are all at the same temperature, and no air is present. Interferential observations of the difference of length of the bars can then be begun, carried on, or repeated, with a degree of convenience amounting almost to luxury. The case is nearly the same if the temperature of boiling water is maintained by the use of steam; variations due to change of barometric pressure produce no inconvenience. But when some intermediate temperature is to be maintained by the use of water heated by leading steam into it, we have constantly to observe the temperature of the water by means of a delicate thermometer immersed in it. This involves vigorous and constant agitation of the water, during which observation of interferential phenomena is impossible on account of vibration. If, therefore, for the determination of the difference of length of two bars at a given temperature, this temperature had to be maintained during the whole operation, the new method could be applied only at temperatures which can be procured by utilizing melting or boiling points.

But, fortunately, the known and measured temperature is required for only a few seconds during the measurement. To illustrate from a simple operation, suppose we desire to measure the length of a steel tape. We have to uncoil it, and lay it on the scale *ab*, Fig. 12, and to keep it stretched with a known tension while we make the end *c* coincide with *a*, and while we then transfer our attention to the point *d* and press the other end of the tape against the scale. During the moment of this transfer of attention, we need to know the tension of the tape; but then, provided we hold *d* in its position, we may, if we please, let the tape be coiled again; we have nothing more to do with the end *c*, nor with the tension of the whole.

So with the interferential measurement. We interrupt the stirring of the water-bath surrounding the bar long enough to verify that *a* and *c*, Fig. 12, coincide; a matter of ten seconds or less. Repeating this verification at intervals, we may still maintain the temperature of the water-bath unchanged; during this time, the air thermometer is read. Now the observer at the telescope transfers his attention to *d*, and the other observer makes his final reading of the air thermometer. After this moment, nothing can interfere with the

Fig. 12.

accuracy of the interferential measurement of the distance *bd*, except some accident which interrupts the observations.

It may be said that, in our apparatus, the temperature of the bar *aa*, Fig. 11, can be observed by means of three thermometers, graduated in fiftieths of a degree, whose bulbs are in contact with the bar. These are useful for procuring and maintaining the desired temperature, and for verifying its constancy; but, in accurate observations, the temperature of the bar must be learned from the air thermometer whose reservoir is in contact with the bar.

In our apparatus, the motion of the bar *a'a'*, Fig. 11, is produced by a screw. This screw is turned by a worm-gear and worm, so that a motion of a small fraction of a wave-length can easily be procured. The experiments have been much delayed by the difficulty we found in getting brass boxes *dd*, *ff*, Fig. 11, which should not leak. One set had to be rejected after the apparatus was completed. A second set has shown leakage only in the stuffing boxes which surround the thermometers and the rods by which the adjustment of the mirrors is accomplished, and this leakage has been so far controlled that experiments have been successfully made with the apparatus.

In the experiments which have been made so far, the expansion of the bar has been determined by counting the whole number of wave-lengths which measures this expansion; the number was nearly 3500. To count 500 wave-lengths is easy, and takes less than ten minutes. But to count 3500 cannot be pronounced easy. We have commonly used the monochromatic light produced by putting a sodium compound in a colorless gas flame. We have sometimes used the green light produced by mercurial vapor in a Plücker's tube, and had no difficulty in making this so bright that we could count wave-lengths in a room where direct sunlight fell near the observer. But, with the means at our disposal, it was not easy to keep the induction coil working well for the period required, so that we returned to the use of the sodium light. Now, owing to the fact that this light is compounded of light of two wave-lengths which differ by about one thousandth part, the distinctness of the interference bands produced by it suffers periodic variations. If we begin to count when the two interfering rays have traversed equivalent paths, then the bands numbered from 450 to 550 are obscure, those from 1450 to 1550 and those from 2450 to 2550 are more obscure, and those from 3450 to 3550 are so obscure as to be very difficult to count. *This* difficulty in counting can readily be overcome by using the green light of incandescent mercury vapor, in which case no variations of distinctness in the interference bands occur within the limits utilized in observations on expansion. Suitable methods of breaking the circuit of the induction coil will make its action sufficiently uniform for many hours;[1] and Plücker tubes containing mercury can be so constructed that they will not fail by leakage due to heating the electrodes.

But even with every convenience for counting 4000 wave-lengths, the operation can hardly occupy less than an hour; so that observations at the temperatures which require the counting of such large numbers could not be readily multiplied. But, *during the actual measurement of expansions*, no count longer than 400 is necessary; in fact, it is possible to reduce the counting required, *during the actual comparison of the length* of two bars, to a hundred, to fifty, or even to twenty wave-lengths.

The methods devised with this end in view form a part of the original plan, which has not yet been carried out. But all the optical, and most of the mechanical, details of the method rest upon experiment, so that no uncertainty is felt as to the result. In order to explain this part of the new method, it will be convenient first to describe what seems to us now to be the best means of supporting the mirrors in contact with the ends of the bars and nearly perpendicular to their axes.

Figure 13 is part of Fig. 1, repeated for convenience. Suppose that h and g are rigorously parallel, and that e' and f' are also rigorously parallel; that is, with an error not more than one second of arc. Suppose, also, that the mirrors and bars are rigidly connected. If, now, we move the bar c' about some point k until f' is parallel to h, e' will also be parallel to g. Let us now move d in the direction of its length, so that f' and h coincide;

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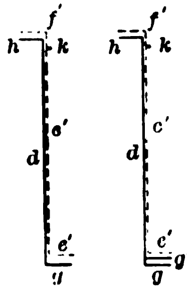
[1] Michelson, Travaux et Mémoires du Bureau International, tome 11, p. 137.

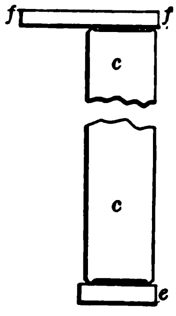
then the difference of length of *c'* and *d* will be equal to the perpendicular let fall upon *g* from ***any point*** in *e'*. It is not necessary, with this adjustment of mirrors and bars, to select that perpendicular which coincides with the axes of the two bars, nor to make these two axes coincide.

Now, imagine *g* replaced by two mirrors *g* and *g'*, Fig. 14, both parallel to *h*. It is obvious that we might make the distance between *g* and *h* equal to the distance between *e'* and *f'* when *c'* is at zero, and the distance between *g'* and *h* equal to the distance between *e'* and *f'* when *c'* is at, say, 100° C.; then the comparison of *c'* and *d* at 0° C. would require only the counting of a small number of wave-lengths, the number depending on the care used in securing equality of the distances involved. The same would be true of the comparisons at 100° C. The distance *gg'* can then be determined, by interferential observations, at leisure, either by direct counting of wave-lengths, or by the elegant method used by Michelson in his interesting and important work "On the Determination of the Value of the Meter in Wave-Lengths."[1]



Figs. 13 and 14.

The use of two mirrors, *g* and *g'*, involves the measurement of perpendiculars connecting the planes *e'* and *g*, *e'* and *g'*, but not coincident with the axes of the bars, because the mirrors *g* and *g'* cannot be superposed. Further, it implies the ability to support all the mirrors *e'*, *f'*, *g*, *g'*, and *h*, in positions which shall remain unchanged throughout all the measurements required for a given bar. For this purpose, it is convenient to dispense with adjusting-screws and movable supports for the mirrors.



Figs. 15 and 16.

Figure 15 shows how the mirror *e* of Fig. 1 would be supported. *c*, Fig. 15, represents part of the bar, whose end is wrought into three points, on which the mirror *e* rests. The adjustment of the mirror so as to be perpendicular to the axis of the bar is effected by a careful reduction of the height of one or another of these points, an operation which practice makes comparatively easy. To *c* is clamped in any convenient way a holder which supports the springs used to apply a constant pressure holding *e* in contact with the three points. Figure 16 shows one way in which the other mirror *ff* may be supported. Figure 17 shows the principle utilized in making *e* and *f* parallel to each other. The bar is attached to an axis bearing pivots at each end, on which it may be made to rotate. This axis might well be made tubular, so that the bar should be placed within it. If now we use a telescope with a collimating eyepiece, we may make successively the two mirrors perpendicular to the axis of revolution, and so parallel to each other. Instead of the telescope, a certain interferential apparatus may be used, with which a considerably greater accuracy of adjustment can be secured if necessary.

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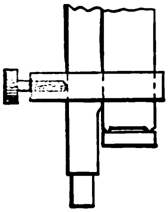
[1] Travaux et Mémoires du Bureau International, tome 11.



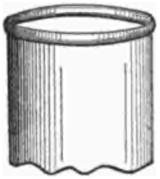
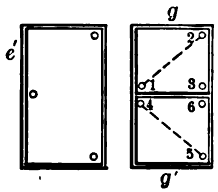


Fig. 17.

When mirrors are thus supported on points which are incapable of slight changes of position, and when care is used to make the pressure of the springs uniform, the constancy of position of the mirrors leaves nothing to be desired. Using supports of this kind, we can adjust the mirrors *g* and *g'*, Fig. 14, with the necessary constancy. Figure 18 represents the mirror *e* of Fig. 15, as seen from the front, the end of the bar, and the three supporting points. The mirrors *g*, *g'*, of Fig. 14, are shown from the front in Fig. 19; *g* is supported on three points wrought on the upper half of the end of the bar, and *g'* on three other points on the lower half. One of these mirrors is first made parallel to *h*, Fig. 14, by the method before explained. Then the second is made parallel to the first by any convenient method. As is seen in the side elevation, Fig. 20, the mirrors are not in the same



Figs. 18 and 19.

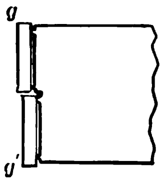


Fig. 20.

plane, and the perpendicular distance between the two reflecting surfaces is to be made approximately equal to the expansion to be measured.

After the mirrors *g*, *g*′ have been made parallel to the mirror *h*, Fig. 14, the points 3 and 6, Fig. 19, are lowered so that the mirrors are made to rotate about lines drawn through 1 and 2, and through 4 and 5. The amount of this rotation may be one minute of arc or less, according to the width of interference bands desired by the observer. If these mirrors were left parallel to the mirror *h*, Fig. 14, the coincidence of one of them with *e*′, Fig. 14, would be shown, in the conditions adopted, by uniform darkness covering the whole face of the mirror, and the counting of wave-lengths would be a counting of alternations of light and darkness. But most observers would prefer to count the march of a series of parallel light and dark bands; hence a suitable inclination is given to *g* and *g*′. In order that this inclination shall introduce no error, we have simply to mark the lines joining 1 with 2 and 3 with 4, and to establish our final coincidence along these lines, and not at some point taken at random.

It may be well to describe the whole course of an observation with this apparatus in the form last described. Let us suppose the bars and mirrors, just before the observation begins, to be placed as in Fig. 21, so that we shall have to move *h* in the direction of the arrow in order to make *h* and *f*′ coincide, and that, afterwards, we shall have to continue the motion in the same direction to make *g* and *e*′ coincide.

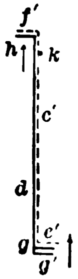


Fig. 21.

By means of the adjustments which move the bar *c*′ in altitude and azimuth about some point *k*, we give to *f*′ and *h* a slight inclination whose amount and direction are arbitrary; let us assume that, if *f*′ and *h* are first made parallel, the observer depresses the larger segment of *c*′ by ten or twenty seconds of arc. With monochromatic light and with the telescope giving distinct vision of the mirrors *f*′ and *h*, we shall see interference bands as shown at *l*, Fig. 22. It is by means of these bands that the amount and direction of the inclination of the mirrors is governed. Next, we illuminate part of the surface of *f*′ and *h*, Fig.

21, with white light; the interference bands of the monochromatic light will be scarcely visible over this part of the field, as shown in *II*, Fig. 22. We now slowly move h nearer to f', Fig. 21; in due time, colored interference bands in white light begin to be seen as shown at *III*, Fig. 22, as seen in an inverting telescope. Continuing the motion of h, we see the colored bands move upwards and cover the whole surface of the mirrors, and we stop when the central black fringe comes near to the center of the mirror, as at *IV*. We now carefully diminish the inclination of the mirrors, which makes the interference bands increase in width, as at *V*; we keep the central black fringe near the center of the mirrors and diminish the inclination till, finally, we have the mirrors parallel and the black fringe covering the whole surface of the mirrors, as at *VI*. We then move h backwards and forwards a tenth of a wave-length, leaving it in the position which gives the maximum of darkness; the planes f' and h now coincide.

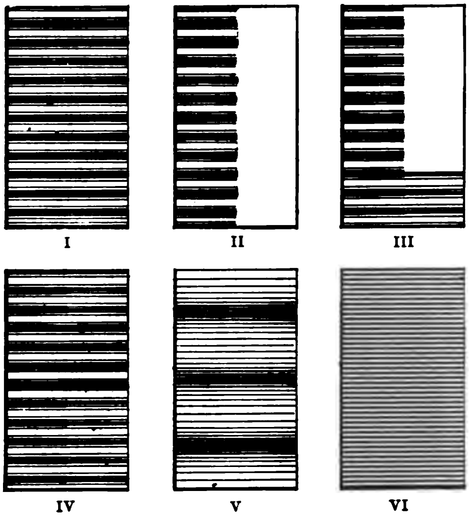
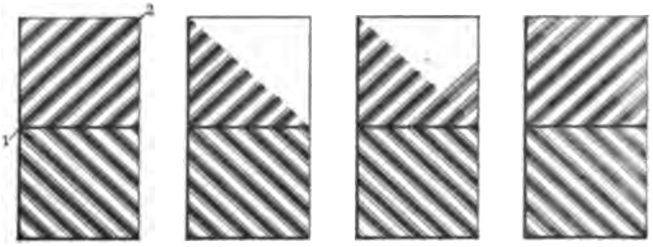


Fig. 22.

We now illuminate the mirrors e' and g, Fig. 21, with monochromatic light, and direct the telescope upon them, and observe a series of parallel interference bands covering both mirrors, as at *I*, Fig. 23. The line connecting the points 1 and 2, Fig. 19, is indicated in Fig. 23. If some one of the black interference bands exactly coincides with this fiducial line, the distance between the planes e' and g, Fig. 21, is a whole number of half-wave-lengths. In the figure, this is the case; if not, we should first measure or estimate the fractional part of a wave-length; measurement is easy, but estimation is sufficient. The observer will have found by trial that, as we bring g nearer c', Fig. 21, the fringes in Fig. 23 will move upwards. The distance from the fiducial line to the fringe which will first cross the line is to be estimated. This fraction is recorded, and is verified after returning to the mirrors $f'h$, and repeating the adjustment for coincidence. Then we introduce white light, as shown at *II*, Fig. 23. We now carefully move g, Fig. 21, nearer e', and count the black bands which pass the fiducial line 1 2, Fig. 23, until we are warned by the appearance of colored bands in white light, as at *III*, that we are near the final coincidence. The count is ended when the black band in white light coincides with the fiducial line, as at *IV*. The whole number counted, together with the fraction first estimated, gives the difference in the effective lengths of the two bars; using the phrase "effective length" to mean the perpendicular distance between the fiducial line and the surface of the remote mirror. This length is made up, when the mirrors are supported as now described, of a length of metal together with a thickness of glass.



I II III IV

Fig. 23.

If we now change the temperature of the bar c, Fig. 1, by a suitable amount, we may repeat the observation, using only the mirror g', Fig. 20, in place of g. The lengths and temperatures may be so chosen that the counting occupies only a minute, and may therefore be repeated as often, and throughout as long a time, as may be thought proper. Afterwards we have to determine the difference of the lengths gh and $g'h$, either by direct count or by Michelson's method; in either way, the determination is easy and accurate.

When we wish to repeat the pair of observations with a different interval of temperature, the distance between the planes g and g' will need to be changed accordingly. But we need not repeat the adjustment of the points of support, Fig. 19. Instead of this, we provide ourselves with a number of plane parallel glass plates of thicknesses varying by a tenth or a twentieth of a millimeter, and we replace the plate g', Fig. 20, by one whose thickness is greater or less by the desired amount.

It is also possible to mount the mirrors gg', Fig. 14, upon sliding carriers, adjust them to the desired position, and clamp them. In this case, there would be left something to be desired as to the fixity of the effective length gh, $g'h$, which would require attention.

If observations by our new method were made in a good vacuum, and if the temperature of one bar were quite constant, the differences of the observed numbers of wave-lengths would represent the real changes of length of the bar whose temperature is made to vary. Neither of these conditions being always obtained, two numerical corrections must be applied to the observed numbers.

(To be concluded.)

THE VISCOSITY OF POLARIZED DIELECTRICS.

By A. Wilmer Duff.

THE list of changes observed in the mechanical properties of a non-conducting liquid on subjecting it to electrostatic stress is a very short one, and that notwithstanding the fact that the subject is one of the greatest importance, because of its bearing on the question of the nature of electrification and electrostatic stress.

Quincke[1] found that some liquid dielectrics expanded in volume and others contracted under electrostatic stress; but Röntgen,[2] on repeating the experiments, failed to find the contractions and explained the expansions as due to heat produced by the passage of electricity through the dielectric; hence no satisfactory conclusion has been reached. Kerr[3] discovered that a liquid dielectric became double refracting under electrostatic stress, some liquids acting "like glass extended in a direction parallel to the lines of electric force" and others "like glass compressed in a direction parallel to the lines of electric force." These phenomena of Kerr are, however, only inferentially mechanical. W. König,[4] Pagliani,[5] Noack,[6] examined the viscosity of the dielectric, but failed to find any alteration of it due to electrostatic stress. Faraday's observation[7] that fibers of silk in the polarized dielectric stretched themselves in the direction of the electric force is an illustration of the effect of different inductive capacities. Hence, it would seem that we still lack positive knowledge as to the effect of electrostatic stress on the mechanical properties (density, compressibility, viscosity, etc.) of liquid dielectrics.

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[1] Wiedemann's Annalen, X., p. 521, 1880.

[2] Wiede. Ann., XI., p. 780, 1880.

[3] Philosophical Magazine, Nov., Dec., 1875; Aug., Sept., 1879.

[4] Wiede. Ann., XXV., p. 618, 1885.

[5] Ac. Torino, 20, p. 615, 1885; 22, p. 1, 1887 (Winkelmann's Physik).

[6] Wiede. Ann., XXVII., p. 289, 1886 (Winkelmann's Physik).

[7] Experimental Researches, XII., 1350.

About a year ago I undertook to reëxamine the question of a possible viscosity variation, but at first failed to find evidence (at least conclusive evidence) of any such effect. The method employed was not that of measuring the flow through a capillary tube placed between condenser plates (König), but one depending on the rate of descent of small spheres through a viscous liquid. Failing to find the effect sought, I have lately repeated the experiments with more care, rather with a view to fixing a superior limit to a possible viscosity variation than with the hope of actually discovering such a variation. The results obtained seem, however, to point clearly to the existence of the variation in question. Not many liquids have as yet been examined, as it seemed of more importance at first to thoroughly test one liquid under varying conditions. The results given in the present paper will, therefore, be limited to two or three liquids; and the extension of the methods employed to other liquids, together with a study of the law of variation of the effect under varying intensity of stress, will be left to a future paper.

The capillary tube method seemed to me, at first, objectionable, because of the difficulty of maintaining a sufficiently constant temperature for the requisite length of time; for, as is well known, the viscosity of a liquid varies very greatly with its temperature (as much as 10 per cent per degree in glycerin), and even a small change of temperature might completely mask other considerable changes of viscosity. This will appear from the following figures given by König, carbon bisulphide being the liquid used.

Temperature.	Viscosity.		Temperature.	Viscosity.	
17.5	0.00413	Plates uncharged.	17.8	0.00422	Plates uncharged.
16.8	0.00447	Plates uncharged.	17.8	0.00420	Plates charged.
17.6	0.00418	Plates charged.	16.2	0.00410	Plates uncharged.
18.6	0.00411	Plates uncharged.	16.1	0.00414	Plates charged.

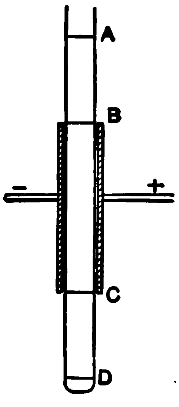
No doubt a much greater constancy of temperature than the above might be readily attained, but there seemed to be more hope in employing some method that would eliminate the effect of temperature variations.

The method adopted was to make use of the formula[1] found by Stokes for the rate of descent of a sphere through a viscous fluid. The steady velocity attained by a sphere falling through a fluid is:

$$V=\frac{2}{9}\frac{\rho-\sigma}{\eta}ga^2,$$

ρ being the density of the sphere, σ that of the fluid, a the radius of the sphere, and η the viscosity of the fluid. This is on the assumption that there is no finite slipping between the sphere and the liquid and that none such exists (at least in the case of such materials as are used in what follows) has been shown by Mr. O. G. Jones.[2] In the remainder of this paper the only assumption made from the above formula is that as the viscosity of a liquid increases, the time of descent of a sphere through a given distance in the medium also increases. (The possibility of a sufficient change in the density of the liquids used in my experiments or of an expansion of the glass tank employed will be adverted to later.)

If, then, a tube be filled with a viscous liquid and the time of descent of a small drop of mercury (the use of which will be justified later) through the length of the tube be noted and the tube be then inverted between the plates of a charged condenser, any change of viscosity would be shown by a change in the time of descent of the mercury drop. This would imply, of course, a constant temperature for the liquid, but such being unattainable, the following modification was made for approximately eliminating the temperature effect. Instead of the whole tube being placed between the condenser plates, the plates were placed opposite the middle half of the tube only, rings being etched around the tube to mark off the upper quarter, *AB*, the middle half, *BC*, and the lower quarter, *CD*, of the length through which the descent was observed. By means of two carefully tested stop watches, the ratio of the time of descent through *BC* to the sum of the



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[1] Lamb's Motion of Fluids (1879), § 184.

[2] Proceedings Physical Society of London, 1894, or Nature, Feb. 22, 1894.

times of descent through *AB* and *CD* was obtained. This ratio would, of course, be independent of the actual size of the drop (providing its steady velocity was attained before reaching *A*), so that the inversion of the tube was unnecessary. Another drop was then allowed to descend, and the above ratio again obtained, the condenser being in this case charged. By this method of taking ratios, we get rid of the effect of any steady variation in temperature affecting the whole tube equally. Hence, barring local inequalities in temperature variation, if any change of the viscosity of the liquid between *B* and *C* were produced by the electrostatic stress, we should find a difference between the ratios obtained above.

Using the above method, I made seven series of observations on glycerin, which, though a very imperfect insulator, was chosen because of its suitable viscosity. The work was carried on in a room not artificially heated. Of the seven series of observations taken, two seemed to point clearly to the existence of the variation sought, three others showed one break each in the consistency of their indications, and the others were quite irregular, though giving a mean result of the same sign as before. These observations, therefore, were, in the absence of any previous knowledge as to such a variation, of little value in themselves; but, as they have become of some importance when taken in connection with later results by a more satisfactory method, I give the two following series as samples, the first being fairly representative of the consistent sets obtained, and the second of the semi-consistent sets. The condenser was kept charged by means of a small influence machine worked with a fair degree of uniformity. The distance between the condenser plates was 3.5 cm. As a rough indication of the potential I give the mean length of spark between the terminals of the influence machine.

Supposing for the moment that those results which were consistent among themselves were not misleading in their indications, the most probable source of the inconsistency of the others was irregular temperature fluctuations. Hence it seemed desirable to continue the observations under the following more favorable conditions: (1) in a room of more constant temperature, (2) with

SERIES I.

GLYCERIN.

Condenser.	T_1 = sum of times of descent through *AB* and *CD*.	T_2 = time of descent through condenser.	T_2/T_1.	Excess of T_2/T_1 when condenser was charged.	Temperature.	Remarks.
	min. sec.	min. sec.				
Uncharged.	4 15.60	4 12.40	0.9875	+0.0074	19.8	
Charged.	4 7.75	4 6.50	0.9949			
Uncharged.	4 28.25	4 26.50	0.9935	+0.0044	19.8	
Charged.	4 4.75	4 4.25	0.9979			
Uncharged.	5 1.75	4 54.75	0.9768	+0.0116	—	
Charged.	3 36.50	3 34.00	0.9884			
Uncharged.	5 4.25	4 54.25	0.9761	+0.0030	20.9	Interval.
Charged.	4 36.00	4 30.25	0.9791			
Uncharged.	3 6.25	3 2.50	0.9799	+0.0076	21.0	
Charged.	4 0.50	3 57.50	0.9875			
Uncharged.	3 31.20	3 28.00	0.9846	+0.0048	21.0	
Charged.	3 9.50	3 7.50	0.9894			
Uncharged.	4 6.20	4 1.50	0.9797	+0.0169	21.2	
Charged.	3 53.00	3 52.20	0.9966			
Uncharged.	3 13.70	3 10.20	0.9819	+0.0083	—	
Charged.	4 1.50	3 59.20	0.9902			

Mean length of spark, 1.6 cm.

a liquid less sensitive to temperature, but still of a suitable viscosity, (3) with the use of a large body of liquid whose temperature might not change so readily. I therefore made a large tank 60 cm. × 30 cm. × 2 cm. (internal dimensions). This was constructed of sheets of plate glass, solidly clamped on an open rectangle of hard wood, with rubber tubing between the glass and the wood. The tank was marked off into an upper quarter, a middle half, and a lower quarter, as in the case of the tube. Sheets of tin-foil glued to the middle halves of the plates served as a condenser, the edges of the tin-foil being thickly coated with paraffin wax. The observations were thenceforward made in a cellar of the Purdue Electrical Building. To protect the tank

SERIES II.

GLYCERIN.

Condenser.	**T_1 = sum of times of descent through AB and CD.**	**T_2 = time of descent through condenser.**	**T_2/T_1.**	**Excess of T_2/T_1 when condenser was charged.**	**Remarks.**
	min. sec.	min. sec.			
Uncharged . . .	1 7.2	1 7.7	1.007	−0.003	
Charged	1 17.2	1 17.5	1.004		
Uncharged . . .	1 36.0	1 36.0	1.000	+0.005	
Charged	1 37.5	1 38.0	1.005		
Uncharged . . .	1 49.0	1 49.5	1.005	+0.007	
Charged	3 11.2	3 13.5	1.012		
Uncharged . . .	1 38.2	1 39.2	1.010	+0.003	
Charged	1 17.2	1 18.2	1.013		
Uncharged . . .	1 31.0	1 32.0	1.011	+0.002	
Charged	1 42.2	1 43.5	1.013		
Uncharged . . .	1 21.7	1 21.7	1.000	+0.003	
Charged	0 58.2	0 58.5	1.003		

Mean length of spark, 1.3 cm.

from air currents, it was surrounded by glass screens. A bright background for the observation of the descending drops was obtained by reflecting sky light from tilted glass plates. The most suitable time for working was found to be between one and four o'clock in the afternoon, during which period the temperature of the cellar usually reached a maximum, and varied but slowly.

The results obtained were at first somewhat irregular, until the following simple modification was made. The drops were started in regular rapid succession, and in calculating, the ratio T_2/T_1, obtained with the condenser uncharged, was compared with the mean of the ratios obtained immediately before and immediately after with the condenser charged. This process nearly eliminated any uniformly progressive temperature variation from above or below. It also eliminated any steady variation in the rates of the stop watches used for taking time. Only days of normal temperature conditions were chosen for work. These conditions being observed, the observations showed thenceforward an almost invariable consistency.

The following series of readings seem to show that castor oil suffers an increase of viscosity under electrostatic stress.

SERIES III.

CASTOR OIL.

Condenser.	T_1 = sum of times of descent through highest and lowest quarters.	T_2 = time of descent through condenser.	T_2/T_1; condenser charged.	Mean of successive values of T_2/T_1; condenser charged.	T_2/T_1; condenser uncharged.	Excess of T_2/T_1; condenser charged.
	min. sec.	min. sec.				
Charged. .	2 37.75	2 31.0	0.9572	—	—	—
Uncharged .	3 40.50	3 30 0	—	0.9564	0.9524	+0.0040
Charged. .	3 40.25	3 30.5	0.9557	—	—	—
Uncharged .	3 7.75	2 58.5	—	0.9546	0.9507	+0.0039
Charged. .	3 18.75	3 9.5	0.9535	—	—	—
Uncharged .	4 14.75	4 2.0	—	0.9534	0.9500	+0.0034
Charged. .	3 39.75	3 29.5	0.9534	—	—	—
Uncharged .	3 21.75	3 11.5	—	0.9536	0.9492	+0.0044
Charged. .	3 20.25	3 11.0	0.9538	—	—	—
Uncharged .	2 58.75	2 49.5	—	0.9537	0.9482	+0 0055
Charged. .	2 31.00	2 24.0	0.9536	—	—	—
Mean					0.9501	+0.0043

Mean increase	0.45 per cent.
Mean spark length	3 cm.
Temperature of oil decreased from	17°.5 to 16°.8
Temperature of air decreased from	17°.5 " 17°.0

Series IV.

CASTOR OIL.

Condenser.	T_1 = sum of times of descent through highest and lowest quarters.	T_2 = time of descent through condenser.	T_2/T_1; condenser charged.	Mean of successive values of T_2/T_1; condenser charged.	T_2/T_1; condenser uncharged.	Excess of T_2/T_1; condenser charged.
	min. sec.	min. sec.				
Charged . .	1 19.25	1 16.25	0.9621	—	—	—
Uncharged .	2 8.00	2 3.00	—	0.9583	0.9609	−0.0026
Charged . .	1 50.00	1 45.00	0.9545	—	—	—
Uncharged .	2 42.00	2 35.00	—	0.9577	0.9568	+0.0009
Charged . . .	1 55.00	1 50.50	0.9609	—	—	—
Uncharged .	1 52.50	1 47.00	—	0.9638	0.9511	+0.0027
Charged . .	1 56.00	1 52.25	0.9668	—	—	—
Uncharged .	2 27.25	2 21.25	—	0.9641	0.9593	+0.0048
Charged . .	2 36.00	2 30.00	0.9615	—	—	—
Uncharged .	2 30.00	2 23.00	—	0.9669	0.9533	+0.0136
Charged . .	3 0.40	2 55.40	0.9723	—	—	—
Uncharged .	2 39.25	2 33.75	—	0.9702	0.9655	+0.0047
Charged . .	2 44.75	2 39.50	0.9681	—	—	—
Uncharged .	2 42.75	2 36.75	—	0.9670	0.9631	+0.0039
Charged . .	2 34.00	2 28.75	0.9659	—	—	—
Uncharged .	2 11.25	2 6.00	—	0.9650	0.9600	+0.0050
Charged . .	1 58.50	1 54.25	0.9641	—	—	—
Uncharged .	1 45.90	2 36.75	—	0.9631	0.9595	+0.0036
Charged . .	2 5.75	2 1.00	0.9622	—	—	—
Uncharged .	2 5.75	2 0.50	—	0.9619	0.9583	+0.0036
Charged . .	2 23.75	2 18.25	0.9617	—	—	—
Mean					0.9587	+0.0040

Mean increase	0.42 per cent.
Temperature of air increased from	13° to 13°.5
Temperature of oil	not taken; light bad.
Mean spark length	3.6 cm.

SERIES V.

CASTOR OIL.

Condenser.	T_1 = sum of times of descent through highest and lowest quarters.	T_2 = time of descent through condenser.	T_2/T_1; condenser charged.	Mean of successive values of T_2/T_1; condenser charged.	T_2/T_1; condenser uncharged.	Excess of T_2/T_1; condenser charged.
	min. sec.	min. sec.				
Charged . .	3 7.75	2 58.75	0.9521	—	—	—
Uncharged .	2 19.00	2 12.50	—	0.9598	0.9532	+0.0066
Charged . .	3 20.00	3 13.50	0.9675	—	—	—
Uncharged .	2 36.00	2 30.00	—	0.9636	0.9616	+0.0020
Charged . .	4 1.75	3 52.00	0.9597	—	—	—
Uncharged .	2 31.75	2 25.50	—	0.9612	0.9588	+0.0024
Charged . .	2 21.00	2 15.75	0.9628	—	—	—
Uncharged .	3 20.75	3 11.00	—	0.9640	0.9515	+0.0125
Charged . .	2 30.75	2 25.20	0.9652	—	—	—
Uncharged .	2 39.75	2 33.25	—	0.9613	0.9593	+0.0020
Charged . .	3 8.25	3 0.25	0.9575	—	—	—
Mean					0.9568	+0.0051

Mean increase . . . . 0.53 per cent.
Mean spark length . . . . 3.2 cm.

In these results it should be noted how extremely good the agreement between the differences is when the mean ratios in the fifth column are changing steadily and not irregularly. Series III. and the latter half of IV. show this especially well. The ratio .9545 (column 4, Series IV.) is so very different from the other ratios in the same column that it must be due to a blunder in taking time. This will account for the only negative difference obtained.

Two other series of readings were made, but were marked "rejected" after calculating the ratios obtained with the condenser uncharged and before calculating those obtained with the condenser charged. This rejection was made on the ground that the ratios obtained with the condenser uncharged varied irregularly among themselves, and hence would be unlikely to give consistent indications. These were all the readings so made.

I then proceeded to inquire into other possible explanations of this apparent increase of viscosity.

(1) Temperature. It is well known that heat is produced by rapidly charging and discharging a condenser. Hence an increase of temperature of the liquid might be produced by fluctuations of the potential to which the condenser was charged. This increase would, however, be very slight, owing to the mass of liquid employed, and to the fact that the influence machine was worked pretty uniformly. Very accurate readings of the temperature of the liquid were not possible in the cellar where I worked, for these would bring the body close to the tank, and thus produce a local heating of the liquid. Rough readings were, however, made, and showed no uniform variations. Any changes of temperature produced in the liquid itself by variations in the dielectric stress acting on it may be rejected at once as a sufficient cause for the phenomenon, for they would produce a decrease of viscosity. Variations in the temperature of the liquid, due to heat produced in the glass, might conceivably produce the effect observed; for, owing to the low conducting power of the liquid, the maximum of the temperature increase produced in the center of the liquid by heat transmitted from the glass might be postponed until the time of descent of the next drop. A glance, however, at the preceding results will show that the time of descent varied so widely that such a fluctuation could not be of a sufficiently periodic nature to account for the results. There were, moreover, frequent short interruptions of work.

An examination of Stokes' formula will show that an alteration of the difference of density of mercury and castor oil due to heating would affect the time of descent; but a short calculation will show that the requisite temperature change would be enormous.

Moreover, the radius of the mercury drop is contained in the formula; but a sufficient alteration of it would require at least 25° change of temperature.

A thermal expansion of the middle halves of the glass plates with a consequent alteration of the apparent time of descent may also be dismissed as insufficient.

(2) The drops might be deflected horizontally by the force of the field. This seemed very improbable, but to test it a small millimeter scale was placed at the bottom of the tank and the distances of the landing places of the drops from the center of the tank noticed. The following were the results:

Plates charged	2½		2½		1½		2
Plates uncharged		2½		2		2	

Hence there was no appreciable horizontal deflection.

(3) The shaking of the table when the machine was being worked to charge the condenser, might possibly affect the velocity of the drops. (The shaking, however, was slight, and the table a heavy one.) Hence, a series of observations were taken with the machine in action all the time during the descent of the alternate drops. The results (all positive) showed that the effect still remained when this possible cause was eliminated. As a further precaution, the rates of the stop watches were compared and found unaffected by the action of the machine. These observations need not be quoted, as the matter is sufficiently settled by the results obtained later with heavy paraffin oil.

(4) The effect might be due to a retardation of the drop where it left the field of the condenser. This seemed also a very improbable explanation, but to guard against it the condenser was discharged before the drop reappeared, the time of reappearance being estimated from the time of descent through the upper quarter of the tank.

(5) The drops of mercury, being conductors, might be deformed by the action of the field. If they became prolate along the lines of force, the time of descent would probably be increased. As this seemed a not unlikely source of fallacy, a series of observations were made with small shot instead of mercury drops. Care was taken to get shot as nearly perfectly spherical as possible, the sphericity being tested by examination with a magnifying glass, and by observing the degree of smoothness with which the shot rolled on a glass plate. The following series of readings seems to dispose of this source of suspicion. It will be noticed

that it gives nearly the same percentage alteration of viscosity as did the mercury drops.

SERIES VI.

CASTOR OIL.

(*Tested by Descent of Small Shot.*)

Condenser.	T_1 = sum of times of descent through highest and lowest quarters.	T_2 = time of descent through condenser.	T_2/T_1; condenser charged.	Mean of successive values of T_2/T_1; condenser charged.	T_2/T_1; condenser uncharged.	Excess of T_2/T_1; condenser charged.
	min. sec.	min. sec.				
Charged. .	2 15.00	2 7.50	0.9444	—	—	—
Uncharged.	1 44.40	1 38.75	—	0.9461	0.9459	+0.0002
Charged. .	1 36.40	1 28.25	0.9479	—	—	—
Uncharged.	2 24.75	2 17.00	—	0.9476	0.9465	+0.0011
Charged. .	2 8.25	2 1.50	0.9474	—	—	—
Uncharged.	2 39.75	2 31.50	—	0.9509	0.9484	+0.0025
Charged. .	2 22.50	2 16.00	0.9544	—	—	—
Uncharged.	2 4.25	1 57.75	—	0.9526	0.9477	+0.0049
Charged. .	3 18.75	3 9.00	0.9509	—	—	—
Uncharged.	2 41.75	2 33.25	—	0.9492	0.9474	+0.0018
Charged. .	2 23.00	2 15.50	0.9476	—	—	—
Uncharged.	2 16.25	2 8.75	—	0.9520	0.9450	+0.0070
Charged. .	2 17.75	2 11.75	0.9564	—	—	—
Uncharged.	2 15.75	2 8.25	—	0.9523	0.9448	+0.0075
Charged. .	2 20.00	2 12.75	0.9482	—	—	—
Mean					0.9465	+0.0036

Mean increase	0.38 per cent.
Mean spark length	2.5 cm.
Temperature of oil increased from	9° to 9°.2
Temperature of air decreased from	10° " 8°.8

(6) The shot, being not perfectly spherical, might, when within the condenser, take an average "set" with the greatest diameter parallel to the lines of force. That this cause could be sufficient seems improbable, the shot being so nearly perfectly spherical. Moreover, it seems very improbable that this and the deformation of the mercury drops should produce so nearly the same percentage change of velocity. If such a "set" of the shot occurred

within the condenser some trace of this would be likely to persist during the remainder of the descent, since the shot ordinarily rotated only very slowly in descending. A lengthy series of readings of the times of descent through the highest and the lowest quarter of the tank failed to show any such effect. A conclusive test would be to replace the shot by spheres of non-conducting material of about the same S.I.C. as the dielectric. To this end many kinds of beads, pills, and seeds were tested, but none were found sufficiently spherical and of suitable density. Finally, some carefully selected imitation pearls (so-called "Roman" pearls), made of hard wax were tried. They gave the following series of results, which seems to finally settle the question.

SERIES VII.

CASTOR OIL.

(Tested by Descent of Imitation Pearls.)

Condenser.	T_1 = sum of times of descent through highest and lowest quarters.	T_2 = time of descent through condenser.	T_2/T_1; condenser uncharged.	Mean of successive values of T_2/T_1; condenser uncharged.	T_2/T_1; condenser charged.	Excess of T_2/T_1; condenser charged.
	min. sec.	min. sec.				
Uncharged.	2 59.00	2 51.75	0.9595	—	—	—
Charged. .	2 35.25	2 29.25	—	0.9577	0.9613	+0.0036
Uncharged.	2 10.25	2 4.50	0.9559	—	—	—
Charged. .	1 58.25	1 53.25	—	0.9554	0.9577	+0.0023
Uncharged.	3 14.00	3 5.25	0.9549	—	—	—
Charged. .	2 20.00	2 13.75	—	0.9547	0.9554	+0.0007
Uncharged.	3 1.25	2 53.00	0.9545	—	—	—
Charged. .	2 33.75	2 28.50	—	0.9606	0.9659	+0.0053
Uncharged.	2 0.50	1 56.25	0.9667	—	—	—
Charged. .	3 5.10	2 58.75	—	0.9621	0.9657	+0.0036
Uncharged.	2 13.25	2 7.75	0.9576	—	—	—
Charged. .	2 59.50	2 51.50	—	0.9549	0.9554	+0.0005
Uncharged.	2 37.75	2 29.75	0.9523	—	—	—
Mean					0.9559	0.0027

Mean increase	0.28 per cent.
Mean spark length	2.6 cm.
Temperature of air increasing from	$8^\circ.75$ to $9^\circ.0$
Temperature of oil increasing from	$8^\circ.65$ " $8^\circ.8$

These results are, as might be expected, less regular than preceding ones. Omitting from the column of differences the numbers which vary widely from the mean, the average percentage increase of viscosity is roughly the same as before. This agreement is a further argument against the existence of finite slipping between the liquid and the descending sphere.

From all the preceding we seem justified in concluding that the viscosity of castor oil is increased by about one half of one per cent when the oil is subjected to the electrostatic stress produced by a potential gradient of about 27,000 volts per centimeter or 90 E.S., C.G.S. units of potential per centimeter.

With reference to the preceding statement, two remarks may be made; first, that the electrostatic field was only steady to the

SERIES VIII.

HEAVY PARAFFIN OIL.

(*Tested by Descent of Mercury Drops.*)

Condenser.	**T_1 = sum of times of descent through highest and lowest quarters.**	**T_2 = time of descent through condenser.**	**T_2/T_1; condenser uncharged.**	**Mean of successive values of T_2/T_1; condenser uncharged.**	**T_2/T_1; condenser charged.**	**Excess of T_2/T_1; condenser charged.**
	min. sec.	min. sec.				
Uncharged .	3 26.6	3 27.8	1.0058	—	—	—
Charged . .	2 40.3	2 42.0	—	1.0069	1.0075	+0.0006
Uncharged .	3 17.2	3 18.8	1.0081	—	—	—
Charged . .	2 7.9	2 9.2	—	1.0087	1.0102	+0.0015
Uncharged .	3 45.6	3 47.7	1.0093	—	—	—
Charged . .	2 47.4	2 48.0	—	1.0084	1.0036	−0.0048
Uncharged .	2 14.0	2 15.0	1.0075	—	—	—
Charged . .	1 51.5	1 52.4	—	1.0062	1.0081	+0.0019
Uncharged .	2 41.8	2 42.6	1.0049	—	—	—
Charged . .	2 20.8	2 21.4	—	1.0067	1.0043	−0.0024
Uncharged .	2 56.8	2 57.8	1.0086	—	—	—
Charged . .	2 7.8	2 8.4	—	1.0056	1.0047	−0.0009
Uncharged .	2 30.4	2 30.8	1.0027	—	—	—
Mean					—	−0.0008

Mean spark length . . . . 3 cm.
Temperature of oil decreasing from . . . . $5^{\circ}.1$ to 4°

extent that the influence machine was worked fairly regularly; second, that the estimate of the field strength, being made from the mean sparking distance and the equation deduced by Chrystal

SERIES IX.

HEAVY PARAFFIN OIL.

(Tested by Descent of Mercury Drops.)

Condenser.	**T_1 = sum of times of descent through highest and lowest quarters.**	**T_2 = time of descent through condenser.**	**T_2/T_1; condenser uncharged.**	**Mean of successive values of T_2/T_1; condenser uncharged.**	**T_2/T_1; condenser charged.**	**Excess of T_2/T_1; condenser charged.**
	min. sec.	min. sec.				
Uncharged .	4 49.4	4 49.0	0.9986	—	—	—
Charged . .	5 4.1	5 4.3	—	0.9993	1.0007	+0.0014
Uncharged .	4 56.0	4 56.0	1.0000	—	—	—
Charged . .	4 33.0	4 32.2	—	0.9986	0.9971	−0.0015
Uncharged .	3 30.8	3 30.2	0.9972	—	—	—
Charged . .	4 14.0	4 14.4	—	1.0002	1.0016	+0.0014
Uncharged .	5 4.0	5 5.0	1.0033	—	—	—
Charged . .	3 37.6	3 37.8	—	1.0040	1.0009	−0.0031
Uncharged .	4 33.7	4 35.0	1.0047	—	—	—
Charged . .	4 0.8	4 0.8	—	1.0028	1.0000	−0.0028
Uncharged .	3 45.0	3 45.2	1.0009	—	—	—
Charged . .	4 30.8	4 31.0	—	1.0004	1.0007	+0.0003
Uncharged .	3 48.0	3 48.0	1.0000	—	—	—
Charged . .	3 25.8	3 25.6	—	1.0015	0.9990	−0.0025
Uncharged .	3 14.4	3 15.0	1.0031	—	—	—
Charged . .	4 32.4	4 32.2	—	0.9981	0.9993	+0.0012
Uncharged .	3 51.6	3 50.0	0.9931	—	—	—
Mean					—	−0.0007

Mean spark length	3.5 cm.
Temperature of air increasing from	7°.3 to 7°.50
Temperature of oil decreasing from	8°.2 " 7°.75

to represent Baille's results for the potential necessary to a given spark length, is only the roughest approximation.

It having now been shown pretty conclusively that at least one dielectric undergoes a change of viscosity under dielectric stress, it becomes an *a priori* probability that other dielectrics are similarly affected. Hence there is no longer any cause to doubt that the earlier experiments on glycerin (Series I. and II.) were reliable in their indications, and that glycerin also has an increased viscosity under electrostatic stress.

The only other liquid so far tested by the present method is heavy paraffin oil (specific gravity = .883). As will be seen by the series of readings VIII. and IX., the evidence as to this liquid is hardly conclusive. Mercury drops were used in this case.

From the above results it seems possible that heavy paraffin oil suffers a decrease of viscosity, the decrease being, however, much less than the increase in the preceding liquids, and much more difficult to determine. It should be noted that the failure to find an increase of viscosity in paraffin oil, or, still more, the discovery of a decrease is a strong evidence of the validity of the method employed in these experiments. The only detraction from this argument is that much smaller drops of mercury were used in the case of paraffin oil than in the case of castor oil. In all other particulars the method was the same.

The method described in this paper can only be applied to very viscous liquids. For testing less viscous liquids I have constructed two separate forms of apparatus, in which capillary tubes are used in such a way as to eliminate temperature effects. These have already been applied to testing several liquids, and to examining the effect of different intensities of dielectric stress and different degrees of unsteadiness or non-uniformity of field. These results will be given in another paper.

PHYSICAL LABORATORY, PURDUE UNIVERSITY,
March, 1896.

NOTE ON THE THEOREM OF CLAUSIUS.

By Edgar Buckingham.

The treatments commonly given of the expression $\int \frac{dQ}{T}$, and the proofs of the existence of a function η, the entropy, such that

$$d\eta = \frac{dQ}{T} \tag{1}$$

seem to me to be very incomplete. Even if we confine ourselves to the consideration of reversible processes, we have difficulty in finding any book where the subject is treated in such a way as to be clear to the ordinary student.

One small difficulty is that most writers only prove that

$$d\eta = \frac{dQ}{T}$$

when the work, w, done on the system by the outside forces is purely mechanical; in fact, when it is of the form

$$dw = -p\,dv, \tag{2}$$

though they afterward apply the theory to problems where electrical and other non-mechanical forms of energy enter. A greater fault is the failure to consider at all systems the state of which is defined by more than two independent variables. Such systems have, indeed, been considered, but work like that of Voigt[1] and Duhem,[2] who assume familiarity with the language at least of geometry of more than three dimensions, is not suitable for the use of most students of physics. If we are to use entropy, free energy, or the thermodynamic potential, we must, to satisfy ourselves, make sure that the entropy and its derivatives are definite functions before we go farther. I have felt, in teaching, the need of a simple but careful treatment of the subject, and as I have not found one, I offer the following remarks mainly in the hope that they may elicit something better on the same theme.

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[1] Kompendium der Theoretischen Physik, Vol. I., Part III., chap. I., § 2.

[2] Journal de Mathematiques, 8, p. 269, and 9, p. 293, 1892 and 1893.

§ 1.

Let our system have throughout the temperature T measured on Thomson's absolute scale. Let its state be completely determined by n independently variable parameters $(c_1 c_2 \cdots c_n)$ in addition to T. Let these parameters be so chosen that a variation of T alone involves no external work. This limitation of the generality of our treatment is practically unimportant, for we always do find it convenient to choose our parameters in this way. In any infinitesimal change involving external work we shall have

$$dw = \frac{\delta w}{\delta c_1} dc_1 + \frac{\delta w}{\delta c_2} dc_2 \cdots + \frac{\delta w}{\delta c_n} dc_n. \tag{3}$$

If we let

$$i_1 = \frac{\delta w}{\delta c_1}, \quad i_2 = \frac{\delta w}{\delta c_2}, \quad i_n = \frac{\delta w}{\delta c_n}, \tag{4}$$

whence (3) reduces to

$$dw = \sum_1^n i_k dc_k, \tag{5}$$

we may call i_k the *force* corresponding to the *parameter* c_k, or the *intensity* corresponding to the *capacity* c_k. The product $(i_k c_k)$ is obviously a quantity of the dimensions of energy.

If we assume, as is done, that it is impossible by any cyclic process or processes to obtain an infinite quantity of work from a system which at the end has exactly the same state as at the beginning, the internal energy, ϵ, is determined by the state of the system, or

$$\epsilon = f(T c_1 c_2 \cdots c_n) \tag{6}$$

so that $\frac{\delta \epsilon}{\delta T}, \frac{\delta \epsilon}{\delta c_1}, \frac{\delta \epsilon}{\delta c_2} \cdots \frac{\delta \epsilon}{\delta c_n}$ are definite functions of $T c_1 c_2 \cdots c_n$.

§ 2.

In the case of Carnot's engine $n = 1$, $c = v$, and $-i = p$, so that we have

$$dw = -pdv^{1}$$

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[1] We may take i and c in any way we please so long as $idc = -pdv$.

For this engine it is easily shown that for any reversible cycle

$$\int\frac{dQ}{T} = 0,$$

whence it at once follows that there exists an entropy function, such that

$$dQ = Td\eta.$$

Since we have the state of the system completely given by two variables, and since

$$p = f(T, v),$$

we may just as well consider p and v as the independent variables and let

$$T = \phi(p, v).$$

We can then represent any process by the movements of an indicator point on a plane diagram with coördinates p and v. An area on the diagram is of the form $\int pdv$ and represents a quantity of work.

§ 3.

Suppose, now, as a first step in advance, that our system is one which does work of *any* sort, but that its state may still be determined by one variable c in addition to the temperature. I have said work of *any* sort, but this must be limited to those forms of energy which are of the same *value* as mechanical energy; *i.e.* forms which are completely convertible, in both directions, with mechanical work.[1]

Among these forms are electrostatic and magnetostatic energy.[2] I shall exclude from consideration all forms of energy, except heat, which do not satisfy this condition. Such a system cannot be coupled directly to a Carnot engine. But if we make the connection by means of a mechanism which is capable of performing this complete mutual transformation of the two forms of energy, Carnot's theorem may be extended at once, and we may say that the efficiency of *any* reversible engine of which the condition is fixed by one variable in addition to the temperature is equal to the efficiency of Carnot's engine. Hence we have at once the extension of the theorem of Clausius to all such engines or systems, and for any reversible cycle

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[1] See F. Wald, Die Energie und ihre Entwertung, Leipzig, W. Englemann, 1889.

[2] The question of the applicability of Carnot's theorem to problems involving radiant energy, electrokinetic energy, etc., involves the preliminary investigation of the question whether these forms are completely convertible into mechanical energy, and *vice versa*.

$$\int \frac{dQ}{T} = 0.$$

Therefore we have

$$dQ = Td\eta,$$

and η is a definite quantity.

The graphic representation of changes in the state of such a system is analogous to that for Carnot's engine. We might use T and c as our coördinates. We might use any two variables, x and y, such that

$$\left.\begin{array}{l} x = f_1(T, c) \\ y = f_2(T, c) \end{array}\right\}, \tag{7}$$

for they fix completely the state of the system, because equations (7) involve the relations

$$\left.\begin{array}{l} T = \phi_1(x, y) \\ c = \phi_2(x, y) \end{array}\right\}, \tag{8}$$

and T and c determine the state of the system. But it is clearest to use as our coördinates c and i, in which case an area on the diagram represents a quantity of work. We may then, if we have sufficient data, draw isothermal and adiabatic lines just as we do on the pv diagram.

A simple example of such a system is a mass of liquid, the condition of which is defined by its temperature, T, and its surface area, S. Here the intensity, i, is evidently the surface tension, σ, and we have

$$\left.\begin{array}{l} c = S \\ i = \sigma \end{array}\right\}. \tag{9}$$

In this particular case the equation $i = f(T, c)$ reduces to $\sigma = f(T)$, since we know that σ is independent of S. The isothermal lines are horizontal if, as usual, we take our force or intensity along the vertical axis. To construct the adiabatic lines we must know the latent heat of isothermal extension, the heat capacity of the system, and $\frac{\delta\sigma}{\delta T}$.

Another somewhat similar case is that of the reversible galvanic cell. Here we may most simply take as our parameter, c, the quantity of electricity, q, which has passed through the cell since the time when it was in some standard state. The intensity, i, is then the electromotive force, E. Here again, if the cell is a constant one, $E = f(T)$, and the isothermals are horizontal. To draw the adiabatics we must know the heat of reaction, the heat capacity of the cell, and $\frac{\delta E}{\delta T}$.

§ 4.

Let us now pass on to the general case of a system the state of which is defined by the $(n+1)$ variables

$$Tc_1c_2 \cdots c_n.$$

In order to represent the state of the system and its changes by the position and motions of one indicator point we must let the point move in space of $(n+1)$ dimensions, so that as soon as $n > 2$ we are left stranded. But we can make the process somewhat more clear by a graphic representation consisting of n plane diagrams. In the equation for the work of the external forces,

$$dw = i_1 dc_1 + i_2 dc_2 + \cdots i_n dc_n, \tag{5}$$

where

$$i_1 = \frac{\delta w}{\delta c_1},\quad i_2 = \frac{\delta w}{\delta c_2} \cdots i_n = \frac{\delta w}{\delta c_n},$$

the forces, i, which keep the system in a state of equilibrium, must depend on the state of the system. There must therefore exist n relations of the form

$$\left.\begin{array}{l} i_1 = f_1(Tc_1c_2 \cdots c_n) \\ i_2 = f_2(Tc_1c_2 \cdots c_n) \\ \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\ i_n = f_n(Tc_1c_2 \cdots c_n) \end{array}\right\}, \tag{10}$$

and since these may be put in the form

$$\left.\begin{array}{l} T = \phi_1(i_1c_1c_2 \cdots c_n) \\ T = \phi_2(i_2c_1c_2 \cdots c_n) \\ \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\ T = \phi_n(i_nc_1c_2 \cdots c_n) \end{array}\right\} \tag{11}$$

we may regard T, which with the c's fixes the state of the system, as fixed by the c's and any one of the i's. Or in general we may take any $(n + 1)$ of the $(2n + 1)$ variables

$$Tc_1c_2 \cdots c_n i_1 i_2 \cdots i_n$$

as arbitrary and consider the others as functions of them.

Let us take the c's as n of the $(n + 1)$ independent variables and let us construct n plane diagrams, each with one of the c's as its abscissa and the corresponding i as its ordinate. Whenever the state of the system is modified in any way, some or all of the indicator points will move and trace curves on their diagrams. Whenever the system returns precisely to its original state, each of the indicator points will have traced a closed curve, the area of which represents the work done by that particular force during the cycle. Each of these diagrams will be analogous to the diagram for the case where $n = 1$. The n diagrams will, however, not be independent, since each i is a function of *all* the c's and of T or one of the remaining i's. However, the state of the system is completely defined by the positions of the indicator points, though of the $2n$ variables only $(n + 1)$ are independent.

We cannot draw any determinate set of isothermals or adiabatics on any diagram until we have stated how the n variables remaining independent shall vary. We may let the c's all vary proportionately; in that case the value of any one c determines the values of all the rest, and therefore, with T, its own i. Hence, for such a change we have a definite set of isothermals. In general, if we fix the mode of variation of the c's so that we have

$$\left.\begin{array}{l} c_1 = f_1(c_k) \\ c_2 = f_2(c_k) \\ \cdot \quad \cdot \quad \cdot \quad \cdot \\ c_n = f_n(c_k) \end{array}\right\}, \qquad (12)$$

any variation of c_k causes a definite motion of each of the indicator points. This amounts, of course, to reducing the number of independent variables from $(n + 1)$ to two; but in any case, as said above, the indicator points describe curves which for cyclic processes are closed, and which by their areas represent quantities of energy of the form $\int i dc$. For an adiabatic change the condition is

$$\frac{\delta\epsilon}{\delta T}dT + \sum_1^n \frac{\delta\epsilon}{\delta c_k}dc_k = \sum_1^n i_k dc_k, \tag{13}$$

or

$$\frac{\delta\epsilon}{\delta T}dT = \sum_1^n \left(i_k - \frac{\delta\epsilon}{\delta c_k}\right)dc_k, \tag{14}$$

where $\frac{\delta\epsilon}{\delta T}$ is the heat capacity, α_c, of the system for constant c's. Since $\frac{\delta\epsilon}{\delta T}$, i_k, and $\frac{\delta\epsilon}{\delta c_k}$ are all definite functions of T and the c's, this equation shows how T varies with the c's, and, therefore, how the i's vary. Hence it determines the adiabatic lines on all our diagrams for any given mode of variation of the c's. We see, then, that in any isothermal-adiabatic process each indicator point describes a curve composed of isothermal and adiabatic pieces.

§ 5.

As an illustration of a system of which the state is defined by more than one parameter beside T, we may take a mass of fluid confined in an enclosure of which the volume depends on the positions of several pistons moving in cylinders. To start with, we will neglect gravity, capillarity, and all other outside forces, except the forces on the pistons, and consider the pressure, P, to be uniform throughout the fluid. As our c's we may take the distances of the pistons from certain normal positions. The i_k corresponding to any c_k is the external force needed to keep that piston in equilibrium, or

$$i_k = A_k P,$$

where A_k is the area of the piston in question. The state of the system is determined by T and the c's. Or we may take the c's and any one of the i's, since any one of the i's determines P, and hence all the other i's, as well as T. The illustration as given is imperfect, for we cannot take more than one of the i's as an independent variable, because there is a necessary connection between the i's which makes them all functions of one. In other words, we do not in this case have the equations (10) as *independent* equations, and the i's do not determine the c's. But we may make our illustration complete by ceasing to disregard gravity. In that case, by reason of the hydrostatic pressure, each i becomes a function of all the c's and of T. We do, therefore, have the equations (10); the c's determine the i's; the i's determine the c's completely, and we may take any $(n + 1)$ of the $(2n + 1)$ variables

$$Tc_1c_2 \cdots c_n, \; i_1i_2 \cdots i_n$$

as independent.

§ 6.

If dQ_k is the heat taken in by the system during a change, dc_k, while T and all the other c's remain constant, we may write

$$dQ_k = d\epsilon_k - dw_k, \tag{15}$$

or

$$dQ_k = \frac{\delta\epsilon}{\delta c_k} dc_k - i_k dc_k. \tag{16}$$

If we let

$$d\eta_k = \frac{dQ_k}{T}, \tag{17}$$

we obtain from (16)

$$d\eta_k = \frac{1}{T}\left(\frac{\delta\epsilon}{\delta c_k} - i_k\right) dc_k, \tag{18}$$

and $d\eta_k$ is a definite quantity.

If the system undergoes *any* infinitesimal reversible modification we have

$$dQ = \alpha_c dT + \sum_1^n \left(\frac{\delta\epsilon}{\delta c_k} - i_k\right) dc_k, \tag{19}$$

and

$$\frac{dQ}{T} = \frac{\alpha_c}{T} dT + \frac{1}{T} \sum_1^n \left(\frac{\delta\epsilon}{\delta c_k} - i_k\right) dc_k. \tag{20}$$

If we let

$$\frac{dQ}{T} = d\eta, \tag{21}$$

we get from (20) and (18)

$$d\eta = \frac{\alpha_c}{T} dT + \sum_1^n d\eta_k. \tag{22}$$

Hence if there exists a function, η, of the variables which determine the state of the system, such that

$$d\eta = \frac{dQ}{T},$$

we may consider its variations as due to the variations of all those variables taken separately, and we may speak of the derivatives of η with regard to the separate variables, defining them by the equations

$$d\eta_k = \frac{\delta\eta}{\delta c_k} dc_k, \tag{23}$$

and

$$\frac{\alpha_c}{T} dT = \frac{\delta\eta}{\delta T} dT. \tag{24}$$

§ 7.

It remains to be shown that the function η, defined (except, of course, for an arbitrary constant) by (21), does exist. For this we have only to show that for any reversible cycle

$$\int \frac{dQ}{T} = 0.$$

We have shown already that for any cycle of the whole system each indicator point describes a closed curve. By substituting for the actual cycle one composed of an infinite number of isothermal and adiabatic elements, differing only infinitesimally from the actually performed cycle, we shall substitute for the curve drawn by each indicator point a broken line composed of isothermal and adiabatic elements, and differing in position from the original curve only infinitesimally. We can thus break up our cycle, as usual, into an infinite number of cycles, each of which is composed of two isothermal and two adiabatic changes, and each of which can be performed separately. Each of these infinitesimal cycles has its n representations on the n plane diagrams. Each cycle may be performed in connection with a Carnot engine working between the same temperatures, and we get by the ordinary reasoning,

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} = 0, \tag{25}$$

or

$$\int \frac{dQ}{T} = 0.$$

The cycle which we substituted for the actual cycle is equivalent to the sum of all these infinitesimal cycles. This is seen by referring to the separate diagrams, where we have all the lines, except those forming the outside boundary, traversed once in each direction, so that the quantities of work and heat involved vanish. We therefore have for the substituted cycle

$$\int \frac{dQ}{T} = 0.$$

For the actual cycle the curves on the n diagrams differ in area by only infinitesimal quantities from the curves for the substituted cycle. The two cycles differ only in respect to infinitesimal quantities of energy, since an area on any diagram means a quantity of energy. For each of the infinitesimal cycles formed by the contours of the actual and the substituted cycles we have $T =$ constant, $\int dQ = \frac{1}{\infty^2}$, and therefore

$$\int \frac{dQ}{T} = \frac{1}{\infty^2}. \tag{26}$$

For the whole number we have

$$\int\int \frac{dQ}{T} = \frac{1}{\infty}. \tag{27}$$

Therefore for the actual cycle, if we neglect the infinitesimal quantity (27), we have

$$\int \frac{dQ}{T} = 0.$$

It follows that there exists a function η, such that for all reversible modifications of the state of the system

$$d\eta = \frac{dQ}{T}.$$

We may therefore use the entropy and its derivatives

$$\frac{\delta\eta}{\delta T}, \frac{\delta\eta}{\delta c_1}, \frac{\delta\eta}{\delta c_2} \cdots \frac{\delta\eta}{\delta c_n},$$

as perfectly well defined quantities in the consideration of any reversible change in the state of a system which is given by any finite number $(n + 1)$ of variables

$$T c_1 c_2 \cdots c_n.$$

Note.— It is perhaps worth while to remark that though the mode of variation of the c's is not in general unimportant for the graphic representation, since we cannot actually construct any diagram for itself independently of the others, yet the change of η depends only on the total changes of T and the c's. For after fixing the mode of variation of the c's as the system passes from a state I. to a state II., we may make the modification in passing back from II. to I. in any way we please (so long, of course, as all the changes are reversible).

THE REFRACTIVE INDEX AND REFLECTING POWER OF WATER AND ALCOHOL FOR ELECTRICAL WAVES.[1]

BY A. D. COLE.

THE electromagnetic theory of light as originally given by Maxwell included only the optics of a single wave-length, and required extension to justify and harmonize the facts of selective absorption and dispersion. This extension was made by Helmholtz. According to him a change in the refractive index with the wave-length is to be expected throughout the complete spectrum; but beyond a certain wave-length these values approximate rapidly to a fixed limiting value, so that the index may be regarded as practically constant for longer waves. The period of this characteristic wave-length is theoretically of the same order of magnitude as that of the slowest possible vibration of the molecule, but it is practically unknown. Yet recent researches render it probable that this close approximation to a constant limiting value is exhibited by wave-lengths much smaller than all electrical waves thus far produced. H. Rubens has shown[2] that several substances which fail to satisfy the fundamental relation of Maxwell, $\mu^2 = K$ (where μ is the refractive index and K the dielectric constant of the substance), for light waves, do fulfil it for radiations in the infra-red only a little below the visible spectrum. For some other substances, —resin oil, olive oil, and certain kinds of glass, —Arons and Rubens[3] have shown that this relation is satisfied with electrical oscillations of 6 meters' wave-length, although not with light waves. E. Cohn[4] has further shown that even with water, whose refractive index for light differs so much from the square root of the dielectric constant, the index for long electrical waves conforms to the law of Maxwell. By entirely different methods Ellinger[1] and Udny Yule[2] have reached similar results. It seems probable that the great change in the value of the refractive index shown by these substances occurs in a part of the spectrum much nearer light-waves than the Hertz oscillations. Still, the researches of Grätz and Fomm[3] and of Drude[4] indicate a certain amount of dispersion even for long electrical waves. It seemed desirable, therefore, to investigate the refractive index of these substances, particularly of water and alcohol, to see if it is the same for very short electrical waves as for long ones.

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[1] Author's abstract from Wiedemann's Annalen der Physik und Chemie. Bd. 57, p. 290, 1896.

[2] Wied. Ann. 45, p. 238, 1892.

[3] Arons u. Rubens, Wied. Ann. 42, p. 581, 1891.

[4] E. Cohn, Wied. Ann. 45, p. 370, 1892.

It seemed first advisable to repeat the measurements of the index for long waves by the method of Cohn with modified apparatus, and to extend the method to alcohol. It will be unnecessary to give more than a very brief account of these experiments, since P. Drude has published the results of a similar research since our work was finished.

I. VELOCITY OF LONG ELECTRICAL WAVES IN WATER AND ALCOHOL BY THE METHOD OF E. COHN.

The apparatus used was very similar to that used by Cohn. An induction coil developed oscillations in two zinc plates 40 cm. square, with attached rods separated by a spark gap. Each of two smaller plates, in front of these, was attached to one end of a long horizontal wire. These wires were stretched parallel, 2.5 cm. apart. They extended several meters through air and then passed through rubber stoppers in the end of a glass trough 68 cm. long, and through the whole length of the trough. The inside of this glass trough was coated with pure silver by chemical deposition. At the inner trough end, and at least two positions outside, bridges of wire were placed across the parallel wires, the proper location being determined when a maximum effect was noticed in two little "Leyden jars" halfway between adjacent bridges, as shown by a Paalzow-Rubens dynamo-bolometer[5] in connection with a re-

[1] Ellinger, Wied. Ann. 46, p. 513, 1892.

[2] Udny Yule, Wied. Ann. 50, p. 742, 1893.

[3] Grätz u. Fomm, Wied. Ann. 54, p. 626, 1895.

[4] P. Drude, Wied. Ann. 54, p. 352, 1895.

[5] A. Paalzow u. H. Rubens, Wied. Ann. 37, p. 769, 1889.

flecting galvanometer. The "Leyden jars," as in the researches of Rubens,[1] consisted of short pieces of thick-walled glass tube threaded on the wires and surrounded by a single turn of fine wire as "outer coating." A second pair of "Leyden jars" was placed within the liquid in the trough, and served to show resonance when the wire bridges were brought into proper positions. These had threads of mercury as outer coatings, insulated by glass from the liquid in the trough. The research differed from that of Cohn in the following respects:

(1) Metallic containing vessels — one of zinc and one of silvered glass — were used. These were provided with metallic covers, as Rubens and Arons have shown that this is important with some substances. For water and alcohol, however, it seems to make little difference, as measurements with the glass trough unsilvered gave about the same result.

(2) The containing vessels were much smaller — from 3.5 to 10 liters capacity, instead of 50. This was made possible by having the parallel wires only 2.5 cm. apart, instead of 7 cm.

(3) The use of at least three bridges outside, and as many inside, gave opportunity to eliminate the influence of the trough end. Cohn assumed that the correction of the wave-length in air due to this cause was the same as for that in the liquid. But this was proved untrue experimentally for our apparatus, and so the values of the waves next the trough end were not used in the calculations.

(4) Four bridge positions were found within the trough of water instead of only two.

The results for distilled water are presented in tabular form. The letters λ_a and λ_w denote half-wave-lengths in air and water respectively, $\frac{\lambda_a}{\lambda_w}$ = ratio of velocities = μ, the refractive index.

The experiments with the silvered trough were much more carefully performed than the others, and the value 8.95 for 19° C. cannot be far wrong. P. Drude has obtained 8.7 for waves 60 cm. long, and Cohn and Zeemann, 8.93 to 8.99.

In similar measurements with commercial absolute alcohol more difficulty was experienced in getting good resonance and well-defined maxima. Only two bridge positions within the liquid could be obtained. The alcohol seemed to have higher absorption for waves of the length used than even quite impure water. Drude has noted the same difficulties. For waves whose half-length in air was 129.4 the refractive index 5.24 at 18° was obtained. Drude found a smaller number, 4.74.

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[1] H. Rubens, Wied. Ann. 41, p. 154, 1890.

Trough used.	λ_a.	λ_w.	$\mu = \frac{\lambda_a}{\lambda_w}$.	Temp. C.
Glass 68 × 7.5 × 7 cm. . . .	261.3	27.8	8.5	18°
Zinc 100 × 10 × 10	299.7	33.5	8.9	16–18°
Glass silvered 68 × 7.5 × 7 cm.	155.7	$\left.\begin{array}{l}17.1\\17.6\\17.4\end{array}\right\}$ 17.4	8.96	18–20°

II. REFRACTIVE INDEX FOR SHORT ELECTRICAL OSCILLATIONS.

The experiments just described demonstrate that the refractive index of water for waves of 300 cm. to 600 cm. is practically constant, and those of Drude carry the limit of constancy down to 60 cm. Our results for alcohol give a higher value for alcohol than his, as we have seen. This may be because we used waves four or five times as long as his. The difference, however, is too small, and the method too uncertain, to justify the inference that the index varies with the wave-length. Later experiments with the short-period oscillations first produced by Righi[1] form a surer basis for such a conclusion.

I first tried to measure the deviation of Righi waves 5 cm. long, produced by a hollow prism, with sides of thin plate glass 15 × 25 cm. and 10° refracting angle. Although measurements could be made in this way with resin oil, the method failed for water and alcohol. The loss of energy by absorption and reflection was so

[1] A. Righi, Rendiconte della R. Accad. de Lincei 2. 1 Sem (11), p. 505.

great that no sparks could be seen in a Righi resonator on the other side.

Next an attempt was made to produce a more sensitive receiving apparatus. Klemenčič[1] had used a thermopile resonator for long waves. This arrangement on a greatly reduced scale gave most excellent results in our work. In Fig. 1, *A* and *B* are bits of copper foil 10 mm. long by 1.5 mm. wide and 4 mm. apart. To each is soldered a fine wire 4 mm. long and bent to an L shape. One is of iron, 0.02 mm. in diameter, the other of German silver, 0.08 mm. in diameter. The wires cross in the middle and are attached to weak copper springs, which maintain a gentle pressure at the point of contact. These springs are connected to a low-resistance reflecting galvanometer. This resonator is mounted on a light wooden frame at the focus of a concave parabolic mirror made of cardboard, covered with three rows of tinfoil strips, 24 mm. long, placed parallel to the focal line of the mirror. As the thermo-element proved very sensitive to heat radiation, it and its mirror were permanently enclosed in a cardboard box, a material which experiment had shown was perfectly transparent to the electrical waves.

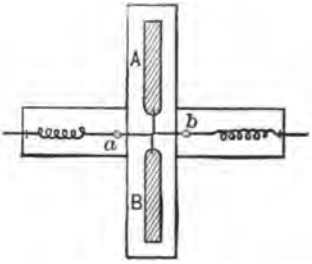

Fig. 1.

A Righi exciter 1 meter away produced galvanometer deflections of 100 to 200 scale divisions. Experiment proved, as theory requires, that deflections were proportional to the energy of the oscillations. Thus the deflection was four times as great when the resonator was 25 cm. distant from the exciter as when it was 50 cm. away. Thus quantitative results were obtainable.

But again the prism experiments failed. A deflection of 100 vanished completely when a prism of water or alcohol was introduced. A flat cell with a layer of water 11 mm. thick allowed only 2 per cent to 3 per cent of the energy to pass. Another with a water film only 0.4 mm. thick showed these results:

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[1] Klemenčič, Wied. Ann. 45, p. 62, 1891.

Empty cell, 65 per cent; filled with alcohol, 57 per cent; with water, 32 per cent.

As the reflection of 5 cm. waves on a film 0.4 mm. thick can be only very slight, even for a liquid with refractive index 9, the large energy loss must be due, directly or indirectly, to absorption.

This experience led to the abandonment of the prism experiments and an effort to measure the refractive index for electrical oscillations by an indirect method. The possession of a receiver capable of giving quantitative results suggested the use of Fresnel's formulas connecting reflected energy with refractive index. Their applicability involves the fulfillment of the condition that the square of the absorption coefficient of the substance be small compared to $(\mu - 1)^2$, where μ is the index, a condition which is certainly met by water and alcohol. Further, for water the independence of the reflecting power and the absorption coefficient was established by experiments in which the latter was made to vary through wide limits without affecting the former materially.

It was not convenient to compare directly the amounts of direct and reflected energy, but it was practically simpler to compare the reflecting power of the substance studied with that of a metal surface, and this amount with that received directly.

Reflecting Power of a Metal for Electrical Waves.

The exciter used by Righi for developing electrical oscillations of short period can be used only in a vertical position. This made another form necessary, and that in Fig. 2 was finally adopted. A and A' are glass tubes 5 cm. long. Their inner ends support two bits of brass wire 8 mm. long, with ends rounded. These glass tubes are held by corks in a wider T-tube, as shown, and allow the distance between the bits of brass to be varied. Two copper wires of 1 mm. diameter, their ends bent into a round loop, are held by the friction of little springs centrally in the glass tubes. The outer ends of these wires are joined to the terminals of an induction coil, whose discharge thus has three spark gaps. The T-tube is filled with heavy petroleum oil, so that the middle spark gap is in oil, the others in air. The tube may be turned about the axis *ab*. A bit of capillary tube *c* allows the gases formed by the decomposition of the oil to escape. For other positions the gases escape through *E*, which is an elbow, connected by a ring of rubber tubing with the third limb of the T-tube. The whole is mounted on a wooden support attached to a spherical concave mirror, so that the middle spark gap remains at the focal point of the mirror when the latter is inclined. The energy radiated was found to vary only slightly when the lengths of the air-gaps were changed, but was much influenced by the length of the oil-gap.

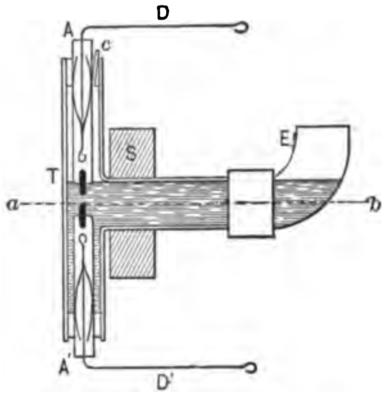

Fig. 2.

Figure 3 shows the arrangement employed to measure the reflecting power of a zinc plate. *C* is the spherical mirror, 50 cm. in diameter, at whose focus *E* the exciter was placed. At a distance of 25 cm. was a pin about which revolved a wooden arm 30 cm. long. At its end the receiving apparatus already described was placed. After four observations of the direct radiation along the axis of the mirror at *R*, the wooden arm was revolved 90° to *R'*, the movable zinc plate *M*, 40 cm. square, placed at an angle of 45° to the axis, and four readings of the reflected energy taken. A series comprised 24 to 32 readings, taken by fours in each position alternately. Three series were taken with the oscillations perpendicular to the incident plane, and eight for those parallel to it. The results were as follows:

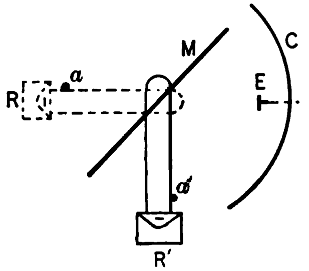

Fig. 3.

Oscillations perpendicular.	Oscillations parallel.
99.5	87.9
101.1	93.1
100.1	88.8
	93.2
	92.9
	95.9
	93.8
	93.5
Mean . 100.2	Mean . 92.4

Thus a metal surface at 45° reflected the radiation without loss if the oscillations were perpendicular to the incident plane, but with about 8 per cent loss if the oscillations were in the incident plane.

Reflection of Electrical Oscillations by Liquid Surfaces.

The axis of the exciter with its mirror was inclined 45° downward, and that of the receiver 45° in the opposite direction. At the intersection of these axes the surface of the liquid to be studied was placed, the liquid being contained in a flat developing tray, 44 × 36 cm. On account of the strong absorption of water and alcohol for these short waves, no interference with reflections from the bottom of the tray was to be feared. A screen of metal, 40 cm. square, was placed between exciter and receiver to cut off any direct radiation. Supports were provided to hold a plate of sheet zinc just above the liquid surface. Three observations of the energy reflected from the metal surface were made, then the metal removed, and an equal number made for the radiation reflected from the liquid. Twenty to thirty readings constituted a series. Five series for oscillations perpendicular, and four for those parallel to the incident plane, were made. An example shows the kind of results obtained. The numbers are galvanometer readings in scale divisions.

	Oscillations perpendicular.			Oscillations parallel.		
	Water surface.		Metal surface.	Water surface.		Metal surface.
	31.0		41.0	18.5		33.0
	31.0	→	41.0	18.5	→	32.0
	30.0		43.0	19.5		30.0
	31.0	↙	42.0	18.5	↙	32.5
	33.0	→	43.0	19.5	→	35.0
	30.0		45.0	21.0		31.0
	30.0	↙	40.0	17.5	↙	31.5
	31.0	→	43.0	18.0	→	32.0
	31.0		43.0	18.0		27.0
	29.0	↙	40.0	15.5	↙	
	33.0	→	43.0	19.0		
	30.0		43.0	17.5		
Mean	30.8		42.3	18.4		31.6

Whence

$$\text{reflecting power (perpendicular)} = \frac{30.8}{42.3} = 73 \text{ per cent,}$$

and

$$\text{reflecting power (parallel)} = \frac{18.4}{31.6} \times 0.924 = 53.7 \text{ per cent,}$$

0.924 being the proportion of the total radiation falling upon it that was reflected by the metal plate at 45°.

The results of the several series are collected in the following table: —

	Oscillations perpendicular.		Oscillations parallel.
	73.6 %		45.6 %
	73.9		53.7
	69.8		57.1
	73.0		54.6
	71.8		
Mean .	71.8 %	Mean .	52.7 %

These values were inserted in the two Fresnel formulas for rays polarized in the two planes,

$$(u_{\text{par}})^2 = \left(\frac{\sqrt{\mu^2 - \sin^2 i} - \cos i}{\sqrt{\mu^2 - \sin^2 i} + \cos i}\right)^2, \qquad (1)$$

and

$$(u_{\text{perp}})^2 = \left(\frac{\mu^2 \cos i - \sqrt{\mu^2 - \sin^2 i}}{\mu^2 \cos i + \sqrt{\mu^2 - \sin^2 i}}\right)^2, \qquad (2)$$

where i is the incident angle 45°, μ the refractive index, and u the amplitude of the oscillations. If the electrical oscillations are perpendicular to the plane of polarization,

$$R \text{ parallel} = (u_{\text{perp}})^2,$$

and

$$R \text{ perpendicular} = (u_{\text{par}})^2.$$

From equation (1) $\mu = 8.8$

(2) $\mu = 8.9$

Mean . 8.85

These results were obtained with distilled water, but the addition of conducting substances, even 50 cc. of H_2SO_4, did not change the amount of energy reflected, although the absorbing power must have been greatly increased. Hence the absorbing power of pure water cannot be great enough to render the Fresnel formulas inapplicable.

Thus practically the same value was found for the refractive index of water for 5 cm. waves as for those 150 cm. to 300 cc. in length. But this was not true of alcohol. Nine series of observations with it gave these results: —

Oscillations parallel.	Oscillations perpendicular.
39.7 %	13.6 %
35.3	17.5
44.3	14.5
42.0	16.2
41.4	
Mean . 40.5 %	Mean . 15.4 %

Inserting these values

Formula (1) gives $(u_{\text{par}})^2 = R_{\text{perp}} = 15.4$; $\mu = 3.25$

$(u_{\text{perp}})^2 = R_{\text{par}} = 40.5$; $\mu = 3.15$

Mean $\mu = 3.2$

Thus the two formulas give the same value for μ, but this value is much lower than that obtained for long waves. This suggests that alcohol possesses strong dispersion for radiation of the wave-lengths used. Apparatus was partially completed for studying oscillations of 1 cm. and of 8 cm. wave-length, but time was lacking to complete the research. We hope to be able to resume the work soon.

The results obtained were so unexpected that the method was tested by applying it to a liquid whose refractive index had been shown to be the same for long electrical waves as for light waves. Refined petroleum (kerosene oil) was taken. A deep vessel was used to avoid disturbing reflections from the bottom. The energy reflected at 45° for oscillations perpendicular was 9.2 per cent, corresponding to $\mu = 1.5$, a value which agrees well with those for light and for long electrical waves.

In conclusion we gather the principal results into one table.

Substance.	Reflecting power.		Refractive index.	
	Oscillations I.	Oscillations II.	Formula (1).	Formula (2).
Metal (zinc) . .	100.0 %	92.4 %	—	—
Water	71.8	52.7	8.8	8.9
Alcohol	40.5	15.4	3.2	3.1
Refined petroleum	9.2	—	1.5	—

Summarizing all the results, we find that

(1) For oscillations of 300 cm. to 600 cm. total wave-length the refractive index for water is 8.95, for alcohol 5.20.

(2) For oscillations of 5 cm. wave-length the two Fresnel formulas give the same value for the refractive index from observations of energy reflected at 45°. For water this value is 8.85, for alcohol 3.2.

(3) The refractive index of alcohol is decidedly greater for long than for short electrical oscillations.

I desire to express my thanks to Dr. Rubens, of the University of Berlin, for numerous suggestions and other help during the whole course of the investigation, and also to Professor Warburg, in whose laboratory the work was done, for his interest and advice.

MINOR CONTRIBUTIONS.

A NEW ELECTROLYTIC GENERATOR FOR OXYGEN AND HYDROGEN.

By W. S. Franklin.

DURING the year 1895 a preliminary test[1] of a generator, of novel design, for the rapid and efficient generation of oxygen and hydrogen by electrolysis has been carried out in the Physical Laboratory of Iowa Agricultural College. The earlier part of this test brought out, as was to be expected, a number of essential imperfections in the preliminary design, which was accordingly modified, and the apparatus twice reconstructed. The experimental work was unexpectedly cut short in October, 1895, before the finally constructed generator was fully tested. The final design, and the more important results of the preliminary tests, are here given. The work is to be continued during 1896.

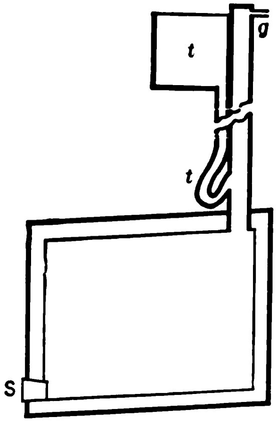

Fig. 1.

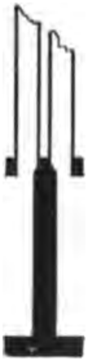

Fig. 2.

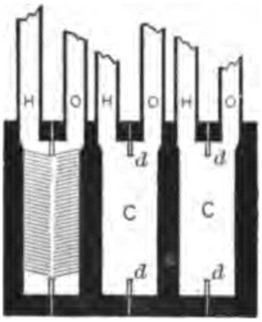

Fig. 3.

The generator consists of a number of massive frames cast of an alloy of 25 parts lead and 1 part antimony. The side view of one of these frames is shown in Fig. 1, and a sectional view in Fig. 2. These frames are bolted together, as shown in Fig. 3, the frames being separated by insulating diaphragms, *dd*, of gum rubber sheet of the kind used for steam packing. The central portions of these diaphragms are cut out as shown in Fig. 4. Two tall lead pipes and one reservoir and trap, *tt*, are fixed to each frame, as shown in Figs. 1 and 2. The water-tight compartments, *cc*, Fig. 3, filled with dilute sulphuric acid (Sp. Gr., 1.17), are broken up by a number of thin glass strips supported at the ends in hard rubber posts, Fig. 5. These glass strips, shown only in one compartment in Fig. 3, prevent the mixing of the two gases. The electric current enters the apparatus at one end-frame and leaves it at the other. The gases collect in the tall lead pipes, pass out through glass tubes attached at *g*, Fig. 1, and are conducted to lead pipes leading to the gas holders. An opening, *S*, Fig. 1 (two in each frame), closed with a rubber stopper, permits of the easy emptying and rinsing of the compartments. The generator is so supported that a vessel may be placed under these openings. The generator in the Physical Laboratory at Ames, now partly finished, is arranged in batteries of five cells each. Each battery weighs about three hundred pounds. The lead frames are 25.5 × 20.4 × 8 cm. outside dimensions, the metal is 1.8 cm. thick, and the other dimensions are as shown in the figures. The openings in the rubber diaphragms are 13 × 19 cm. Such a battery of five cells will stand a continued current of considerably more than 22.5 amperes, without overheating, requiring about 3.8 volts of E.M.F. per cell. The gases produced are not perceptibly mixed.[1]

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[1] By Messrs. C. R. Cave and D. M. Hosford. Thesis for 1895. MS. in the College Library, Ames, Iowa.

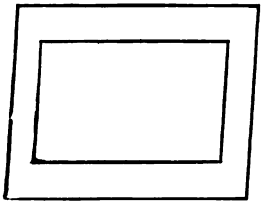
Fig. 4.

Fig. 5.

In the preliminary tests a battery of five cells of the above dimensions was used. *The holes in the diaphragms were only* 7.5 × 15.2 *cm., and no glass strips were used to prevent mixing of gases.*

With this battery the E.M.F. per cell, the final temperature rise above room temperature, and the purity of the gases were determined for currents ranging from 5.9 amperes to 22.5 amperes. The results are exhibited in the following table: —

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[1] This point, alone, has been tested with the finally constructed apparatus. The estimates as to heating and required E.M.F. are based upon the preliminary tests. See below.

Number of run.	Current (amperes).	E.M.F. per cell (volts).	Temp. rise (centigrade).	Purity of H (per cent).	Purity of O (per cent).
1	5.9	3.0	2°.5	91.6	75.0
2	7.6	3.0	2°.0	92.7	78.2
3	8.8	3.2	6°.5	93.9	83.0
4	11.2	3.4	13°.5	94.4	83.2
5	13.6	3.5	15°.5	94.8	85.0
6	17.3	3.6	16°.5	94.8	85.0
7	20.0	3.7	21°.8	94.8	85.5
8	22.5	3.8	24°.0	94.7	86.2

The diaphragms for the first battery of five cells which was set up were made of best quality hard rubber sheet, 1.5 mm. thick. After about a month these diaphragms were found to be very badly broken. The battery was then reconstructed, using gum rubber sheet 1.5 mm. thick, as mentioned above.

The efficiency of a hydrogen and oxygen generator requiring E volts per cell is calculated as follows: During the passage of one coulomb of electricity EQ joules of energy are expended, and .00001039 Q grams of hydrogen, and a corresponding quantity of oxygen are generated. The recombination of this H and O would give $34200 \times 4.2 \times .00001039 \times Q$ joules; 34200 gram-calories of heat being generated by the combustion of 1 gram of hydrogen in oxygen, and 1 gram-calorie being equal to 4.2 joules. The efficiency is therefore $\eta = 34200 \times 4.2 \times .00001039 \times Q + EQ$, or

$$\eta = \frac{1.45}{E}.$$

Therefore a cell used under such conditions as to require 3.8 volts has an efficiency of 38.2 per cent. The generator described[1] by E. L. Nichols and G. S. Moler required, under the conditions of their test, 19 volts per cell, the current through each cell being 24.6 amperes. The great efficiency of the generator here described is due to its low resistance; the anodes and kathodes have each 500 qcm. area, and the distance between them is 6 cm.

One great advantage of a low E.M.F. is that it is impossible for gas to be generated upon the detached sediment of PbO_2, which is gradually formed in the cell. The rapidity of formation of this sediment is, however, not great enough to seriously limit the life of the generator; for during the passage of 1240 ampere-hours through the generator, which was tested, the amount of PbO_2 formed was not great enough to be estimated. The test was continued at frequent intervals for more than a month, so that it seems that no serious corrosion of the frames could take place in less than fifteen or twenty years of use.

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[1] PHYSICAL REVIEW, Vol. I., p. 373.

At 15 cents per kilowatt-hour for electrical energy, 660 liters H and 330 liters O would cost about 80 cents, counting 4 volts per cell of generator. This is sufficient to run a strong lime light for three hours. Counting oxygen only as of value, its cost would be $2.40 per kiloliter (about 8 cents per cubic foot). It seems, therefore, with a generator efficiency of $1.45 \div 4 = 36$ per cent, that the electrolytic generation of H and O for laboratory purposes is not only the most convenient, but perhaps also the cheapest method of production.

AN APPARATUS FOR ILLUSTRATING THE LAWS OF FALLING BODIES.

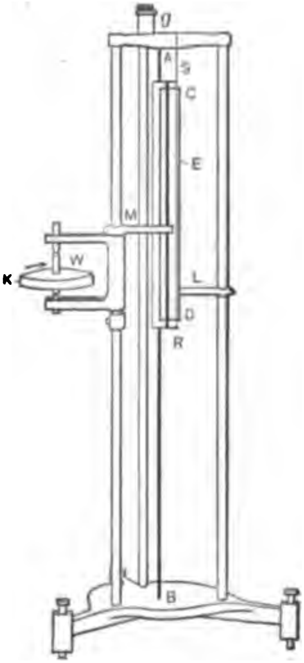
By H. M. Randall and W. A. Markey.

THE problem of uniformly accelerated motion as illustrated by falling bodies has always been a difficult one for laboratory experiment. The piece of apparatus here described was devised for the purpose of obtaining the necessary data from a practically freely falling body, and is simple both in idea and in manipulation.

The apparatus is shown in the accompanying figure and will be seen to possess considerable stability, the top and the base being of cast iron held rigidly together by three pieces of steel shafting. The falling body is quite heavy, and consists of three iron plates, but one shown in the figure, twenty inches in length, soldered to triangular end pieces. Through the centers of these triangular ends passes the guiding rod *AB*. One of the faces of the falling body is furnished with a wide rubber band *E*, which holds in position a strip of stiff paper *CD*. The body is held in suspension at about the height shown in figure by a thread *RS*. This thread is so tied to the rod *R*, which projects from the base of the falling body, that it passes over the forward surface of the paper very near its outer edge. The thread is furnished by the spool *G*, which can be clamped in any position by the thumb screw on its top.

A heavy brass wheel *W* is carefully mounted in an iron frame which turns about the shaft as an axis. The wheel is furnished with a knife edge made from a stout knitting needle. This knife *K* projects about a sixteenth of an inch beyond the circumference of the wheel. The rod *M* is so clamped to the shafting that it arrests the motion of the iron frame when turned toward the suspended body at such a point that the circumference of the wheel just escapes the outer edge of the paper strip. The knife is then adjusted so that if the wheel was rotated it would touch the paper just before it would cut the thread.

The wheel, drawn back from paper, is set in rapid rotation, in the direction indicated by the arrow, by a heavy cord wound around the upper part of its axle. It is then brought opposite the paper. The knife cuts the thread and at the same time makes a fine cut upon the forward surface of the paper, about an eighth of an inch long, so that the first line is made the instant the body begins to fall. With each succeeding rotation of the wheel the knife makes a fine cut on the edge of the paper. The distances between the cut lines are evidently the distances the body has fallen during the corresponding revolutions of the wheel or units of time. Since the rotation of the wheel, while the body is falling its own length, may be regarded as practically uniform, these units of time are considered equal. The number of records obtained on a strip of paper eighteen inches long varies from six to fifteen, according to the velocity of the wheel.



The body strikes on a pad on the tripod. The paper is taken from under the rubber band, and the distances between the lines are measured. As the knife edge is very sharp, the lines are fine and permit very accurate measurement. The rod *L* is so adjusted that it just touches the back side of the iron plate upon which the paper is fastened, thus preventing any rotation of the falling body that might be caused by the knife.

The apparatus is being used at present in the laboratory, and the results obtained are uniformly good. The following tables express the results obtained in two experiments, the unit of length being the thirty-second part of an inch.

Measured acceleration.	Computed acceleration.	Difference.	Measured S'.	Computed S'.	Difference.	Measured S.	Computed S.	Difference.
	17.54	+0.05	8.9	8.77	+0.13	8.9	8.77	+0.13
17.4	17.54	−0.15	26.2	26.31	−0.11	35.1	35.08	−0.02
17.5	17.54	−0.05	43.7	43.85	−0.5	78.8	78.93	−0.13
17.6	17.54	+0.05	61.3	61.39	−0.09	140.1	140.32	−0.22
17.6	17.54	+0.05	78.9	78.93	−0.03	219.0	219.25	−0.25
17.6	17.54	+0.05	96.5	96.47	+0.03	315.5	315.72	−0.22
17.7	17.54	+0.15	114.2	114.01	+0.09	429.7	427.73	−0.03
17.5	17.54	−0.05	131.7	131.55	+0.15	561.4	561.28	+0.12
18.0	17.76	+0.24	9.0	8.88	+0.12	9.0	8.88	+0.02
17.6	17.76	−0.16	26.6	26.64	−0.04	35.6	35.52	+0.08
17.7	17.76	−0.06	44.3	44.40	−0.1	79.97	79.92	+0.05
17.9	17.76	+0.14	62.2	62.16	+0.04	142.1	142.08	+0.02
18.0	17.76	+0.24	80.2	79.92	+0.28	222.3	222.0	+0.3
17.7	17.76	−0.06	97.9	97.88	+0.02	320.2	319.68	+0.52
17.6	17.76	−0.16	115.5	115.44	+0.06	435.7	435.12	+0.58
17.8	17.76	+0.04	133.3	133.2	+0.1	569.0	568.32	+0.68

The distances the body has fallen during the succeeding units of time are measured and placed in the column headed "Measured S'." Subtracting each distance from the succeeding distance, the values in the first column are obtained, which are the measured accelerations and should be uniform. The average of these accelerations is taken as the computed acceleration in the second column. The third column contains the difference between the measured and computed accelerations. The fifth column contains the distances the body should fall during the succeeding units of time, according to the formula $S' = \frac{1}{2}a(2t - 1)$, where a is the average acceleration. The sixth column contains the difference between the measured and the computed values. The seventh column contains the total distances the body has fallen in the corresponding units of time. The eighth shows the distances the body should have fallen according to the formula $S = \frac{1}{2}at^2$, where a is the average acceleration. The ninth column contains the difference between the measured and the computed values of the total spaces.

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NEW BOOKS.

A Treatise on Bessel Functions. By **Andrew Gray** and **G. B. Mathews**. London and New York, Macmillan & Co.

The purpose of this treatise is to supply such a treatment of the theory of the Bessel and allied functions as shall make them practically useful to physicists. The larger portion of the treatise is therefore devoted to illustrative physical problems. It is, however, very complete in the analytical treatment, and includes some of the latest researches on the theory of the Bessel functions. The authors have shown that almost all of this theory is useful in the solution of various physical problems. Some parts are, however, more instructive on the mathematical than on the physical side, notably the chapter on the complex theory.

The book is neatly printed, well written, and remarkably free from errors. I have noted a few errors of reference, however, and on page 8, line 2, $(2n + s)$ should be $(2n + 2s)$.

The first chapter is devoted to three physical problems in which the Bessel functions first presented themselves, viz., the small oscillations of a flexible chain suspended by one end, the flow of heat in a solid circular cylinder, and Bessel's original problem, the determination of the eccentric anomaly of a planet in terms of the time.

In the second chapter we find the solution of the differential equation that defines the Bessel functions, viz.,

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0.$$

If r be the *lowest* power of x in a series solution of this equation, it is shown that $r^2 = n^2$, so that there are only two such series, beginning respectively with x^n and x^{-n}, where n is positive. These series, with appropriate constant coefficients, are denoted by $J_n(x)$ and $J_{-n}(x)$ respectively, and differ from each other only by the sign of n. When n approaches an integral value, the second series approaches $(-)^n$ times the first series, so that another solution of the differential equation is required when n is integral. This is supplied by either of two functions of the form $J_n(x) \log x$ + an infinite series beginning with x^{-n}.

In following the demonstrations the reader will need to make liberal use of pencil and paper, since only the principal results, and the steps necessary to take to obtain them, are given by the authors. The book is on this account more valuable for reference, since it is not encumbered with a large number of intermediate equations. The reader will find, also, that in following a piece of work himself with pencil and paper, he will become more quickly familiar with the subject and its leading methods than if he were saved the labor of doing this by having every equation supplied in the text. One finds after reading in this way the second chapter of only ten pages, that he is prepared to enter at once upon the physical problems, only referring to the omitted eight chapters whenever necessary.

In order that the character of the work may be fully realized by physical students, we give outlines of some of the problems which it contains.

The first physical problem considered is that of the small vibrations of circular membranes. The equations of motion are first obtained with respect to rectangular axes, the axis of z being perpendicular to the plane of the fixed circular boundary. These equations are then transformed to cylindrical co-ordinates by the substitutions $x = r\cos\theta, y = r\sin\theta$. This is the usual transformation in the various physical problems in order to put them in form suitable for solutions in Bessel functions. The transformations are readily made in the case of u, $\frac{du}{dx}$, $\frac{du}{dy}$, etc., by considering that u is a function of r, $= \sqrt{x^2 + y^2}$, and $\theta = \tan^{-1}\frac{y}{x}$, so that

$$\frac{du}{dx} = \frac{du}{dr}\frac{dr}{dx} + \frac{du}{d\theta}\frac{d\theta}{dx} = \frac{du}{dr}\cos\theta - \frac{du}{d\theta}\frac{\sin\theta}{r}, \text{ etc.}$$

After the transformation to cylindrical co-ordinates, the normal types of vibration are sought by putting $z = v\cos n\theta\cos pt$, where v is a function of r only. The resulting equation shows that v is a Bessel function, which becomes, after introducing the boundary conditions, $v = A\cdot J_n(\kappa r)$. The constant κ is such that v vanishes at the boundary, or when $r = a$, the radius of the boundary. This equation for κ has an infinite number of roots for each integral value of n. Each integral value of n thus corresponds to a series of normal types of vibration in which there are 0, 1, 2, 3, . . . , concentric circular nodes within the circular boundary, and n diametric nodes that divide the circle into equal sectors. It is further shown how any possible type of vibration may be determined as a composition of normal types. Annular membranes bounded by concentric circles are similarly treated.

In the chapter on hydrodynamics several interesting problems are solved. The first is a case of steady motion of an incompressible fluid, bounded by two coaxial cylinders $r = a$, $r = b$, and two planes $z = h$, $z = -h$. The motion is such that each particle always moves in a plane through the axis, and possesses a molecular rotation that is inversely proportional to its distance from the axis. The stream lines are shown to be the intersections of the planes $\theta = \text{const.}$, with the surface $\psi = \text{const.}$, where ψ is Stoke's current function and is independent of θ. The value of ψ is found in terms of functions that have the same relation to Bessel functions that hyperbolic functions have to circular functions. These functions are considered in the chapter on the complex theory, since they are Bessel functions with imaginary arguments. They may be considered independently, however, as solutions of the Bessel differential equation, in which the coefficient $(x^2 - n^2)$ of y is changed to $-(x^2 + n^2)$.

The solution for ψ shows that the stream lines in any plane through the axis consist of the rectangular section of the boundary, corresponding to $\psi = 0$, and of a series of closed curves within the rectangle that are symmetrical with respect to the axis $z = 0$. If c be the maximum value of ψ within this rectangle, then $\psi = c$ is the equation of the circular core or filament of this interesting vortex ring.

The authors next consider the small oscillations of a cylindrical vortex rotating approximately as a rigid body, and a hollow cylindrical vortex having approximately an *irrotational* rotation about the axis ($r^2\theta = \text{const.}$), and also a combination of these two cases. The solution consists in determining the velocity systems, or the radii of the approximately circular but vibratory stream lines, and the varying pressures. Two problems of wave motion and the application of Bessel functions to the two-dimensional motion of a viscous liquid complete the chapter on hydrodynamics.

In the chapter on the steady flow of electricity, the following expression is obtained for the potential at the point (r, z) due to a circular disk of radius r_1, situated at the origin and perpendicular to the z axis on which there is a charge of electricity in equilibrium at potential c,

$$V = \frac{2c}{\pi}\int_0^\infty c^{\lambda[z]} \sin(\lambda r_1) J_0(\lambda r)\frac{d\lambda}{\lambda},$$

where $[z]$ denotes the positive value of z.

This result leads to the consideration of several important problems, such as the flow of electricity from the disk when immersed in an unlimited conducting medium or laid on the bounding surface of a conductor, or the resistance between such a disk source and sink when placed on a conducting surface at such distance apart as not to interfere with each other. Also the problem of the potential and resistance of an infinite conductor bounded by parallel plane faces, the disk source and sink being applied at opposite points of the face; also when this solid instead of being infinite is bounded by a circular cylinder with elements perpendicular to the plane faces, the disks being situated at the centers of the faces, and the whole surrounded by non-conducting matter. The problem of the resistance due to the passage of electricity from a large metal plate through a thin stratum of slightly conducting material to a second conductor of relatively smaller conductivity than the first, etc., is likewise considered.

In the chapter on the propagation of electro-magnetic waves along wires, the authors begin with the modified forms of the equations of the electro-magnetic field used by Hertz and Heaviside. These are that the variation $\left(4\pi k + \kappa\frac{\delta}{\delta t}\right)$ of the electric force equals double the rotation of the magnetic force, and that the variation $-\mu\frac{\delta}{\delta t}$ of the magnetic force equals double the rotation of the electric force, where k is the conductivity, κ the electric inductive capacity, and μ the magnetic inductive capacity of the medium. By the rotation of a force at any point is meant the molecular rotation in the hydrodynamic analogue in which force corresponds to velocity. Using dilatation in the same sense, it follows from the above relations that the dilatations of the electric and magnetic forces are each zero. It also follows that the electric and magnetic forces satisfy the same differential equation of propagation. In the case of propagation along a straight wire, the field is symmetrical round the wire at every instant, so that transforming to cylindrical co-ordinates x, ρ, θ, the variable θ disappears, and we have two components of electric force P, R along x, ρ, and one component of magnetic force H, perpendicular to x, ρ. The transformed differential equations of propagation give two equations, one satisfied by P, and the other satisfied by R, H.

The problem considered is essentially that of a cable, viz., a wire of finite radius surrounded by an envelope of insulating isotropic substance, and the whole surrounded by an infinite conducting material. For the insulating envelope $k = 0$, and taking the normal type of simply periodic forces the corresponding differential equations for P and R, H become Bessel equations of zero and first order respectively. In the conductors $\frac{\kappa}{k}$ is supposed small, and similar differential equations are obtained for those mediums. The general values of P in the three mediums being thus found, the values of R, H are found from differential equations connecting them with P, and the determination of constants from the boundary conditions completes the solution. This solution is fully completed for waves of slow vibrations in a cable of small radius. The case of rapid vibrations is also discussed, and the theory of Hertz's vibrator is considered.

The authors have introduced an unusually long chapter on the diffraction of light, with several diagrams and numerical computations, the object being, as stated in the preface, to attract more general attention to the valuable and interesting results contained in Lommel's memoirs. The

Bessel functions are applicable, of course, only to the case of symmetry round an axis, and the specific problem considered is that of the diffraction produced by a small circular opening on which light falls from a point-source that lies on a perpendicular erected at the centre of the opening. The quantity that it is desired to find is the intensity of the illumination at any point of a plane screen parallel to the plane of the opening, corresponding to any given wave-length of source.

Various miscellaneous applications are given in Chapter XV. This chapter might very well be read by the student before the other more difficult chapters on general subjects. A long list of examples for practice also follows this chapter.

The book closes with over 40 pages of tables of the numerical values of the Bessel functions, a short bibliography of three pages, and a page of the graphs of $J_0(x)$ and $J_1(x)$.

From the preceding outline of contents the reader will see that the treatise is one that cannot fail to be of great value to the student of physics. Physical investigations have now become so far advanced that it is necessary for the student who wishes to keep up with his times to be as familiar with several important functions of higher order as he is with the trigonometric and logarithmic functions, and the present work can be relied upon to furnish him with accurate knowledge of all the properties of Bessel functions that are suitable for their practical application. There is, in fact, no other book of this kind in the English language that gives this information in such a direct, compact, and well-arranged form, and with such a wealth of illustrative examples.

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Physikalische Krystallographie und Einleitung in die krystallographische Kenntniss der wichtigeren Substanzen. Von P. GROTH. Third edition, rewritten. pp. xvi + 783. Leipzig, W. Engelmann, 1895.

The first edition of this book appeared in 1876. The second edition appeared in 1885, and was considered at that time to be "the most satisfactory work of its kind in any language." It must be a great pleasure, in expressing one's estimate of such a book as this third edition, to be able to acknowledge an old debt of gratitude to the author.[1] The present writer finds the book simply admirable, and certainly as satisfactory at this time as the second edition was in 1885. It is distinctly a treatise on physical crystallography, and chiefly satisfactory in being such.

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[1] See the excellent review of this book of Groth's by Mr. L. Fletcher, F.R.S. Nature, January 30, 1896.

Professor Groth points out that crystal form, inasmuch as it is the result of crystal growth, is essentially a physical property, so that several extensive chapters are rightly devoted to the geometrical properties of crystals. It is in these chapters that the most striking changes from the second edition occur; changes which grow out of the unequivocal adoption of the conception of the *axis of symmetry.*

A very important feature of the work is the manner in which the theoretical portions, *e.g.* the theory of crystal structure and geometrical crystallography, are based upon experimental facts.

The chapter on the theory of the molecular structure of crystals is chiefly of speculative value, inasmuch as very few, if any, experimental facts have ever been foreshadowed thereby; and placed as it is between the chapters on the physical properties of crystals and the chapters on geometrical crystallography, this chapter certainly disturbs the simplicity of the development. The molecular theory can indeed be used as a basis for geometrical crystallography, as many of us know too well. Professor Groth does not so use it, and such use of it must be misleading and unsatisfactory from its necessarily uncertain experimental foundation.

This molecular theory has great value,[1] however, in its simpler aspects, in giving concrete representations of such experimental abstractions as the *law of crystal homogeneity*, the *law of constant angles*, the *law of rational indices*, and the *law of symmetry*. The theory should by all means be used in this connection, especially in a text for students who cannot be sufficiently familiar with real crystals to enable them to dispense with it. Indeed it must be admitted that such an abstraction as the law of rational indices, for example, cannot mean anything to any one except in so far as it is thought of in connection with a geometrical complex which has that property.

Geometrical crystallography has been unsatisfactory heretofore, — although often spoken of as one of the most complete of the sciences! — chiefly because its connection with physical crystallography has been too much ignored. The following is a brief statement of the physical foundation and of the physical signification of geometrical crystallography as one would get it by reading Groth with this point in view: —

(*a*) Every part of a normal crystal is exactly similar in its physical properties to every other part. (Law of homogeneity.)

Remark. — An abnormal crystal is a more or less fine-grained intergrowth of differently oriented crystals.

(*b*) The physical behavior of a crystal is different in different directions. (Law of ælotropy.)

Remark. — A crystal is a body which is homogeneous and ælotropic.

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[1] See p. 145, Vol. I., Text-Book of Physics, Nichols and Franklin.

The external form of a crystal is merely an indicator of its growth. (See *c*.)

(*c*) Certain directions in a crystal are physically similar.

Remark. — The number of distinct directions in a given crystal which show similarity, depends upon the physical property which is made use of in testing it. Professor Groth gives a good *résumé* of the physical properties of crystals in this connection (pp. 242–243). In the classification of crystals it is, therefore, important to make use of those physical properties which show always the least number of similar directions in a crystal. Crystal growth and crystal disaggregation, under the action of a solvent or by vaporization, show in every case the least number of similar directions in a crystal. Therefore, two directions in a crystal are admittedly similar when the crystal grows or dissolves similarly in those directions. In some cases the growth of a crystal (indicated, of course, by its form) is similar in two directions, which are shown to be different by dissolution, or *etching*, as it is technically called. Therefore, etching is the ultimate criterion in determining the symmetry (see *h*) of a crystal.

(*d*) During slow growth a crystal becomes a polyhedron, a solid bounded by plane[1] faces. For example, if the faces of a given crystal be obliterated by cutting, they will reappear in their original positions if the defaced crystal is placed in its mother liquor and allowed to grow.

(*e*) The relative amount of deposition of material upon the different faces of a growing crystal is largely determined by *external* circumstances, and cannot, therefore, be taken as an indicator of the physical similarity of the crystal in different directions. *The angles between the various faces of a crystal* (or between a set of similar directions in the crystal) *are, at a given temperature and pressure, invariable.* (Law of constant angles.)

(*f*) *Similar faces of a crystal.* The faces of a given crystal which are similar polygons (polygons of equal angles) are similar. A set of similar faces is called a crystal form. The minimum number of faces in such a set is one, the maximum number forty-eight (a result of *g* and *h*). This criterion of similar faces sometimes fails, inasmuch as etch figures on two faces of such a set are sometimes different. Similar faces may therefore be defined as faces which dissolve similarly.

Remark. — Those directions in a crystal which are similarly situated with respect to similar faces, are similar with respect to every physical property. It is possible to derive, from this fact, a statement of the law of rational indices (*g*) referring to *sets of similar directions* in a crystal.

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[1] Some crystals have curved faces. The conception of similar faces applies to such crystals and also the law of rational indices, for this law relates strictly to the directions in a crystal which are physically similar, and cleavage and all physical properties of such crystals are in definite directions.

(*g*) Choose three edges of a crystal as axes of reference. Consider the three intercepts, a, b, and c, on these axes, of a plane parallel to a given face of the crystal. The two ratios $\frac{b}{a}$ and $\frac{c}{a}$, associated with the axes b and c respectively, are called the indices of the given face. *The respective indices of all faces of a crystal are integral multiples of two parameters.* (Law of rational indices.)

(*h*) Consider a set of similar directions in a crystal. Imagine the crystal to be moved in such a way that this set of directions comes into congruence with its initial position. Such a displacement is called a congruence displacement.

Any translatory displacement is a congruence displacement, from (*a*).

In some crystals a rotation less than 2π about an axis may be a congruence displacement. Such an axis is called an *axis of symmetry.* A screw displacement need not be considered. In the molecular theory of crystal structure, certain translations only are congruence displacements, in which case screw displacements must be considered as involving something not included in independent rotary and translatory displacements.

A set of similar directions in a crystal may in some cases be brought into congruence with their initial position by the ideal process of inversion with respect to a plane, an operation which is exemplified in the formation of an image of an object in a plane mirror. Such a plane is called a *plane of symmetry.*

This, (*a*) to (*h*), is the experimental basis of geometrical crystallography.[1] The elements of crystal symmetry are the *axis of symmetry* and the *plane of symmetry.* Certain combinations only, of these elements, are consistent with the law of rational indices, and all possible combinations may be rigorously derived from that law. This derivation is perhaps not suited to a text of the character and scope of this of Professor Groth's; at least, in his discussion of the thirty-two combinations of axes and planes of symmetry, including the case where neither occur, he does little more than prove the possibility of the combinations without showing that no other combinations are possible.

It is indeed a question whether planes of symmetry do exist.

Any crystal which has at least two axes of symmetry not in the same direction must show symmetry to three mutually perpendicular planes in its thermal and electric conduction, elastic and optic properties (except optic activity), electric and magnetic polarization, etc. Such physical properties from their nonpolar character always lead to the notion of symmetry to planes when there is any symmetry at all.

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[1] The claim of Mr. William Barlow, Nature, Nov 15, 1894, that the conception of Homogeneity pure and simple leads to all the varieties of symmetry presented by crystals, is, in the opinion of the writer, not justifiable.

On the other hand, absence of symmetry to planes is definitely indicated by such physical properties as pyroelectric, piezoelectric, and optic activity, by asymmetric growth, and by asymmetric dissolution or etching. Some crystals, however, show absence of symmetry to planes by etching, but do not show optic activity. Other crystals show absence of symmetry to planes by etching, but to all appearances exhibit symmetry of growth. Whether this is true also for pyroelectric and piezoelectric activities the writer cannot say; but pyroelectric and piezoelectric effects are very difficult to observe, and a failure to detect them would in no case be very significant. Therefore the ultimate criterion of symmetry to planes is by etching, and this test, although in many cases unequivocal, requires in some cases the closest scrutiny to use it with certainty, and in many cases it gives no result at all, and then we have symmetry to planes.

In his development of the theory[1] of the molecular structure of crystals, Sohncke scarcely mentions symmetry to planes. Indeed, point-systems or cell-systems in space can have symmetry to planes only when the elementary points or cells possess that kind of symmetry themselves, and point-systems which are not symmetrical can be built up of such elements. Asymmetric molecules may be grouped in large nuclei (crystal molecules) which are approximately symmetrical, and in the detachment of such approximately symmetrical crystal molecules one would not expect a solvent to produce noticeably asymmetric etch-figures.

From these facts, together with the physical impossibility, not to say utter lack of meaning, of the operation of *inversion*, the writer is inclined to look upon the conception of the plane of symmetry as uncertain, to say the least. Crystal forms which are thought to show symmetry to planes are easily thought of as combination forms. However, the question whether we are to accept planes of symmetry is not very important, because what we know of crystallography at present cannot decide it. The plane of symmetry is a permissible notion and is very convenient.

In regard to Professor Groth's choice of the *composite axis and plane* of symmetry instead of the *center of symmetry*, of which Mr. L. Fletcher (*loc. cit.*) does not see the justification, it may be said that Professor Groth's conception includes the two distinct cases: the composite double-axis and plane, and the composite quadruple-axis and plane, Professor Groth's second and sixth classes of crystal forms respectively; while the conception of the center of symmetry leaves the latter case unprovided for.

Crystallography is certainly growing in importance to the chemist — many chemists would be surprised to know that the theory of the molecular structure of crystals is in almost complete harmony with *all known physical properties of crystals*, and that it includes the whole of stereo-chemistry as a special case — and no serious student of theoretical chemistry should remain without an intimate knowledge of the subject. It is to be hoped that Professor Groth's promised work on chemical crystallography will soon be in our hands.

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[1] Theorie der Krystallstructur, Dr. Leonhard Sohncke, Leipzig, 1879.

During the careful perusal of Groth's book, and of others bearing upon the subject, the writer's thoughts have, perhaps naturally, reverted often to the recent writings on Energetics; in particular to the writings of the great thinker, Ostwald, which have, indeed, a fine *metaphysical* quality. It is with no little pains that one turns from the easy and enticing sweep of such writing to the turgid hardness of such a work as this of Groth's, which is sternly *physical.* Most of us have, indeed, read the former, and many of us await compulsion in taking up the latter. The quality of the advancement of learning is strained! Such easy writings, indeed, " can have only the result: to excite those young in science to dilletantish speculation instead of to deeper study of the masterpieces." We cannot do without geometry yet awhile. Mechanics is geometry! Our theories of light, sound, electricity, and magnetism are geometry! The molecular theory, even in its simplest aspects, is geometry! However, it has been said of the older geometry that " Es ist nur eine ungerechtfertigte Gewohnung Euklidische Geometrie für streng wahr zu halten," and the wisdom of this remark applies to the newer geometries also.

W. S. FRANKLIN.

AMES, IOWA, March, 1896.

Grundriss der Electrochemie. By HANS JAHN. Royal 8vo, pp. 312. Vienna, Alfred Hölder, 1895.

The material contained in this book was presented in a course of lectures at the University of Berlin during the winter semester of 1894 and 1895. It was the reviewer's privilege to hear this lecture course, and to gain inspiration from Dr. Jahn's clear and enthusiastic presentation of the subject. The book, too, is not disappointing in that respect. There is a certain vivacity that adds much to the interest of the reader. The presentation is usually clear, but in a few cases one is at a loss to know what the significance of the symbols is until one has read a half-page farther. Such things occasion much loss of time. A writer should scarcely use *one* symbol in a single topic to express *three* quantities, even if those quantities are such simple expressions as "constant," "concentration," and "temperature."

A knowledge of the differential and integral calculus, and some knowledge of thermodynamics, is essential in order to understand the presentation of the subject. One would also labor through the treatise with difficulty had one no previous knowledge of the subject.

The author may have intended to provide against this latter difficulty, as the first chapter upon "The Foundation Laws of Electrochemistry" is treated in a very elementary manner. The Kohlrausch law, together with the independent wandering of the ions and their absolute velocity, are about the only points in this chapter which would suggest that the reader would soon be introduced to the Arrhenius theory of electrolytic dissociation. This theory is the subject of the second chapter — the longest one in the volume. With this chapter the book proper may be considered to begin, and from this point on one could reasonably call the book a treatise upon electrochemistry based upon thermodynamics. Planck's formula for equilibrium is developed from the second law of thermodynamics. Indeed, all the theories of the new electrochemistry are developed from the principles of energy and entropy. This seems to be a most desirable basis, as the objection which is most frequently raised is that the whole theory is contrary to the doctrine of the conservation of energy. This objection generally comes from persons who are not familiar with the "sharp weapons of mathematical analysis." It suggests the necessity of their acquiring such familiarity, if they would not, as the author says in his preface, "assume the danger of losing all understanding of the developments of theoretical chemistry." The fact, too, that chemists, and not physicists, have generally raised the above objection, confirms the author's statement.

In Chapter II. the tension of the saturated vapor of a solvent in contact with a solution, lowering of the freezing-point of the solvent, osmotic pressure, Ostwald's law of dilution, heat of dissociation, influence of pressure upon conductivity, and the Nernst theory of diffusion are some of the subjects which are given a thermodynamic treatment. Finally we are given Planck's demonstration of current conduction in electrolytes as a mechanical process.

The next chapter treats of the transformation of energy by electrochemical processes. In discussing the mechanical equivalent of heat it is surprising that the determinations made by Rowland, and some of the other more recent and careful determinations, are not mentioned.

The theory of Kelvin and Helmholtz that the chemical and electrical energy of a galvanic cell are equal, is shown to be wrong. That the electrical energy may be greater or less than the chemical energy, according as the temperature coefficient of the cell is positive or negative, is shown by the later Helmholtz equation, an equation developed from the principle of entropy.

The Helmholtz theory of free (*i.e.* available) energy is discussed, and upon it are based some conclusions of great interest concerning the variations of the electromotive force with pressure, and Helmholtz' theory of the variation of current with varying concentration. While this enables us to understand that a difference of potential exists between different concentrations of the same salt, it teaches us nothing concerning the mechanics of the subject. The theory of Nernst — one of the greatest fruits of the dissociation theory — has given a more satisfactory solution of this question by assuming that the Helmholtz concentration current arises from a difference of ionic concentration and a difference of osmotic pressure in the two solutions. Planck first discussed concentration series as a process of diffusion, and showed from the dissociation theory that the energy appearing as an electromotive force is due to diffusion alone. This theory of Planck is presented, and shown to agree with the results of the above methods.

Air and vacuum electrodes are discussed and the theory of Warburg presented. In a series of the form

$$\mathrm{Hg} \mid \underset{\text{In air}}{H_2SO_4 + XH_2O} \mid \underset{\text{In vacuum}}{H_2SO_4 + XH_2O} \mid \mathrm{Hg},$$

the oxygen of the air oxidizes the mercury, which in turn is acted upon by the sulphuric acid. This would bring mercury ions into the solution. The Warburg theory demands that mercury ions must pass into solution at the other electrode, but in much smaller quantity. We may say that all substances are at least sparingly soluble. The theory finds extensive application in the use of platinum electrodes. It explains, too, the capillary electric phenomena which the Helmholtz theory of opposite charges upon the two contact strata left in a rather unsatisfactory state.

The Daniell cell is taken as a type of cells reversible as to the kathion.

The electromotive force is localized at the electrodes as an osmotic process, and at the contact of the electrolytes as a process of diffusion. The author speaks of series reversible as to the anion, and of series reversible as to a kathion and an anion. Following Goodwin, the author attempts to prove that the latter class is essentially reversible in respect to the kathion.

The Lippmann electrometer and the drop electrode method of measuring single potential differences are discussed at some length. Some sources of error of different observers are pointed out. The work and precautions of Paschen are mentioned. This experimenter succeeded in locating forty-five per cent of the electromotive force of a Daniell cell at the contact of the two solutions, which is entirely contrary to the theory of Nernst. One can scarcely help forming the opinion that the drop electrode method of measuring single potential is very uncertain, and that much is left to be learned concerning the action of such electrodes.

Chapter IV. treats of polarization. The variation of free (*i.e.* available) energy at the kathode is discussed, and the conclusion drawn, "that the kathode polarization is dependent only upon the chemical nature of the separated metal, and the concentration of the solution; and that it reaches a limit equal to the potential difference, demanded by the theory of Nernst, between metal and solution." At the anode the process becomes more complicated; but in the case of strong acids, for which the concentration of the hydrogen ion is nearly constant, the anode polarization is independent of the chemical nature of the ion. After the kathode polarization has reached its limit, the increase of total polarization is to be attributed to an increase in the anode polarization. This takes place rather rapidly until the solution at the anode is saturated with oxygen, when the development of gas begins. From this point the polarization increases very slowly, and there is a more rapid increase of current strength. Polarization in dilute acids, alkaline solutions, and bases, as well as variation of polarization with a change in temperature, are discussed at some length. It follows from the formula, since the concentration of the ions does not enter, that all oxygen acids must have the same polarization.

Only two pages are devoted to accumulators. The author's statement that their action is complicated, and not completely explained, will meet with general approval.

Chapter V., upon the decomposition of some of the most important chemical combinations by means of an electric current, and some applications of electrolysis, concludes the book.

Numerous and valuable tables of data are scattered throughout the book. A small index would have made these tables more accessible for reference. Had such a table been provided, and had numerous references to the original literature been made, this volume would have been very complete.

B. E. MOORE.

The Theory of Light. By THOMAS PRESTON. Second edition. London, Macmillan & Co., 1895.

The present edition of this well-known work differs in general but little from the previous one. Considerably more space has been devoted to the discussion of the rectilinear propagation of light as well as to the explanation of graphical methods. The vibration spiral is considered earlier in the work, although its application is hardly evident until the subject of diffraction is reached.

In the treatment of reflection and refraction at spherical surfaces, the idea of "impressed curvature" has been introduced. While this method is extremely convenient in the development of approximate formulæ, yet the degree of approximation involved is apt to be lost sight of unless the limitation to small apertures is kept constantly in mind.

The discussion of diffraction by graphic methods has been considerably improved. The diagrams showing the methods of strip divisions will doubtless aid the reader in the formation of a clear idea of the meaning of the spirals employed, as well as in better understanding the physical conceptions which underlie the rather complicated analytical expressions usually encountered in that subject. The treatment of the diffraction grating is hardly as complete as might be expected, being limited to rather more perfect gratings than are likely to be produced.

Some additions have been made to the chapter on rotatory polarization, but the greater part of the new matter will be found in connection with the determination of the velocity of light and the subject of relative motion of matter and ether, thus making this portion of the work as complete as one could wish.

The book is, on the whole, quite free from errors or misprints. There are, however, a few, as, for example, on page 93, where the angles are rather confusing.

While the reader will doubtless share the regret of the author that time has not permitted him to "bring all parts of the work up to the standard demanded by the present state of the science" (especially along the lines of the electromagnetic theory), yet it may be expected that the new edition will continue to receive the very favorable consideration extended to the first. It is to be regretted that the author did not append a bibliography of the subject for the convenience of those who may have access to original memoirs only briefly mentioned in the text.

J. S. SHEARER.

Synopsis of Current Electrical Literature for 1895. Compiled by MAX OSTERBERG. pp. 143. New York, D. Van Nostrand Co.

This synopsis is compiled chiefly from the technical journals and magazines for the year, being practically a revision of the synopsis published monthly in *Electric Power.* The synopsis falls midway between an index and a digest, brief mention of the subject-matter of each paper referred to being made, so that the reader may know whether or not to consult the original article.

Volume IV. *September–October, 1896.* *Number 2.*

THE

PHYSICAL REVIEW.

THE VELOCITY OF ELECTRIC WAVES.

BY CLARENCE A. SAUNDERS.

THE velocity of propagation of the front of electric disturbances along metallic conductors is, according to present acceptance, equal to the velocity of light. This was indicated by the old theories; it is a direct conclusion from Maxwell's theory, and on the supposition that k, in the expression for the electrodynamic potential of two current elements upon each other, is equal to unity, is derivable from the equations of Helmholtz.[1] Nearly all of the early experiments made to determine "the velocity of electricity," so called, dealt with the phenomena of current diffusion. The trail left by the electric perturbation in its propagation greatly affected the results, so that they were widely divergent. A great impetus was given to the subject by the publication of Maxwell's electro-magnetic theory, as the demonstration of the equality of light-wave and electric-wave velocities seemed a strong verification of that theory.

The methods that have been employed for the determination of the velocity of electric perturbations may be divided into two classes.

(1) Direct methods, where the time taken by the disturbance to traverse a certain distance is observed, or where the two factors λ and T in the equation $\lambda = vT$ are directly measured.

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[1] Helmholtz, Wiss. Abh., Vol. I., pp. 539–544; Blondin, La Lumière Électrique, Sept. 29, 1894; No. 3, p. 121.

(2) Indirect methods, where the velocity depends in any way on a factor that is not observed, but calculated.

With only one exception all the methods employing standing waves on wires have belonged to the latter class, for although the wave-length has been experimentally determined, the period has been calculated from Lord Kelvin's well-known formula, $T = 2\pi\sqrt{LK}$. The values of L and K inserted in this formula have usually been those obtained under ordinary conditions, and not under the actual conditions of the experiment.

The following investigation, undertaken at the suggestion of Professor Arthur G. Webster of Clark University, is also based on the phenomena of resonance. It is, however, a direct method, since both λ and T are experimentally determined. Professor Trowbridge and Mr. Duane[1] have recently published an article on the same subject, in which the same principle is employed. The present piece of work had been commenced, however, some time before, and was carried on without any knowledge of their being occupied in a similar line.

Historical.

In the course of the last fifty years numerous attempts have been made to directly determine the velocity of propagation of electric disturbances. Only a few such attempts, however, require to be mentioned here.

In most cases telegraph wires were used, and as the phenomena dealt with were those of diffusion, the high permeability and specific resistance of the iron exercised a considerable retarding influence. In several instances, as, for example, in the case of submarine cables, the circuit possessed very great capacity. In such cases the velocity, being a function of the capacity, had no fixed value. For the detection of current, instruments with magnets were frequently employed, but the time required for these magnets to act could not be properly eliminated. The same objection holds against the use of galvanometers. Owing to the lack of sensitiveness of the instruments, the arrival of the maximum, or of a neighboring part of the wave of diffusion, and *not* of its front, was indicated.

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[1] American Journal of Science, XLIX., p. 297. April, 1895.

The earliest attempt to measure the speed of propagation was that of Wheatstone[1] in 1834. The circuit consisted of a copper wire 731.5 m. long, and extended backward and forward so as to form twenty parallel lines 15 cm. apart. Three spark gaps were inserted, one at each end, and one in the center. These were arranged vertically one above another in front of a rotating mirror.

Upon discharging a condenser through the circuit, the image of the central spark was found to be displaced with reference to the other two. The amount of this displacement, together with the speed of the mirror, furnished the data for computing the speed of propagation. The value found by Wheatstone was 463,000 Km. per second, or more than one and one-half times the velocity of light. It seems probable that this high value is to be accounted for by the form given to the circuit and the uncertainty in the measurements of displacement.

In 1850 Fizeau,[2] in conjunction with Gounelle, made a series of experiments with wires, both of copper and of iron, the method employed being somewhat similar in principle to that used by him in the determination of the velocity of light. The circuits used were the telegraph lines between Paris and Amiens (314 Km.) and between Paris and Rouen (288 Km.). Fizeau's measurements give a velocity of 101,700 Km. per second for iron wires, and 172,200 Km. per second for copper.

In 1875 W. Siemens[3] made a determination of the velocity by a method far superior to any hitherto used. His apparatus consisted of two Leyden jars, whose external armatures were united through a short wire. One internal armature was connected by a short wire to a metallic point placed opposite a rotating steel cylinder covered with smoke-black; the other was connected with a long telegraph wire, which ended in a point opposite the cylinder near the first. The jars were charged, then discharged by joining the external armatures. The electricity of the internal armatures suddenly became free, and sparks took place — with an interval of time between their formation — between the points and the cylinder. Knowing the angular distance separating the sparks and the velocity of rotation of the cylinder, the time necessary for the discharge to traverse the length of the long telegraph wire could be calculated. Siemens's experiments were made with iron wires, and gave values of v ranging from 202,600 to 250,600.

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[1] Phil. Trans., Part II., p. 583, 1834; Pogg. Ann., 34, p. 464.

[2] Pogg. Ann., 80, p. 158; Comptes Rendus, 30, p. 437, 1850.

[3] Gesammelte Abhandlungen, p. 365; Ann. d Phys. und Chem., 157, p. 309.

The method employed by M. Blondlot[1] in 1893 was practically that of Siemens. Refinement in details, especially in the method of determining the interval of time between the two sparks, constitutes the chief difference.

With a copper wire 1029 m. long the mean of the results obtained gave a velocity equal to 296,400 Km. per second. In another series of experiments on a line 1821.4 m. long the mean velocity was 298,000 Km. per second.

In 1895 Professor Trowbridge and Mr. Duane[2] employed a method depending upon the principle of resonance. A condenser with its armatures connected by a short wire with an inserted spark-gap formed the primary. Opposite the primary condenser plates, and separated from them by plates of hard rubber, were two secondary plates, to which were attached two wires. These extended for a considerable distance parallel to one another, and ended in a secondary spark-gap. The primary and secondary were "tuned" to resonance, and the nodes and loops of the stationary waves set up in the secondary were measured by means of a bolometer. The period of these waves was obtained by photographing the secondary spark after reflection from a rotating concave mirror. The wave-length and the period being thus known, the velocity of propagation was directly determined. The average of the results gave a velocity of 2.816×10^{10} cm. per second. Later experiments by Messrs. Trowbridge and Duane,[3] in which the method employed was the same, give the value 3.0024×10^{10} cm. per second for the velocity of propagation.

[1] Comptes Rendus, 117, p. 543. October, 1893.

[2] American Journal of Science, XLIX., p. 297. April, 1895.

[3] *Ibid.*, Vol. L., p. 104. August, 1895.

For the benefit of those who wish to enter somewhat fully into the history of the general subject of electric propagation an extended list of references is appended.

EXPERIMENTAL.

Wheatstone,	Phil. Trans. Part II., p. 583, 1834.
Wheatstone,	Pogg. Ann. 34, p. 464.
Walker,	Sill. Jour. 7, p. 206; 8, p. 142, 1849.
Fizeau and Gounelle,	Pogg. Ann. 80, p. 158, 1850.
Fizeau and Gounelle,	C. R. 30, p. 437.
Mitchell,	Pogg. Ann. 80, p. 161, 1850.
Gould and Walker,	Sill. Jour. 11, pp. 67, 153.
Gould and Walker,	Pogg. Ann. Ergänzungsband III., p. 374.
Faraday,	Exp. Res. 3, p. 508.
Faraday,	Phil. Mag. 7, p. 197, 1854.
Faraday,	Pogg. Ann. 92, p. 152.
Whitehouse,	Report British Assoc., 1855.
Guillemin,	Ann. d Chim. et Phys. 60, p. 385, 1860.
Plantamour and Hirsch,	Mém. de la Soc. de Phys. de Gen. 17, p. 289, 1864.
Bezold,	Pogg. Ann. 140, p. 541, 1870.
Lovering,	Sill. Jour. 11, p. 211.
W. Siemens,	Pogg. Ann. 157, p. 309, 1876.
Löwy and Stefan,	Ann. de l'Observ. de Marseille, 1, 1878.
Albrecht,	Astron. Nachr. 91, p. 228; 93, p. 257, 1878; 94, p. 189.
Lorenz,	Wied. Ann. 7, p. 161, 1879.
Colley,	Wied. Ann. 26, p. 432; 28, p. 1, 1885.
Föppl,	Wied. Ann. 27, p. 410, 1886.
Hagenbach,	Wied. Ann. 29, p. 377, 1886.
Hertz,	Wied. Ann. 37, p. 395, 1889.
Lecher,	Wien. Ber., 1890; Phil. Mag. 30, p. 128.
J. J. Thomson,	Phil. Mag. 30, p. 129, 1890.
Sarasin and de la Rive,	C. R. 112, p. 658; 115, pp. 1277, 1400, 1891.
Arons and Rubens	Wied. Ann. 44, p. 206, 1891.
Blondlot	C. R. 113, p. 628, 1891.
Sarasin and de la Rive,	Arch. de Sc. Phys. et Nat. Genève, XXIX., pp. 358, 441, 1893.
Blondlot,	C. R. 117, p. 543, 1893.
Dufour,	C. R. 118, p. 1039, 1894.
Lodge and Glazebrook,	La Lum. Électr. 34, p. 120.
Trowbridge and Duane,	A. J. Sc. XLIX., p. 297, 1895.

THEORETICAL.

Kirchoff,	Pogg. Ann. 100, p. 193; 102, p. 529, 1857.
Sir William Thomson,	Collected papers, Vol. II., p. 135, 1860.
Bertrand,	C. R. 86, p. 916, 1878.
Mascart,	C. R. 86, p. 965, 1878.
Cornu,	C. R. 86, p. 1120, 1878.
Heaviside,	Electrician, Jan. 3, 1885; Sept., 1887; also Dec. 31, 1887; Jan. 2, 1891.

Heaviside,	Phil. Mag. p. 146, 1888.
Stefan,	Wien. Ber., Apr. 1891; Phil. Mag. 31, p. 519; 32, p. 480.
Blondin,	La Lum. Électr. 41, p. 101, 1891.
Hertz,	La Lum. Électr. 41, p. 251, 1891.
Raveau,	La Lum. Électr. 41, pp. 166–609, 1891.
Poincaré,	C. R. 114, pp. 645, 1046, 1229, 1892.
Poincaré,	C. R. 117, p. 1027, 1893.
Picard,	C. R. 118, p. 16, 1894.
Blondin,	La Lum. Électr. 51, p. 401, 1894.
J. J. Thomson,	Recent Researches in Electricity and Magnetism.

Method and Apparatus.

Resonance is employed in the present experiments in the way indicated by Professor Oliver Lodge[1] in his experiments on the "recoil kick." It was thought that, by refinement in the details and by the exercise of great care, this method of obtaining resonance could be made to furnish good results, and the sequel has confirmed the expectation.

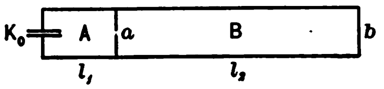

Fig. 1.

From the condenser K_0 (Fig. 1) two wires are stretched parallel to each other with spark-gaps at *a* and *b*, thus forming a primary circuit *A* and a secondary *B*. If oscillatory sparks are formed at *a* by means of a Holtz machine, or an induction coil, connected to the armatures of the condenser, oscillations are set up in the secondary *B*. The period of the oscillations in the primary is governed by the self-induction and the capacity of the circuit, and can therefore be varied by moving *a* back and forward. When the period of the primary approaches the natural period of the secondary, we have the phenomena of resonance, and violent oscillations are set up in *B* so that the *b* gap can be widened greatly before the sparks cease there. Maximum resonance is obtained when these periods are equal. If the circuit *B* act like an organ-pipe open at both ends, $l_2 = \frac{\lambda}{2}$ (Oliver Lodge). The current is zero at the ends, and the potential a maximum. If, however, the circuit act like a pipe closed at one end, $l_2 = \frac{\lambda}{4}$. The current at one end is a maximum, and at the other zero. The same is true of the potential. This latter case is the one my experiment deals with. The wave formed is the fundamental.

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[1] Proc. Roy. Soc., 50, p. 2, 1891.

A series of preliminary experiments was carried out with short wires. A spark micrometer was used at b, and the number of turns of the screw between the lengths of the b spark, when the discharge indifferently chose b and a, and when the sparks ceased at b altogether, gave a measure of the resonance. The wavelength can be calculated from Lord Kelvin's formula $\lambda = 2\pi\sqrt{LK}$, where L is measured in electromagnetic, and K in electrostatic units. The value of L can be obtained for parallel wires from Lord Rayleigh's formula for very rapid oscillations: $L = 4l\log\frac{b}{a}$, where b is the distance separating the wires, and a is their radius. For our rectangular primary circuit, L can approximately be considered to consist of two parts: one for two wires of length l_1 and distant apart b say, and the other for two wires of length b, and distant l_1 from one another. The capacity of the condensers is probably somewhat different in the circumstances of the experiment from that measured by ordinary methods. Its value here is that under very rapid oscillations, while its determination is made under relatively slow alternations. The capacities of our different condensers — gallon Leyden jars of best Bohemian glass — were roughly obtained by comparison with a one-half microfarad condenser, and also very carefully in electro-magnetic measure by the method described in Maxwell's "Electricity and Magnetism," § 776, and somewhat modified by J. J. Thomson.[1] When a condenser is discharged n times per second through a galvanometer, the relation between its capacity and the resistance that will produce the same galvanometer deflection when inserted in the circuit instead of the condenser, is given by $nK = \frac{1}{R}$. By the use of Wheatstone's bridge we can obtain this relation at once. The pairs of opposite points AB and BC of the bridge are respectively connected through the galvanometer and the battery. The arm AC is not closed, but the points AC are connected with two poles R and S, between which a vibrator *P* oscillates rapidly. The plates of the condenser are joined with *A* and *P*, so that when *P* is in contact with *S*, the condenser is charged — a momentary current through the galvanometer resulting — and when with *R* it is short-circuited. When the resistances are so adjusted that the deflection of the galvanometer due to the momentary current is just balanced by the deflection due to the steady current, the above formula very approximately holds. For the vibrator *P* a commutator with a lever was used, each of whose ends oscillated between contact points on two steel springs. The lever was worked by an arm fixed to its center that pressed against an eccentric attached to the end of the axle of the self-regulating motors to be described below. Through the contacts made by the lever, the condenser was charged and discharged once every revolution of the motor. Twenty-five storage cells were employed with the bridge.

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[1] Phil. Trans., 1883, Part III., p. 707.

The following data were obtained in the resonance experiments with short wires. The numbers under the heading "recoil" represent the number of turns of micrometer screw; the other column gives the length of the primary wires l_1. Much better resonance was obtained with the jars of large capacity. The influence of the condition of the surface of the balls, between which the sparks passed, was very great; the surface had to be always kept as clean as possible. Far sharper resonance was obtained with the Toepler-Holtz machine used, than with the induction coil. The distance between the wires is denoted by *d*.

d = 60.

Jar I.				Jar II.		Jar III.		Jar IV.	
Cm.	Recoil.	Cm.	Recoil.	Cm.	Recoil.	Cm.	Recoil.	Cm.	Recoil.
38	20	58	35.5	48	37	48	14	118	6.0
48	29	59	35.5	53	44	58	23	128	10.0
58	36	60	36.0	55	46	64	32	138	13.0
63	40	61	38.0	56	48	66	38	142	14.0
68	36	62	39.0	57	53	67	39	143	14.0+
78	15	63	41.0	58	54	68	40	144	14.5
88	5	64	42.0	59	51	69	42	145	15.0+
		+65	42.5	61	45	70	44	146	15.5
		66	42.0−	63	40	+71	49	147	16.0
		67	42.0−	68	19	72	48	148	16.5
		68	39.0			73	47	+149	17.0+
						74	47	150	16.5
						75	46	151	16.0−
						76	43	152	15.5
						77	37	153	15.0+
						78	33	154	15.0
						88	6	158	15.0−
								168	6.0

d = 60.

Jars I. and II.		Jars I. and III.		Jars I. and IV.		Jars II. and III.		Jars II. and IV.		Jars III. and IV.	
Cm.	Recoil.	Cm.	Recoil.	Cm.	Recoil.	Cm.	Recoil.	Cm.	Recoil.	Cm.	Recoil.
148	7.0	138	6.0	228	9.0	138	3.5	204	0.0	208	4.0
155	10.5	148	9.0	238	11.0	148	8.5	219	5.0	218	5.0
156	15.0	158	10.0	240	11.5	158	9.5	224	6.0	228	7.0
+157	16.0	162	11.5	242	12.0	161	10.0	229	6.5	238	8.0
158	15.5	164	12.0−	244	12.0+	162	11.0	234	8.0−	248	9.0
159	15.0	166	12.0−	246	13.0	163	12.0	239	8.0+	250	9.5
160	13.5	170	12.0+	+248	13.5	164	12.0	244	10.0	252	9.75
161	12.5	+172	12.5+	250	12.0	165	11.0	245	10.0	254	10.0
163	11.5	174	12.5−	252	11.5	166	14.0	246	10.0	256	10.0+
		176	12.0	258	10.0	+167	14.0+	247	11.0	258	10.5
		178	11.5	268	6.0	168	13.0	+248	11.5	−260	11.0
		188	7.5			169	12.0	249	11.0	+262	11.0
		198	3.0			178	6.5	250	10.0	264	8.0
								254	5.0	266	7.0
								259	3.0	268	6 0
										278	4.0

$d = 30$.

Jar I.		**Jar II.**		**Jar III.**	
Cm.	**Recoil.**	**Cm.**	**Recoil.**	**Cm.**	**Recoil.**
68	5.0	63	5.5	78	5.5
73	6.5	68	8.0	88	9.0
78	8.0	73	8.5	93	10.5
83	10.5	78	11.0	98	11.5
88	11.5	81	13.0	99	11.5
90	13.5	82	14.0	100	12.0
92	15.0	83	15.0	101	12.0+
94	15.5	84	15.0+	102	13.0
+96	15.75	85	15.5	103	14.0
98	15.0	86	16.5	+104	14.5
100	13.0	+87	17.0	105	14.0
102	12.0	88	16.0	106	12.5
108	5.5	89	14.0	108	11.0
		90	12.5	113	7.0
		92	10.0	118	1.5
		93	8.5		

The curves plotted from these data for the separate jars are given in Fig. 2. The ordinates represent the recoil; the abscissæ

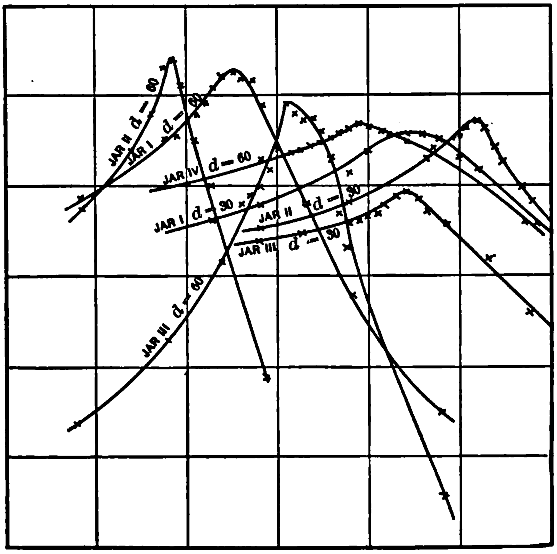

Fig. 2.

the length of the *A* circuit. It is seen that the curves rise somewhat less rapidly on the left than they fall on the right. In general, where there is less capacity there is less marked resonance; hence the curves plotted for the pairs of jars joined in series would be more flattened.

If the wave-lengths be calculated from the self-induction and capacity in the way indicated above, we obtain the following results. The radius of the wires is equal to 0.079 cm. Brass rods were used for the cross-connections with radius equal to 0.28 cm.

Jar.	b	l_1	l_2	L for A circuit.	K for A circuit.	λ calculated.	$4l_2$
I.	60	65	5097	3048	3492	20,498	20,388
	30	96	5078	2977	3492	20,258	20,312
II.	60	58	5104	2834	3767	20,530	20,416
	30	87	5087	2753	3767	20,230	20,348
III.	60	71	5091	3262	3255	20,470	20,364
	30	104	5070	3178	3255	19,980	20,280
IV.	60	149	5013	5518	1906	20,329	20,052
I. and II.	60	157	5005	5676	1812	20,151	20,020
I. " III.	60	172	4990	6093	1685	20,130	19,960
I. " IV.	60	248	4914	8198	1230	19,954	19,656
II. " III.	60	167	4995	5956	1746	20,260	19,980
II. " IV.	60	248	4914	8177	1265	20,214	19,656
III. " IV.	60	261	4901	8346	1202	19,902	19,604

The last two columns give the wave-lengths as calculated from the capacity and self-induction of the primary circuit, and also as directly obtained from the length of the secondary.

After these preliminary experiments, long wires were stretched from the physical to the chemical laboratory and back three times, a distance in all of about 580 m. A very careful measurement of their length was made beforehand in the following way. A portion was subjected to a tension approximately equal to that the wire would bear when finally suspended, and was left under this tension for some time. It was then carefully measured with a steel tape. Another portion was treated in the same way without severing the wire, and the process was continued for the whole length. Very great care had to be taken in insulating the wires;

in fact, it was only after long, glazed porcelain knobs were adopted that anything like good resonance could be obtained. The wires were extended at a distance of about six meters from the ground. Four Leyden jars with a total capacity of 17,552 cm. were used as the condenser. Resonance was first obtained with a Toepler-Holtz machine, and then the machine was replaced by a Ruhmkorff coil. The resonance on this long circuit was naturally not so sharp as on a shorter one; the maximum was determined to one part in 150 of the length of the primary. In the case of the two shorter circuits used in obtaining the final results, the maximum was determined to one part in 400. As the maximum obtained affects almost entirely the factor L, in the expression for the wave-length, and as this occurs under the square root, the wave-length is determined to one part in 800. Again, it is to be noted that an error in the period of the primary does not carry along with it an error of equal magnitude in the period of the secondary. The period of the oscillations in the secondary will be determined by the dimensions of its circuit, along which the wave is actually traveling, and these oscillations natural to the secondary will be set up when they approximate in period to those of the primary.

In order to obtain the period of the oscillations, photographs were taken of the secondary spark by means of a revolving plane mirror and lens. The mirror (see Plate opposite) was constructed by J. A. Brashear, of Allegheny, for Professor Michelson after the latter's experiments on the velocity of light. The mirror is made from a single piece of steel of a thickness sufficient to guard against lateral distortion. It has four circular polished plane surfaces at right angles to one another, each 38 mm. in diameter. The amount of light that can be employed is thus doubled, and the air-vortex caused by the oblique motion of the reflecting surfaces lessened. The top and bottom of the mirror terminate in pivots which are slightly conical. The bottom of the box, in which the mirror rests, is pierced by a screw-hole, through which passes a hollow cylinder (R) having both its internal and external surfaces cut into a screw. This screw can be adjusted in height so as to bring the revolving mirror vertically into the proper position. Its opening widens conically at the top, so that the conical surface of the lower pivot of the mirror fits accurately into the top of the opening. An interior screw, *T*, turned by the head, passes through the opening. This screw terminates on the top in a diamond plane. When the screw *R* has been properly adjusted, *T* is screwed in till the diamond touches the end of the steel pivot of the revolving mirror, so as just to support the weight of the latter. The proper position is indicated by the mirror's freedom of motion. A binding-screw clamps the screws when adjusted. An identically similar arrangement is provided for the top of the mirror. The mirror is placed in a strong frame of iron tubes, through which air is driven. The air issues from orifices opposite to turbine grooves, with which the axle of the mirror is provided above and below, and sets the mirror in rapid rotation. To control the mirror and to render its motion uniform, a valve turned by the hand, and a large wind-chest, 60 cm. x 80 cm., provided with heavy weights, were respectively used. The blast of air was furnished by an 18-inch Sturtevant fan blower, driven by a three horse-power motor.

In the plate the central figure represents one of the two boxes between which the mirror rotates. The air blast enters by the four cylinder terminals. To the right of the box is *T* and at the extreme right *R*. *T* screws into *R*, *R* into the central hole in the box, and both *T* and *R* are screwed in from above and from below until they properly touch the conical bearing of the mirror.

The determination of the velocity of the mirror when regulated was a matter of very great importance. Optical comparison with some fixed standard appears to be the readiest and most accurate method. Professor Michelson compared the rotation-period of the mirror with the vibration-period of a standard tuning fork, but the smallness of the amplitude of the fork's vibration renders the observation somewhat difficult. The standard used in the present case was a motor (*e*, Fig. 3) accurately regulated, as already described by Professor A. G. Webster,[1] by a synchronous alternating motor (*g*), rotating on the same axle. To this axle was attached a disk (*y*) with holes symmetrically situated on the circumference. The light from a lamp (*k*) passed through a large condensing lens

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[1] Electrical World, Vol. XXII., No. 10. Sept. 2, 1893.

(*h*), and after passing through the holes in the disk (*y*) fall on the rotating mirror (*l*). The holes in the disk through which the light passes move vertically, whilst the rotating mirror moves horizontally. The number of bands of light seen in the mirror gave the ratio of the rotation velocities of the motor and the mirror. If the mirror were regulated so as to run at a fixed ratio, the bands remained stationary in the field. Fluctuations in the velocity were most delicately indicated by motion of the bands up or down. The field of the governing motor was independently excited. The current through the armature was made intermittent by means of an electrically maintained tuning fork (*u*) provided with a mercury break. The mercury cup was kept cool and the waste products were carried away by causing a current of water to continually pass over. The fork made 25 vibrations per second. The light from a lamp (*a*) passed through a small rectangular slit (*w*), was reflected from a mirror attached to the electrically driven tuning fork (*u*), and was then reflected from a mirror placed at the end of the axle of the motors, and somewhat inclined to the axle. The eye at (*d*) would see, when the motor was rotating and the fork at rest, a continuous circle of light, and, when the fork vibrated, a series of bright arcs. These arcs reduced in number to two when there was synchronism between fork and motor. The speed of the driving motor was varied by means of a fluid resistance in the circuit of its field magnets until this synchronism was attained The controlling current through the alternator was then thrown on by turning the switch at (*f*). The motors would then continue

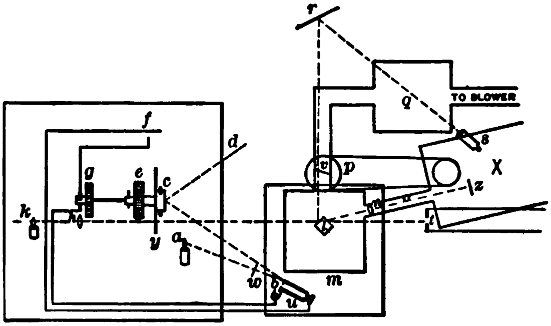

Fig. 3.

to run with uniform speed. If then the absolute frequency of the fork be determined, we know the absolute velocity of rotation of the motor, and hence that of the mirror.

The large iron frame which held the mirror, the regulating motors, and the smaller pieces of apparatus were placed on heavy stone piers, which rested on a deeply laid brick foundation. Disturbing vibrations due to external causes were thus carefully guarded against.

The spark-gap, in which the long wires end, was placed at (*t*). The light was reflected from the mirror (*l*) down the dark chamber (*o*), passing through the long-focus Alvan Clark lens (*n*). The sensitive plate was exposed at (*z*). (*x*) is a small dark room, where the observer was seated. The valve (*v*) was put under the control of the observer by means of a wheel attached to the valve, and by means of a cord with pulleys. The light from the lamp (*k*), after traversing the holes in the disk, was reflected from the rotating mirror, and then from the fixed mirror (*r*) into the telescope (*s*), placed at the eye of the observer. A pasteboard hood was placed over the frame of the mirror, with small rectangular slits inserted, so that the light falling on the mirror from the front could not pass down the dark chamber at the back. A large induction coil was used to produce the sparks. Sharp breaks were obtained by having the primary current pass through two brushes, which pressed against the circumference of a wheel (*c*) 10 cm. in diameter attached to the end of the axle of the regulating motors. Part of the circumference of the wheel was copper, and part hard rubber. Where the current was broken a strip of mica was inserted. This arrangement for obtaining sharp sparks worked very satisfactorily. A switch in the chamber (*x*) permitted the throwing on of the primary current and the production of sparks at will.

The modus operandi of the apparatus is then as follows: The fork (*u*) and the current of water through the mercury cup are set going. The motors are brought into synchronism with the fork. The blower is started. The observer takes his place in the dark chamber. By means of the valve and the bands of light seen in the telescope (*s*) the mirror is regulated. When the lines of light on the face of the mirror stand perfectly stationary, the switch is thrown on to produce the sparks, and the sensitive plate is exposed.

The rate of vibration of the tuning fork was determined by the method of comparing with a clock introduced by Lord Rayleigh.[1] The principle employed was to accurately govern an electro-magnetic engine by an interrupter fork. The accompanying figure exhibits the arrangement. To a horizontal shaft revolving on steel points were attached two hard rubber disks which carried a number of soft iron bars disposed symmetrically on the circumference. Directly beneath these bars was placed an electro-magnet excited intermittently by means of a battery (*a*) and a fork (*b*). If the magnet is excited just as one of the bars approaches, the latter is attracted; so that if the passages of the bars exactly synchronize with the excitations, and therefore with the vibrations of the fork, the wheel is maintained in rotation. To check the oscillations which set in when any disturbance is communicated to the wheel, one of the disks had a channel hollowed out on the inside near the circumference, and this groove was filled with mercury. When the wheel rotates rapidly, the mercury acts as a rigid body, and tends to check any moderate variations of speed. For the 25-fork four bars were used. The wheel was set in rotation by the hand, and the proper speed was indicated by the apparent standing still

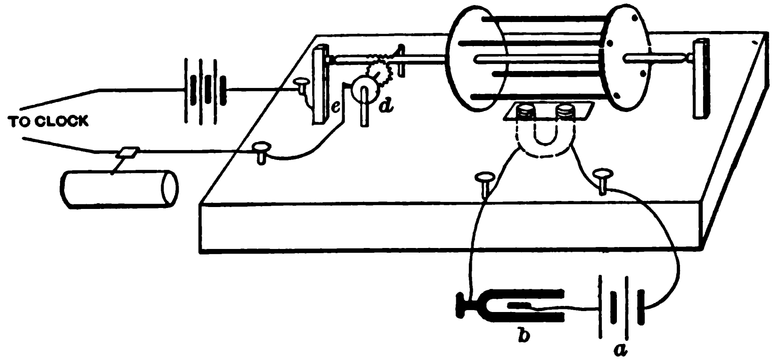

Fig. 4.

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[1] Phil. Trans., 1883, Part I., p. 316.

of the bars when viewed in a mirror on the end of the fork. Engagement can also be very readily detected by the ear.

A screw-gear, carrying a small wheel (*d*), was attached to the shaft. The clock, a chronograph, and the wheel (*d*) with a small piece of hard rubber inserted in its circumference for a break, were put in series with a battery. Contact was made with the wheel (*d*) by a small spring (*e*). The circuit was broken every second by the clock, and once in every revolution by the small wheel. The time of revolution of (*d*) can thus be, with great accuracy, objectively determined by comparison with the seconds' record on the chronograph sheet. The screw on the axle gives the ratio between (*d*) and the phonic wheel, and the ratio between the phonic wheel and the fork is known from the number of the bars. If $n =$ number of seconds in the time of rotation of (*d*); $a =$ ratio between (*d*) and the shaft; $c =$ ratio between shaft and fork $=$ number of bars.

Wheel (*d*) makes $\frac{1}{n}$ revolutions per second.

Shaft makes $\frac{a}{n}$ revolutions per second.

Fork makes $\frac{ac}{n}$ vibrations per second.

In our case $a = 68$, $c = 4$, $n = 11$, approximately.

In order to secure as great regularity in the seconds' breaks of the clock as possible, the circuit was made through a mercury drop, and was broken by a strip of mica attached to the lower end of the pendulum. The mercury drop formed the horizontal connection between two small glass tubes filled with mercury. When by careful adjustment the drop was not brushed away by the mica strip in its passage, sharp regular breaks were obtained. The clock employed was constructed by Howard & Co., Boston.

To determine the absolute pitch of the fork, the daily rate of gain or loss of the clock was obtained by astronomical observations. A series of observations with the transit instrument, extending over the period of the experiments, was made, and the clock was found to gain 6.5 seconds per day.

The accuracy of the method can be seen from the following series of results:

Rotations of wheel (d).	No. of seconds.	Rate of fork.
24	263.1929	24.8031
29	318.0273	24.8029
27	296.0932	24.8030
27	296.0908	24.8032
31	339.9575	24.8031
29	318.0286	24.8028
25	274.1549	24.8035
27	296.0944	24.8029

Average = 24.8031.

The relation between the displacement on the photographic plate and the velocity of propagation can be found in the following way. We will suppose, preliminarily, that the axis of rotation of the mirror lies in its face. Let s (Fig. 5) be the spark-gap, and xx' the two positions of the mirror corresponding to the beginning and end of the displacement. While the mirror turns through the angle α, the image of s turns from a to b through the angle 2α. The photographic arrangement for obtaining an image of ab, taken as an object, is to be considered as entirely separate. The greater the distance of the spark-gap from the mirror, the greater is ab. With a given distance of spark-gap from mirror, the shorter the focal length of the lens that can be used the better, because the shorter-focus lens can be placed nearer the mirror, and will therefore gather more of the light. The sum of the distances of the mirror from the lens and from the spark-gap was made equal to twice the focal length of the lens. Then the displacement on the plate $a'b'$ was equal to ab, and there was no magnification.

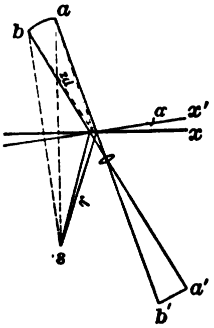


Fig. 5.

If n = number of turns of mirror per second the angular velocity = $2\pi n$.

Then in the time $T = \frac{l}{V}$ the mirror turns through the angle $2\pi n \frac{l}{V} = \alpha$ say.

But $ab = r \cdot 2\alpha$.

$$\therefore\ a'b' = ab = 4\pi n r \frac{l}{V}$$

Suppose now the mirror, as in our experiment, does not rotate about a point on its surface. It will be readily seen from Fig. 6 that the locus of the image of a fixed point in the rotating mirror is an epitrochoid (limaçon). S is the spark-gap. M is one position of the mirror, O being the point about which it rotates. The angle α is measured from the fixed line OS. The angle $SOO' =$ the angle $S'O'O$. O', the center of a circle C' equal to the circle C, moves to its different positions as C' rolls upon C. The locus therefore of S', the image of S, is an epitrochoid, and its center of curvature is the center of the mirror Q, the point of contact of the two circles. The displacement $S'S''$ of the image, due to the rotation of the mirror through the angle $\delta\alpha$, subtends at the center of the mirror the angle $2\,\delta\alpha$. Consequently the formula obtained above still holds good so long as $\delta\alpha$ is sufficiently small. In our experiment $\delta\alpha$ was approximately equal to only one-half a minute and therefore the same law of the double angle holds as in the previous case.

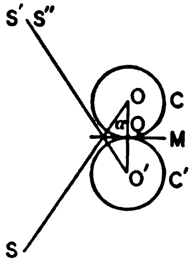


Fig. 6.

The spark-gap was placed at a distance of 186 cm. from the mirror in our experiments. All the distances were carefully measured by means of two sliding rods. Great care was exercised in the determination of the focal length of the lens, which was equal to 100.66 cm. A transparent scale divided into mm. was used, and the method followed was that indicated in Kohlrausch's *Physical Measurements*, No. 5.

Effect of the Resistance of the Wires.

If the wires were perfect conductors, the velocity of propagation would be the same as that of a wave in the dielectric. The question may be asked whether the resistance influences the velocity.

The equation for the propagation of the current or potential is, as first shown by Heaviside (*Phil. Mag.*, 1876, "Electrical Papers," Vol. I., p. 54),

$$L\frac{\partial^2 V}{\partial t^2}+2R\frac{\partial V}{\partial t}=\frac{1}{K}\frac{\partial^2 V}{\partial x^2}$$

where L is the self-induction of the pair of parallel wires per unit of length, K the capacity per unit of length, and R the resistance of a single wire per unit of length. If $R=0$, this becomes

$$\frac{\partial^2 V}{\partial t^2}=\frac{1}{KL}\frac{\partial^2 V}{\partial x^2},$$

which is the equation of propagation with the velocity

$$v=\frac{1}{\sqrt{KL}}.$$

If R is not zero, put $V=we^{-\frac{R}{L}t}$ and the equation becomes

$$\frac{\partial^2 w}{\partial t^2}=v^2\frac{\partial^2 w}{\partial x^2}+\frac{R^2}{L^2}w.$$

This is satisfied by the periodic solution

$$w=\sin\frac{2\pi}{\lambda}(x-v't)$$

of wave-length λ and velocity of propagation v', where

$$v'^2=v^2-\frac{R^2}{L^2}\frac{\lambda^2}{4\pi^2}.$$

For R we must put the resistance as affected by the imperfect penetration of the periodic current. According to Lord Rayleigh, when n, the frequency of the oscillations is very great

$$R'=\sqrt{\pi n\mu R}.$$

In the case of our longest circuit $n=125{,}000$, roughly,

$$R\frac{\rho}{\pi r^2}=\frac{1620}{\pi(0.079)^2}\quad R'=\sqrt{\frac{n\rho}{r^2}}=1.8\times 10^6,$$

$$\therefore\ v'^2=v^2-\frac{R^2\lambda^2}{L^2 4\pi^2}=9.10^{20}-6.10^{16}.$$

The error, therefore, in the square of the velocity due to the neglect of the second term is only one part in 15,000, or in the velocity itself, one part in 30,000. This correction is too small to be taken into account.

Final Results.

After a determination of the velocity for the long circuit had been made, the wires were cut and results obtained for new lengths. The speed of the mirror was varied in each case. The primary spark took place between balls 9 mm. in diameter; the secondary spark was formed between magnesium points which were separated beyond the equivalent sparking distance so as to have true resonance. The rate of the fork was taken daily, but it was found to be practically constant: the room was kept at a uniform temperature, so that no temperature coefficient had to be applied. There were no special peculiarities in the photographs, except that a large number of plates were obtained with relatively very great distances between the oscillations. These distances too were variable, being different for different plates, with no apparent law existing. It may have been that these long oscillations were due to the direct action of the primary with its large capacity. The oscillations were measured on a dividing engine with a glass of low magnifying power.

The following tables exhibit the results.

Circuit I.

Resonance: $l_1 = 2701$ cm.; $l_2 = 55,592$ cm.; $\lambda = 222,368$ cm.

Plate.	No. of oscillations.	Total displacement.	Plate.	No. of oscillations.	Total displacement.
		centimeters			centimeters
1	3	2.653	7	3	2.648
2	4	3.531	8	4	3.521
3	4	3.544	9	4	3.538
4	4	3.531	10	5	4.410
5	5	4.428	11	5	4.421
6	3	2.661	12	4	3.526

Speed of mirror = 49.7420
Total number of oscillations = 48.0000
Sum of displacements = 42.4120
Displacement per oscillation = 0.8836

$V = 2.934 \times 10^{10}$ cm.

Plate.	No of oscillations.	Total displacement.	Plate.	No. of oscillations.	Total displacement.
		centimeters			centimeters
1	2	2.640	7	3	3.949
2	2	2.636	8	3	3.959
3	3	3.949	9	4	5.270
4	3	3.941	10	2	2.626
5	4	5.256	11	3	3.945
6	2	2.637			

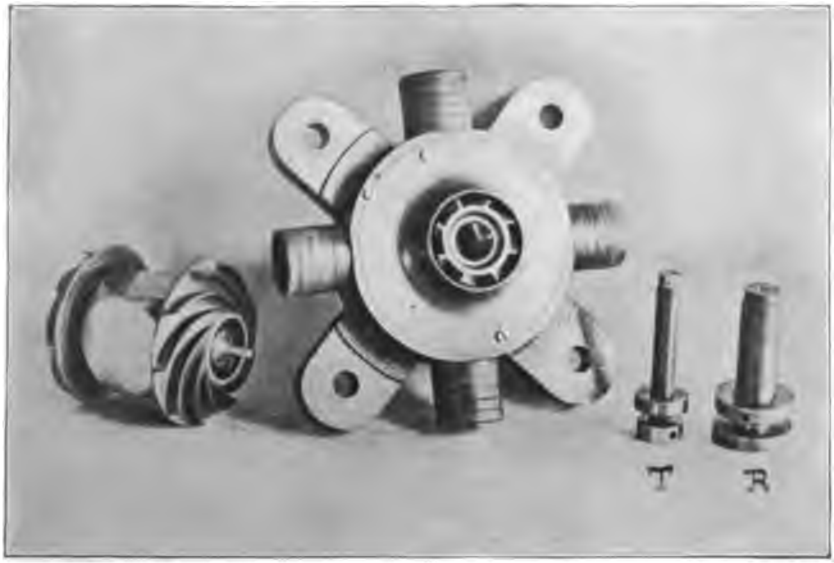
Speed of mirror = 74.6130
Total number of oscillations = 31.0000
Sum of displacements = 40.8080
Displacement per oscillation = 1.3164
$V = 2.954 \times 10^{10}$ cm.

Circuit II.

Length of wire removed = 15,240 cm.
Length of wire left = 43,053 cm.
Resonance: $l_1 = 1484$ cm.; $l_2 = 41,569$ cm.; $\lambda = 166,276$ cm.

Plate.	No. of oscillations.	Total displacement.	Plate.	No. of oscillations.	Total displacement.
		centimeters			centimeters
1	4	2.582	9	5	3.224
2	4	2.579	10	4	2.580
3	3	1.933	11	4	2.581
4	3	1.936	12	4	2.579
5	6	3.871	13	5	3.224
6	5	3.228	14	3	1.935
7	5	3.223	15	5	3.227
8	4	2.580			

Speed of mirror = 49.552
Total number of oscillations = 64.000
Sum of displacements = 41.282
Displacement per oscillation = 0.645
$V = 2.994 \times 10^{10}$ cm.



SAUNDERS: ELECTRIC WAVES.

Plate.	No. of oscillations.	Total displacement.	Plate.	No. of oscillations.	Total displacement.
		centimeters			centimeters
1	3	2.896	8	4	3.868
2	3	2.899	9	4	3.866
3	3	2.901	10	4	3.870
4	4	3.868	11	3	2.899
5	4	3.865	12	3	2.900
6	3	2.896	13	4	3.865
7	4	3.865	14	4	3.864

Speed of mirror = 74.328
Total number of oscillations = 50.000
Sum of displacements = 48.322
Displacement per oscillation = 0.966
$V = 2.998 \times 10^{10}$ cm.

Circuit III.

Length of wire removed = 14,909 cm.
Length of wire left = 28,144 cm.
Resonance: $l_1 = 614$ cm.; $l_2 = 27{,}530$ cm.; $\lambda = 110{,}120$ cm.

Plate.	No. of oscillations.	Total displacement.	Plate.	No. of oscillations.	Total displacement.
		centimeters			centimeters
1	6	2.562	8	5	2.134
2	6	2.558	9	4	1.708
3	4	1.705	10	4	1.706
4	3	1.279	11	5	2.131
5	5	2.134	12	5	2.134
6	5	2.130	13	6	2.559
7	5	2.133			

Speed of mirror = 49.5520
Total number of oscillations = 63.0000
Sum of displacements = 26.8730
Displacement per oscillation = 0.4266
$V = 2.998 \times 10^{10}$ cm.

Plate.	No. of oscillations.	Total displacement.	Plate.	No. of oscillations.	Total displacement.
		centimeters			centimeters
1	5	3.204	9	5	3.201
2	5	3.202	10	4	2.561
3	5	3.203	11	4	2.562
4	4	2.564	12	5	3.201
5	5	3.200	13	5	3.200
6	6	3.844	14	6	3.845
7	6	3.840	15	5	3.202
8	5	3.205			

Speed of mirror = 74.3280
Total number of oscillations = 75.0000
Sum of displacements = 48.0340
Displacement per oscillation = 0.6405
$V = 2.995 \times 10^{10}$ cm.

Plate.	No. of oscillations.	Total displacement.	Plate.	No. of oscillations.	Total displacement.
		centimeters			centimeters
1	3	2.556	4	3	2.558
2	4	3.410	5	4	3.414
3	3	2.559			

Speed of mirror = 99.1040
Total number of oscillations = 17.0000
Sum of displacements = 14.4970
Displacement per oscillation = 0.8528
$V = 2.999 \times 10^{10}$ cm.

The value of the velocity calculated from the formula is corrected so as to be referred in the above to mean solar time. Why the longer circuit gives a lower velocity than the other two is not quite evident. The difference may probably be accounted for partly by the fact that the resonance was not so sharp in the case of this circuit and the maximum could not be determined to the same degree of precision. The oscillations, it is noticed too, are not so constant in length as in the case of the shorter circuits.

The value of the average velocity for the three circuits is 2.982×10^{10} cm. per second, and for the last two 2.997×10^{10} cm. per second.

I wish to express special thanks to Prof. A. G. Webster for the kindness shown and the advice given continually throughout the progress of this work.

LABORATORY OF CLARK UNIVERSITY.

[The above research was completed in July, 1895, but various circumstances have prevented its earlier presentation for publication. — A. G. W.]

ON THE MEASUREMENT OF THE EXPANSION OF METALS BY THE INTERFERENTIAL METHOD.

BY EDWARD W. MORLEY AND WILLIAM A. ROGERS.

PART II.

EXPERIMENTAL DATA AND REDUCTIONS.

THE bars to be compared have the following dimensions:

The bronze bar, which is mounted in the west box, has a length of 1023 mm.

The bar of Jessop's steel, mounted in the north box, has the same length.

Each bar has a cross section of $1\frac{1}{4} \times \frac{1}{2}$ inches.

Since the thickness of the mirrors is 8 mm., the total length measured is 1031 mm.

We have, therefore, for the excess of wave-lengths in air over the wave-lengths in a vacuum,

$$D = 1031 \times 3393.8 \times 0.0002923 \text{ fringes.}$$

$$\log D = 3.009782.$$

I. CORRECTION OF OBSERVATIONS FOR THE EFFECT OF THE RESIDUAL AIR.

When the air surrounding the two bars is at the same temperature, the numbers of wave lengths contained in the lengths of the two bars *in vacuo* are multiplied by the same factor: as the numbers are nearly identical and the factor nearly equal to unity, this is equivalent to the addition of equals; the observed differences, therefore, do not require correction. But when the air around the two bars differs in temperature, the factor depending on the refraction of the air differs for the two bars. If we put

λ = wave-length, *in vacuo*, of light employed;

l = the *effective* length of the bar, or the distance between the reflecting surfaces of the two mirrors applied to the bar;

S = number of half-wave-lengths in l, if the air in the apparatus had everywhere the same density;

t = the temperature of the residual air around the heated bar, in centigrade degrees;

p = the pressure of the residual air;

S_t = the number of half-wave-lengths in air of temperature t° and pressure p;

n = the refractive index of air at 0° and 30 in.;

we shall have: $$S = \frac{2l}{\lambda}\left[1 + \frac{p}{30}(n-1)\right];$$

$$S_t = \frac{2l}{\lambda}\left[1 + \frac{p}{30} \cdot \frac{273}{273+t}(n-1)\right]$$

$$\therefore\ S = S_t + \frac{2l}{\lambda} \cdot \frac{t}{273+t} \cdot \frac{p}{30}(n-1).$$

If we tabulate the factor

$$F_t = \frac{2l}{\lambda}\frac{t}{273+t}\frac{1}{30}(n-1).$$

we shall have for suitable values of t,

$$S = S_t + pF_t. \tag{1}$$

It should be remembered that the subscript t refers to the temperature of the *air* in the apparatus.

From Bell's determination of wave-lengths we have for sodium light:

$$D_1 = 5.896 \times 10^{-5},\quad D_2 = 5.890 \times 10^{-5},\quad D_m = 5.893 \times 10^{-5}.$$

Hence the number of waves in 1 mm. $= 1696.9\lambda = 3393.8\frac{\lambda}{2}$.

VALUES OF THE FACTOR $F_t = \frac{1}{30} \cdot \frac{2l}{\lambda} \cdot \frac{t}{273+t}(n-1)$.

t	F_t	t	F_t	t	F_t	t	F_t	t	F_t	t	F_t
1°	$0.13\frac{\lambda}{2}$	9°	1.09	18°	2.11	27°	3.07	54°	5.64	63°	6.40
		10	1.21	19	2.22	28	3.18	55	5.73	64	6.48
2	0.25	11	1.32	20	2.33	29	3.28	56	5.81	65	6.57
3	0.37	12	1.44	21	2.44	30	3.38	57	5.90	. . .	. . .
4	0.49	13	1.55	22	2.55	. . .	. . .	58	5.98	96	8.88
5	0.61	14	1.67	23	2.65	50	5.29	59	6.07	97	8.96
6	0.73	15	1.78	24	2.76	51	5.38	60	6.15	98	9.02
7	0.85	16	1.89	25	2.86	52	5.46	61	6.24	99	9.09
8	0.97	17	2.00	26	2.97	53	5.55	62	6.32	100	9.15

II. CORRECTION OF OBSERVATIONS FOR CHANGES OF TEMPERATURE OF THE BAR WHOSE TEMPERATURE IS KEPT NEARLY CONSTANT.

Let us put

l_1 = length of the bar;
g = thickness of glass mirror at nearer end of the bar;
T = temperature of the bar;
C = coefficient of expansion of bar;
c = coefficient of expansion of glass of mirror;
B_0 = number of half-wave-lengths in $l_1 + g$ at 0°;
B_T = number of half-wave-lengths in $l_1 + g$ at T°.

We shall then have:

$$B_0 = \frac{2\,(l_1 + g)}{\lambda}\left[1 + \frac{p}{30}(n - 1)\right],$$

and,
$$B_T = \frac{2}{\lambda}\Big[l_1(1 + Ct) + g(1 + ct)\Big]\left[1 + \frac{p}{30}(n - 1)\frac{273}{273 + t}\right].$$

Hence,

$$B_0 = B_T - \frac{2}{\lambda}Ct\left(l_1 + \frac{c}{C}g\right) + \frac{2\,(l_1 + g)}{\lambda}\cdot\frac{p}{30}\cdot\frac{t}{273 + t}(n - 1) + \cdots$$

If we tabulate the quantity $f = \frac{2}{\lambda}Ct\left(l_1 + \frac{c}{C}g\right)$, we shall have

$$B_0 = B_t - tf + pF_T, \tag{2}$$

of which, however, the last term is negligible.

VALUES OF THE QUANTITY $f = \frac{2}{\lambda}Ct(l_1 + \frac{c}{C}g)$.

t	f	t	f	t	f	t	f	t	f	t	f
0.00°	$0.0\frac{\lambda}{2}$	0.08°	4.9	0.17°	10.3	0.26°	15.8	0.35°	21.2	0.43°	26.1
		0.09	5.5	0.18	10.9	0.27	16.4	0.36	21.8	0.44	26.7
0.01	0.6	0.10	6.1	0.19	11.5	0.28	17.0	0.37	22.4	0.45	27.3
0.02	1.2	0.11	6.7	0.20	12.1	0.29	17.6	0.38	23.0	0.46	27.9
0.03	1.8	0.12	7.3	0.21	12.7	0.30	18.2	0.39	23.7	0.47	28.5
0.04	2.4	0.13	7.9	0.22	13.3	0.31	18.8	0.40	24.3	0.48	29.1
0.05	3.0	0.14	8.5	0.23	13.9	0.32	19.4	0.41	24.9	0.49	29.7
0.06	3.6	0.15	9.1	0.24	14.6	0.33	20.0	0.42	25.5	0.50	30.3
0.07	4.2	0.16	9.7	0.25	15.2	0.34	20.6				

III. APPLICATION OF CORRECTIONS TO OBSERVED NUMBERS.

The direct result of observation is

$$N = B_t - S_t. \tag{3}$$

From equations (1), (2), and (3) we obtain

$$B_0 - S = N - tf - pF_t + pF_0,$$

which expresses, in wave-lengths, the difference of length of the two bars reduced to the conditions that the observations are made *in vacuo*, and that the temperature of one is kept constant. The last term is negligible.

IV. THERMOMETERS EMPLOYED AND THEIR CORRECTIONS.

It was expected that the temperature of the steel bar, whose expansion we wished to determine, would be measured with the air thermometer. Since an unexpected difficulty was not remedied in time, we were disappointed in this. In order to utilize the determinations of expansions, we decided to investigate the mercurial thermometers which had been intended, not to measure temperatures, but to serve in maintaining constant temperatures. Thermometers Green 8573, G. 8574, G. 8613, Gerhardt 2403, G.' 2405 and Tonnelot, No. 62, were employed in this investigation.

It therefore became necessary to make a study of the corrections required for these thermometers.

Since March, 1893, the basis of all thermometric observations adopted by the writer depends upon the system of corrections to the observed readings of Tonnelot, No. 62, given on page 119. These corrections result:

(*a*) From a long series of comparisons of Green 1503 loaned to the writer by the U. S. Weather Bureau; subsequently, a careful study of this thermometer was again made by the Bureau;

(*b*) From two investigations of the relation between G. 2292 and G. 2262, made by the Weather Bureau in 1890 and 1893;

(*c*) From a comparison of Green 2292 and Tonnelot 62 with the standard of the International Bureau, reduced to the hydrogen scale by the United States Coast and Geodetic Survey in September, 1890.

Since the scale of Tonnelot 62 reaches only to 80° C., it was impossible to obtain the absolute corrections of this thermometer without first obtaining the absolute corrections of some other thermometer. Two thermometers were selected for this purpose, viz. G.′ 2403 and G.′ 2405. The discussion for G.′ 2403 was made by Messrs. Cole and Durgan, and will be found in an article by these gentlemen, entitled "An Example in Thermometry," which will appear in the next number of the REVIEW. For the details of the method employed, the reader is referred to this paper.

A photograph giving a view of the apparatus with which the observations were made, is inserted at the close of the article just referred to.

The general formula for the computation of the temperature t, as given in the *Memoirs* of the International Bureau, is (see Vol. I., page D, 11):

$$t = \eta + x_n + \gamma_1 + \gamma_2 - \zeta + \eta k,$$

in which

$\eta = \eta + x_n + \gamma_1 + \gamma_2 - \zeta$,

$n =$ the observed reading,

$x_n =$ the calibration correction,

$\gamma_1 =$ the correction due to external pressure,

$\gamma_2 =$ the correction due to internal pressure,

$\zeta = n_0 + x_n + \gamma_1 + \gamma_2$,

$$k = \frac{t_2 - t_0}{n_2 + x_n + \gamma_1 + \gamma_2 - \zeta} - 1,$$

n_2, n_0, being the readings for two known temperatures t_2, t_0, preferably 100° and 0°.

Our first inquiry will relate to the determination of the values of x_n.

I.

Calibration by the Method of Marek.

OBSERVED DATA IN UNITS OF THE THIRD DECIMAL PLACE.

	Degrees.	λ_{10}	**Degrees.**	λ_9	**Degrees.**	λ_8	**Degrees.**	λ_7	**Degrees.**	λ_6
x_1-x_2	0… 10	− 9	0… 20	+44	0… 30	+33	0… 40	+19	0… 50	+25
x_2-x_3	10… 20	+45	10… 30	+14	10… 40	+27	10… 50	+12	10… 60	+21
x_3-x_4	20… 30	−31	24… 40	−30	20… 50	−27	20… 60	−22	20… 70	−20
x_4-x_5	30… 40	+ 2	30… 50	−20	30… 60	+ 3	30… 70	+13	30… 80	−35
x_5-x_6	40… 50	−19	40… 60	−10	40… 70	+ 8	40… 80	−37	40… 90	− 6
x_6-x_7	50… 60	+ 6	50… 70	+19	50… 80	−28	50… 90	+ 5	50…100	+15
x_7-x_8	60… 70	+10	60… 80	−31	60… 90	+ 2	60…100	+10		
x_8-x_9	70… 80	−33	70… 90	−15	70…100	−18				
x_9-x_{10}	80… 90	+34	80…100	+29						
$x_{10}-x_{11}$	90…100	− 5								

	Degrees.	λ_5	**Degrees.**	γ_4	**Degrees.**	λ_3	**Degrees.**	λ_2		
x_1-x_2	0… 60	+18	0… 70	+12	0… 80	+ 6	0… 90	−3		
x_2-x_3	10… 70	+49	10… 80	+27	10… 90	+25	10…100	+3		
x_3-x_4	20… 80	−68	20… 90	−42	20…100	−31				
x_4-x_5	30… 90	+ 1	30…100	+ 3						
x_5-x_6	40…100	+ 0								

For the solution of the equations thus formed see Vol. V., page 18; also an article by the writer, "On the Cumulative Errors of a Graduated Scale," Vol. XV., *Transactions of Mechanical Engineers.* The following numerical results follow from this solution:

$x_1 = 0$
$x_2 = -8$
$x_3 = +46$
$x_4 = +13$
$x_5 = +12$
$x_6 = +2$
$x_7 = +10$
$x_8 = +18$
$x_9 = -25$
$x_{10} = +1$
$x_{11} = +0$

II.

Calibration Corrections derived from the independent *Measures of the Equal Space Corrections and of the Corresponding Equal Volume Corrections for the Detached Mercury Columns.*

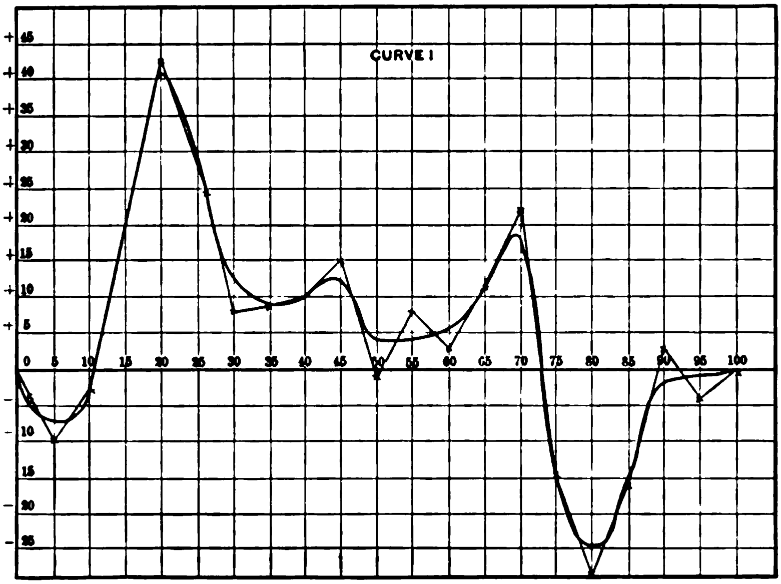
This method depends upon the requirement that the corrected positions of the scale divisions must be identical with the corresponding positions of equal volumes occupied by the detached mercury thread. We have simply to measure, in any unit desired, the values of the successive aliquot divisions of the entire length of the scale, compared with any constant value of this interval that we may adopt. The same remark applies to the measures for the equal volume points. In this investigation the "stop" method was employed. In this case, when the movement of the micrometer wire is apparently from left to right for an increasing reading of the index of the screw, it indicates that the measured space is too short. Hence, when the mean of all the values is subtracted from each value, the residuals express corrections and not errors.

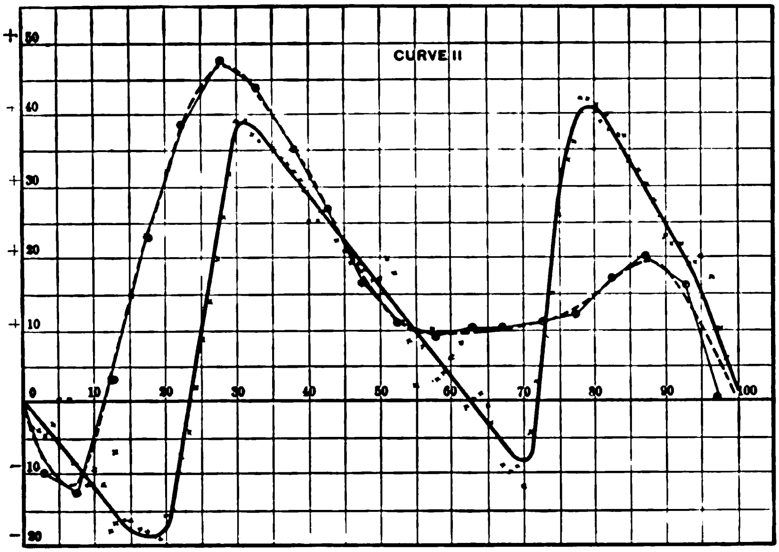
The numerical results obtained are found in the following Table. For the values of x_n which refer to G.′ 2403; see article by Cole and Durgan.

INDEPENDENT MEASURES OF THE 5° SPACES AND OF THE CORRESPONDING MERCURY COLUMNS.

	Scale divisions.			**Mercury columns.**			
	Readings.	**Relative corrections.**	**Summed series = Σ.**	**Readings.**	**Relative correction.**	**Summed series = Σ.**	**Calibration correction.**
deg.	deg.			deg.			
0- 5	5.020	+000	+000	5.010	−010	−010	−010
5- 10	5.010	−010	−010	5.017	−003	−013	−003
10- 15	5.013	−007	−017	5.036	+016	+003	+020
15- 20	5.017	−003	−020	5.040	+020	+023	+043
20- 25	5.049	+029	+009	5.035	+015	+038	+029
25- 30	5.050	+030	+039	5.029	+009	+047	+008
30- 35	5.016	−004	+035	5.017	−003	+044	+009
35- 40	5.010	−010	+025	5.011	−009	+035	+010
40- 45	5.016	−004	+021	5.011	−009	+026	+005
45- 50	5.016	−004	+017	5.010	−010	+016	−001
50- 55	5.005	−015	+002	5.014	−006	+010	+008
55- 60	5.024	+004	+006	5.019	−001	+009	+003
60- 65	5.013	−007	−001	5.021	+001	+010	+011
65- 70	5.016	−011	−012	5.020	+000	+010	+022
70- 75	5.058	+038	+026	5.021	+001	+011	−015
75- 80	5.035	+015	+041	5.021	+001	+012	−029
80- 85	5.012	−008	+033	5.025	+005	+017	−016
85- 90	5.010	−010	+023	5.023	+003	+020	−003
90- 95	5.017	−003	+020	5.010	−010	+016	−004
95-100	5.000	−020	+000	5.004	−016	+000	+000
Mean .	5.020		Mean .	5.020			

In Curve I. the residuals of the last column are laid off as ordinates from which the final calibration corrections are determined. The unit of the scale is one small square, which represents one unit in the third decimal place.





It will be found preferable, however, to lay off the curves for the equal scale divisions and the equal column divisions independently, and to derive the calibration correction by measuring the vertical distance between the two curves corresponding to the same values of the horizontal argument. These two curves are drawn in Curve II. The full line curve represents equal scale corrections, and the dotted line represents the equal volume corrections. The unit is the same as in Curve I. The horizontal argument for the dotted curve is taken as the middle point of the 5° scale.

Instead of deriving the equal space curve from the residuals at the 5° point, the corrections for the single degree points were obtained by direct measurement. The details of these measures are omitted for lack of space.

In order that we may judge whether anything is gained by making the equal volume points correspond to the middle points of the mercury thread, the results for the 5° points, together with the independent determination by the method of Marek, are placed side by side in the following table:

	Curve I.	Curve II.	Method of Marek.		Curve I.	Curve II.	Method of Marek.
0°- 5°	− 7	− 5		5°- 55°	+ 4	+ 1	
10	− 3	+ 6	− 8	60	+ 6	+ 6	+10
15	+20	+32		65	+11	+14	
20	+41	+48	+46	70	+18	+17	+18
25	+29	+33		75	−15	−18	
30	+13	+ 8	+13	80	−25	−25	−25
35	+ 9	+ 8		85	−15	−16	
40	+10	+ 2	+12	90	− 2	− 6	+ 1
45	+12	+ 0		95	− 1	− 5	
50	+ 4	− 2	+ 2	100	+ 0	− 0	+ 0

While in this case both methods of reduction give substantially the same results, the double curve method has decided advantages when the bore of the thermometer investigated varies greatly in diameter at different points.

The value of the calibration corrections of G. 2405 expressed in units of the third decimal place, are given in the following table:

T.	Corr.	T.	Corr.	T.	Corr.	T.	Corr.	T.	Corr.
0°	0	21°	+48	41°	+2	61°	+ 8	81°	−24
1	− 2	22	+48	42	+2	62	+ 9	82	−22
2	− 3	23	+48	43	+1	63	+11	83	−19
3	− 4	24	+43	44	+0	64	+13	84	−17
4	− 5	25	+33	45	+0	65	+14	85	−16
5	− 5	26	+30	46	−1	66	+17	86	−12
6	− 4	27	+25	47	−1	67	+17	87	−10
7	− 3	28	+15	48	−2	68	+18	88	− 8
8	− 1	29	+13	49	−2	69	+18	89	− 7
9	+ 3	30	+ 8	50	−2	70	+17	90	− 6
10	+ 6	31	+ 7	51	−2	71	+14	91	− 6
11	+12	32	+ 6	52	−1	72	+ 7	92	− 6
12	+18	33	+ 6	53	−1	73	+ 0	93	− 5
13	+22	34	+ 5	54	−1	74	− 9	94	− 5
14	+28	35	+ 5	55	−1	75	−22	95	− 5
15	+32	36	+ 3	56	+2	76	−23	96	− 4
16	+35	37	+ 3	57	+3	77	−25	97	− 3
17	+38	38	+ 2	58	+4	78	−26	98	− 2
18	+44	39	+ 2	59	+5	79	−26	99	− 1
19	+47	40	+ 2	60	+6	80	−25	100	+ 0
20	+48								

The values of x_n for T_n are given below:

T.	x_n	T.	x_n	T.	x_n	T.	x_n	T.	x_n
1°	−0°.10	4°	−0°.40	15°	−0°.50	30°	−0°.68	60°	−0°.70
2	−0°.21	5	−0°.45	20	−0°.50	40	−0°.68	70	−0°.70
3	−0°.29	10	−0°.50	25	−0°.58	50	−0°.70	80	−0°.70

Determination of the Coefficients of External and of Internal Pressure.

Reference is made to Plate 5 for a description of the method by which the coefficient of external pressure for the thermometers employed in this investigation was determined. It is to be noted that care must be taken in these observations to allow the thermometers to remain a considerable time after the exhaustion takes place, before being read, in order that the glass of the bulb may reach a normal molecular condition. If this precaution is not observed, too large values of the coefficient of pressure will be obtained.

The coefficients of internal pressure were obtained from a comparison of the readings of the same thermometer for a vertical and for a horizontal position. The conclusion reached at Breteuil is confirmed by these observations; viz. that the coefficients of external and of internal pressure are so nearly identical that no sensible error will be introduced by the assumption of their equality. The numerical values of these coefficients for the different thermometers employed are given below. The data necessary for the computation of the pressures are added.

Name.	Change for 30 inches.	Change for 1 mm.	Distance from center of bulb to 0° = *x*.	Value of 1° in mm.
G. 8573	0°.19	0.00025	328 mm.	26.0
G. 8574	0°.22	0.00030	328	26.0
G. 8613 F.	0°.32	0.00045	294 (to 50° F.)	3.50
G.′ 2403	0°.05	0.00005	69	4.47
G.′ 2405	0°.13	0.00016	60	3.37
T_{62}.	0°.10	0.00014	63	3.69

The correction for G. 8613 F., the graduations of which extend from 50° to 85°, is − 0.04 for the entire interval.

The graduation of G. 8573 and of G. 8574 are to fiftieths of a degree.

Determination of the Fundamental Interval of a Thermometer.

If we assume the fundamental interval of a thermometer to be 1°, we have

$$k = \frac{t_2 - t_0}{[n_2 + x_n + \gamma_1 + \gamma_2] - [n_0 + x_n + \gamma_1 + \gamma_2]} - 1 = \frac{t_2 - t_0}{n'' - \zeta} - 1, \quad (1)$$

in which t_2 is to be derived either from the table given in Vol. I. of the *Memoirs*, page 47; or from the reading of a thermometer whose elements are completely known.

The following equivalent method of reduction will be found to have the advantage of a more exact knowledge of the temperature of the steam under the condition of normal pressure.

Let b = the observed reading of the water manometer.

y = the value of 1 mm. expressed in degrees, for the middle point between the observed pressure and 760 mm.

B' = the reduced reading of the barometer

$$= B\left[\frac{1 + 0.0000186 \times T}{1 + 0.0001815 \times T}\right] 0.99996 \text{ for } \phi = 44^\circ\ 30'.$$

Then, with sufficient accuracy, we have,

$$k = \frac{\left[[n_{100} + x_n + \gamma_1 + \gamma_2] - \frac{by}{13.6} + (760 - B')y - [n_0 + x_n + \gamma_1 + \gamma_2]\right]}{100} - 1. \qquad (2)$$

Thus, if for G.′ 2405, we have

$B' = 751.68$, $y = 0.0370^\circ$, $b = 6.2$ mm., $n_{100} = 99.801$, $n_0 = 0.000$,

we have

$$k = \frac{[99.801 + 0.000 + 0.001 + 0.073]}{100}$$
$$- \frac{6.2}{13.6 \times 100} \times 0.0370 + \frac{832}{100} \times 0.0370$$
$$- \frac{0.000 + 0.000 + 0.001 + 0.010}{100} - 1.00$$

$k = -0.00155$.

By formula (1) we have

$$k = \frac{99.708}{99.875 - 0.011} - 1 = -0.00156.$$

For the value of t corresponding to the pressure 752.13, see Vol. I. of the *Memoirs*, p. A 47.

The error most likely to be introduced in the determination of the temperature at or near 100°, is due to the erroneous reading of the thermometer for the pressure of the steam indicated by the water manometer, if sufficient care is not taken to allow the thermometer to remain in the steam for a considerable interval of time.

It will be found that the reading of the temperatures will gradually increase for fifteen or twenty minutes after the immersion in steam has been made. Experience has shown that the readings can safely be made only after the pressure has remained constant for at least fifteen minutes. It is the evidence furnished by the observations for pressure which enables one to judge whether the thermometer indicates the true temperature after the corrected reading of the manometer has been applied. Under this limitation formula (2) is to be preferred.

The essential constancy of the zero point and of the 100° point, for G.′ 2405, will be seen from the following results:

$$k = \frac{[99.801 + 0.000 + 0.001 + 0.073]}{100}$$

$$- \frac{6.2}{13.6 \times 100} \times 0.0370 + \frac{832}{100} \times 0.0370$$

$$- \frac{0.000 + 0.000 + 0.001 + 0.010}{100} - 1.00.$$

$$k = -0.00155.$$

Date.	Reduced barometer readings.	Exposure to steam.		Depression of zero-point.	Corrected reading in steam at 760 mm.
		Before.	After.		
1896.	mm.	°	°	°	°
Jan. 17	769.7	+0.010	−0.017	+ 0.027	100.127
Jan. 20	768.0	+0.028	+0.004	+ 0.024	100.114
Jan. 20	768.0	+0.017	−0.003	+ 0.020	100.120
Jan. 21	764.9	+0.034	+0.013	+ 0.021	100.108
Jan. 21	763.4	+0.023	−0.002	+ 0.025	100.163
Jan. 22	766.1	+0.020	−0.002	+ 0.022	100.109
Jan. 22	766.9	+0.013	−0.004	+ 0.017	100.121
Jan. 23	771.6	+0.024	+0.008	+ 0.016	100.140
Jan. 23	770.6	+0.008	−0.003	+ 0.011	100.139
Feb. 10	746.3	+0.010	−0.013	+ 0.023	100.119
Feb. 11	741.9	−0.095	−0.022	−[0.073]	100.106
Feb. 12	751.7	+0.024	+0.000	+ 0.024	100.154
Feb. 13	762.7	+0.006	−0.003	+ 0.009	100.142
Feb. 14	746.4	+0.028	+0.016	+ 0.012	100.132
Feb. 16	759.4	+0.023	+0.003	+ 0.020	100.120

We conclude this portion of the investigation with a comparison of G.′ 2403, G.′ 2405, and T_{62}. All the thermometers were submerged in water to a depth such that the distance from the center of the bulb to the surface of the water was $h = 510$ mm. = 33 mm. of mercury. The details of the comparisons are given only for 30° C.

	2403.	2405.	T_{62}.
n	30°.104	30°.090	30°.720
x^n	+0.013	+0.033	−0.068
γ_1	+0.001	+0.002	+0.001
γ_1	−0.001	−0.003	−0.002
γ_2	+0.010	+0.031	+0.027
Sum.	30°.127	30°.153	30°.678
$-\zeta$	+0.015	+ 0.000	−0.461
η	30°.142	30°.153	30°.207
ηk	−0.043	−0.038	−0.105
t	30°.099	30°.105	30°.102
t	25°.188	25°.184	25°.169
t	20°.090	20°.095	20°.082
t	15°.761	15°.791	15°.775
t	7°.135	7°.114	7°.119

The direct comparison of the corrections for T_{62} may now be made with the corrections assumed previous to this investigation.

Designating the corrections to T_{62} adopted in 1893 by C_1, and the corrections derived from the present investigation by C_2, we have

T.	C_1	C_2	T.	C_1	C_2	T.	C_1	C_2	T.	C_1	C_2
°	°	°	°	°	°	°	°	°	°	°	°
0	−0.43	−0.46	6	−0.49	−0.52	12	−0.52	−0.54	20	−0.58	−0.56
1	−0.44	−0.47	7	−0.50	−0.52	13	−0.53	−0.54	25	−0.61	−0.59
2	−0.45	−0.48	8	−0.50	−0.52	14	−0.54	−0.54	30	−0.62	−0.62
3	−0.46	−0.49	9	−0.51	−0.53	15	−0.55	−0.55	35	−0.65	−0.64
4	−0.47	−0.50	10	−0.51	−0.53	16	−0.56	−0.55	40	−0.66	−0.65
5	−0.48	−0.51	11	−0.52	−0.53	17	−0.56	−0.55	45	−0.67	−0.67

Since the completion of this investigation an opportunity has occurred for the direct comparison of G′. 2403 with the standard of the International Bureau through the medium of standards of the U.S. Bureau of Weights and Measures, with the following results:

T.	Corr. to G. 2403.		T.	Corr. to G. 2403.	
	Observer C and D.	**Observer U. S. Bureau.**		**Observer C and D.**	**Observer U. S. Bureau.**
0°	+0°.000	+0°.000	30°	−0°.051	−0°.047
5	−0°.020	−0°.019	35	−0°.054	−0°.088
10	−0°.019	−0°.034	40	−0°.052	−0°.050
15	−0°.023	−0°.056	45	−0°.048	−0°.055
20	−0°.029	−0°.030	50	−0°.056	−0°.026
25	−0°.18	−0°.020			

V. OBSERVATIONS ON THE EXPANSION OF THE STEEL BAR.

We give in the following table the observed data, from which the final determination of the coefficient of expansion of the steel bar will be derived. This table will be followed by other observations which will not enter into this determination, but which will furnish the necessary evidence that the proper temperature conditions have been established.

The first and second columns give the time of the observations; the third gives the pressure of the air remaining in the apparatus; the fourth gives the observed difference of length of the two bars; the fifth and sixth give the readings of the thermometers which were in contact with the bronze and the steel bars respectively.

EXPERIMENTAL DATA.

Series I. — Table I.

Date.	Time of observation.	Reading of manometer.	Number of fringes.	Reading of 8574 W. box.	Reading of 8573 N. box.
1895.	h. m.	in.		deg.	deg.
July 17	10 0	2.0	682.0	−0.01 C.	−0.10 C.
	10 30	1.9	682.8	−0.00	−0.10
	10 50	1.8	682.0	−0.01	−0.10
					No. 8613 F.
July 19	11 25	1.1	+ 7.6	−0.03	64.90
	11 30	0.6	+ 7.6	−0.03	64.90
	11 35	0.6	+ 7.7	−0.04	64.90
	11 38	0.6	+ 7.7	−0.06	64.90
	11 42	0.7	+ 7.0	−0.05	64.90
	11 44	0.6	+ 7.1	−0.05	64.90
	11 46	0.6	+ 7.0	−0.05	64.90
	12 38	0.9	+ 8.0	−0.06	64.91
	12 42	0.9	+ 8.2	−0.05	64.91
	12 45	0.9	+ 8.1	−0.04	64.92
	12 55	1.6	+ 10.0	+0.01	64.92
	1 7	1.3	+ 8.8	+0.00	64.92
July 21	10 20	1.2	− 78.0	−0.04	69.49
	10 25	1.2	− 78.3	−0.04	69.50
	10 30	1.5	− 78.8	−0.08	69.60
	10 45	1.5	− 78.8	−0.08	69.60
	11 0	1.4	− 81.0	−0.06	69.60
	11 5	1.1	− 81.3	−0.06	69.63
July 22	3 55	1.1	−359.0	−0.04	83.01
	4 20	1.1	359.0	−0.05	83.05
	4 30	1.0	359.0	+0.02	83.11
	5 7	1.0	358.5	+0.02	83.27
	5 25	0.9	358.7	+0.00	83.20
July 23	11 15	0.9	−325.0	+0.06	81.95
	11 45	1.1	−326.0	+0.10	82.11
	12 10	0.8	−330.0	+0.08	82.20

SERIES II.

Date.	Time of observation.	Reading of manometer.	Number of fringes.	Reading of 8574 W. box.	Reading of 8574 N. box.
1895.	h. m.	in.		deg.	deg.
July 29	8 44	3.5	+ 256.8+	+0.01 C.	−0.09 C.
	8 52	3.5	257.0	+0.00	−0.09
	9 23	2.6	257.0	−0.02	−0.11
	9 28	2.8	257.7	−0.02	−0.09
	9 38	2.8	257.6	−0.02	−0.09
	10 5	2.9	+257.1	−0.02	−0.09
	10 15	3.0	257.3	−0.02	−0.09
	10 30	2.5	257.5	−0.05	−0.11
	10 46	3.0	258.1	−0.05	−0.11
	10 51	2.5	258.0	−0.04	−0.09
	11 7	2.6	+ 258.3	−0.03	−0.09
	11 16	2.8	258.5	−0.02	−0.09
	11 50	2.8	258.7	−0.02	−0.06
	11 55	2.5	258.0	−0.04	−0.09
	12 7	2.9	258.0	−0.02	−0.11
	1 30	2.9	+ 257.8	−0.01	−0.15
	1 40	2.9	256.8	−0.01	−0.13
	1 53	2.6	257.0	−0.03	−0.14
	2 5	2.6	257.5	−0.03	−0.14
	2 16	2.5	257.3	−0.02	−0.14
	P.M.			No. 8573.	No. 8613.
July 30	3 2	2.2	−1037.2	+0.10	71.41 F.
	8 15	2.8	−1035.6	+0.05	71.04
	8 44	2.8	−1034.0	+0.06	70.90
					No. 2405.
July 31	4 10	2.9	−2572.0	+0.07	63.80 C.
Aug. 1	11 20	2.6	−2575.0	+0.07	63.93
	3 30	2.4	−2575.1	+0.11	63.93
					No. 8574.
Aug. 2	4 10	2.8	− 257.0	+0.11	+0.10
	5 10	2.8	− 257.1	+0.10	+0.12
	5 20	2.8	− 257.5	+0.10	+0.12

Notes in Regard to Observations preceding those given above.

July 16, 5 h. 50 m. P.M. — Put 500 lbs. of ice in each box. 6 h. 25 m. — Thermometer in N. box = 0°.33.

July 17, 5 h. 58 m. A.M. — Found that considerable of the ice had melted during the night. N. box = 0°.44. W. box = 0°.39. Filled both boxes with ice. Drew off the water melted during the night. Exhausted the air in the boxes at 8 h. 32 m.

July 18. — Refilled the W. box with ice at 4 h. 55 m. M. Turned steam into the N. box at 5 h. 10 m. At 5 h. 0 m., N. box = 70° F. The number of fringes = – 220. At 8 h. 50 m. N. box = 67°.8 F. The number of fringes = 70. At 9 h. 0 m. refilled the W. box with ice.

July 19, 5 h. 15 m. — Before filling the W. box with ice the temperature = 0°.98; N. box = 65°.20. 5 h. 20 m. — Filled W. box with ice. At 9 h. 30 m., N. box = 64°.80. Number of fringes = + 22. At 10 h. 0 m., N. box = 64°.90. Number of fringes = 22.0. At 10 h. 30 m., N. box = 64°.90. Number of fringes = + 16.1. At 10 h. 45 m., N. box = 64°.90. Number of fringes = + 12.0. At 11 h. 15 m., N. box = 64°.90. Number of fringes = + 8.6. At 11 h. 25 m., stationary point apparently reached.

July 20, 10 h. 0 m. — W. box = 1°.16, N. box = 68° .55. Turned steam heat into the air space surrounding the room in which the refractometer is mounted. 2 h. 0 m. — Filled W. ice-box. Room = 68° F. 7 h. 40 m. — Filled ice-box. Room = 78°. 7 h. 45 m. — Started air pump.

OBSERVATIONS TO DETERMINE THE LENGTH OF TIME REQUIRED AFTER THE AIR IS EXHAUSTED BEFORE THE STATIONARY POINT IS REACHED.

Time.	Number of fringes.	W. box.	N. box.	Time.	Number of fringes.	W. box.	N. box.
h. m.				h. m.			
7 55	– 0.0	+0.00	68.46	8 16	–17.4	–0.04	68.57
8 0	– 3.0	+0.00	68.48	8 24	–22.7	–0.02	68.61
8 3	– 5.6	+0.00	68.52	8 26	–26.7	–0.02	68.61
8 5	– 7.1	–0.01	68.52	8 33	–32.5	–0.03	68.63
8 7	– 9.0	–0.03	68.54	8 35	–35.0	–0.00	68.65
8 9	–11.9	–0.05	68.54	8 40	–38.0	–0.00	68.67
8 14	–15.0	–0.04	68.57				

July 21, 5 h. 15 m. A.M. — Before filling with ice found the reading of W. box = + 0°.80, N. box = 69°.60. 5 h. 30 m. — Filled the W. box with ice and turned off about half of the water melted during the night.

MADE THE FOLLOWING OBSERVATIONS TO ASCERTAIN WHEN THE STATIONARY POINT WAS REACHED.

Time.	Number of fringes.	W. box.	N. box.	Time.	Number of fringes.	W. box.	N. box.
h. m.				h. m.			
6 40	−41.4	+0.11	69.28	8 50	−63.8	+0.03	+ 69.50
6 50	−47.4	+0.11	69.30	9 5	−74.5	−0.05	69.46
7 2	−53.0	+0.02	69.30	9 30	−75.0	−0.02	69.48
7 12	−59.0	+0.05	69.30	9 40	−76.5	−0.01	69.48
7 20	−63.2	+0.05	69.32	10 0	−76.8	−0.01	69.45

10 h. 10 m. Stationary point apparently reached.

July 22, 8 h. 30 m. M. — Exhausted the air from the boxes.

Time.	Number of fringes.	W. box.	N. box.	Time.	Number of fringes.	W. box.	N. box.
h. m.				h. m.			
10 1	−77.8	+0.17	70.66	11 1	− 97.5	+0.00	70.69
10 16	−86.0	+0.10	70.69	11 10	− 99.5	+0.00	70.69
10 31	−89.5	+0.05	70.70	11 21	−102.0	−0.01	70.69
10 41	−94.5	−0.03	70.69	11 31	−102.0	+0.00	70.69
10 37	−96.0	+0.00	70.69	11 41	−102.0	−0.01	70.69

11 h. 45 m. — Turned steam into N. box. Temperature of N. box = 83°.16. Number of fringes = − 298. 2 h. 20 m. — N. box = 83°.49. Number of fringes = − 370. 3 h. 37 m. — N. box = 83°.08. Number of fringes = − 365. Stationary point reached.

July 23, 5 h. 15 m. — Filled W. box with ice. Stationary point reached at 11 h. 15 m. Found that oil had filtered through one of the boxes, and that a drop had fallen on one of the mirrors, compelling the abandonment of the series at this point.

July 27. — New series. The mirrors have been cleaned and remounted. 5 h. 0 m. — Filled both boxes with ice.

July 28. — Packed both boxes closely with ice. Kept both boxes filled with ice during the day.

July 29, 4 h. 8 m. — Refilled both boxes with ice. Stationary point reached at 8 h. 40 m. 2 h. 20 m. — Turned steam into N. box and raised the temperature to about 70° F.

July 30, 5 h. 30 m. — Filled W. box with ice. Stationary point reached at 11 h. 5 h. 55 m. — Turned steam into N. box until a temperature of 73° C. was obtained. Packed the N. box with blankets for the night.

July 31, 6 h. 12 m. — Found the temperature of N. box = 50° C. 7 h. 20 m. — Raised the temperature of the N. box to 64° C. Maintained this temperature within 0°.1 from this time until 4 h. 10 m. P.M.

August 1. — Maintained the temperature at 64°.7 until 11 h. 20 m.

August 2, 6 h. 30 m. — Reduced the temperature of the N. box to 0° C.

VI. REDUCTION OF THE OBSERVATIONS RECORDED IN TABLE I.

SERIES I.

Date.	Reading of manometer.	No. fringes.	Reading of 8574; W. box.	Manometer $\times\frac{0^\circ.22}{30}$.	Constant.	Corrected readings.	Reading of 8573; N. box.	Manometer $\times\frac{0^\circ.19}{30}$.	Constant.	Corrected readings.	F. reduced to C.	Corrections γt + etc.	Corrected temperature.	Constants from page 24.	Correction I. in fringes.	Correction II. in fringes.	Corrected No. fringes.
July 17	1.9	+ 682.3	−0.01	−0.01	+0.15	+0.13	−0.10 C.	−0.01	+0.12	+0.01	+0.01	0.00	+0.01	—	− 0.0	− 7.9	674.4
July 19	0.7	+ 7.7	−0.04	−0.01	+0.15	+0.10	64.90 F.	−0.01	+0.36	65.25	18.47	−0.02	18.45	2.16	− 1.5	− 6.1	+ 0.1
July 19	0.6	+ 7.0	−0.05	−0.00	+0.15	+0.10	64.90 F.	−0.01	+0.36	65.25	18.47	−0.02	18.45	2.16	− 1.3	− 6.1	− 0.4
July 19	1.1	+ 8.8	−0.03	−0.01	+0.15	+0.11	64.92 F.	−0.01	+0.36	65.27	18.48	−0.02	18.46	2.16	− 2.4	− 6.7	− 0.3
July 21	1.3	− 79.4	−0.05	−0.01	+0.15	+0.09	69.57 F.	−0.01	+0.36	69.92	21.07	−0.02	21.05	2.45	− 3.2	− 6.5	− 88.1
July 22	1.0	− 358.8	−0.01	−0.01	+0.15	+0.13	82.20 F.	−0.01	+0.36	83.55	28.64	−0.02	28.62	3.24	− 3.2	− 7.9	− 369.9
July 23	0.9	− 327.1	+0.08	−0.01	+0.15	+0.22	82.09 F.	−0.01	+0.36	82.44	28.02	−0.02	28.00	3.18	− 2.8	−13.7	− 343.2

SERIES II.

July 29	3.0	− 257.2	−0.02	−0.02	+0.12	+0.08	−0.10 C.	−0.02	+0.15	+0.03 C.	+0.03	+0.00	+0.03	—	− 0.0	− 4.9	− 262.1
July 29	2.8	− 257.6	−0.04	−0.02	+0.12	+0.06	−0.10 C.	−0.02	+0.15	+0.03	+0.03	+0.00	+0.03	—	− 0.0	− 3.6	− 262.2
July 29	2.7	− 258.3	−0.03	−0.02	+0.12	+0.07	−0.08 C.	−0.02	+0.15	+0.04	+0.04	+0.00	+0.04	—	− 0.0	− 4.2	− 262.1
July 29	2.7	− 257.3	−0.02	−0.02	+0.12	+0.08	−0.14 C.	−0.02	+0.15	−0.01	−0.01	+0.00	−0.01	—	− 0.0	− 4.9	− 262.2
July 30	2.5	−1035.6	+0.07	−0.02	+0.12	+0.17	71.12 F.	−0.02	+0.36	71.46 F.	21.92	−0.02	21.90	2.51	− 6.3	−10.3	−1052.2
July 31 } Aug. 1 }	2.6	−2577.0	+0.08	−0.02	+0.12	+0.18	63.89 C.	−0.02	+0.13	64.00 C.	64.00	−0.05	63.95	6.48	−16.8	−10.9	−2604.7
Aug. 2	2.8	− 257.2	−0.02	−0.02	+0.12	+0.08	−0.11 C.	−0.02	+0.15	+0.12	+0.12	+0.00	+0.12	—	− 0.0	− 4.9	− 262.1

From the data given on the previous page the following equations of condition are formed:

Series I.	Series II.
$+674.4 = a + 0.01\,b$	$-262.1 = a + 0.03\,b$
$+0.1 = a + 18.45\,b$	$-262.2 = a + 0.03\,b$
$-0.4 = a + 18.45\,b$	$-262.1 = a + 0.04\,b$
$-0.3 = a + 18.46\,b$	$-262.2 = a - 0.01\,b$
$-88.1 = a + 21.05\,b$	$-1052.2 = a + 21.90\,b$
$-369.9 = a + 28.62\,b$	$-2601.7 = a + 63.95\,b$
$-343.2 = a + 28.00\,b$	$-262.1 = a + 0.12\,b$

From the solution of these equations by the process of least squares, we obtain

Series I. $b = 36.41$ fringes. Series II. $b = 36.58$ fringes.
Mean $b = 36.49$ fringes.

Reducing to the corresponding value for 100 cm., we have

$$b = 10.45\,\mu,$$

that is, the expansion of a bar of Jessop's steel, 100 cm. in length, is 10.45 μ per degree centigrade.

It may be added that the mean of a large number of determinations of this coefficient, obtained by comparing a line-measure bar at varying temperatures with an end-measure bar in melting ice, gave, for the limits $-5^\circ \cdots +25^\circ$ C., $b = 10.51\ \mu$.

It may also be added that the value of this coefficient, determined at Sêvres for a selected specimen of Jessop's steel, was found by the method of Fizeau to be

$$10.387\,\mu + 0.00595\,\mu t.$$

For 32° C. this becomes $\cdots b = 10.58\,\mu$.

The result which we have obtained is to be regarded as tentative, for our determinations of temperature have been inferior in accuracy to our measurements of expansion. This was owing (1) to our inability to get our air thermometer ready for use; and (2) to our partial failure to maintain the polish of our mirrors. It is our intention to renew the observations under conditions in which these difficulties will be avoided.

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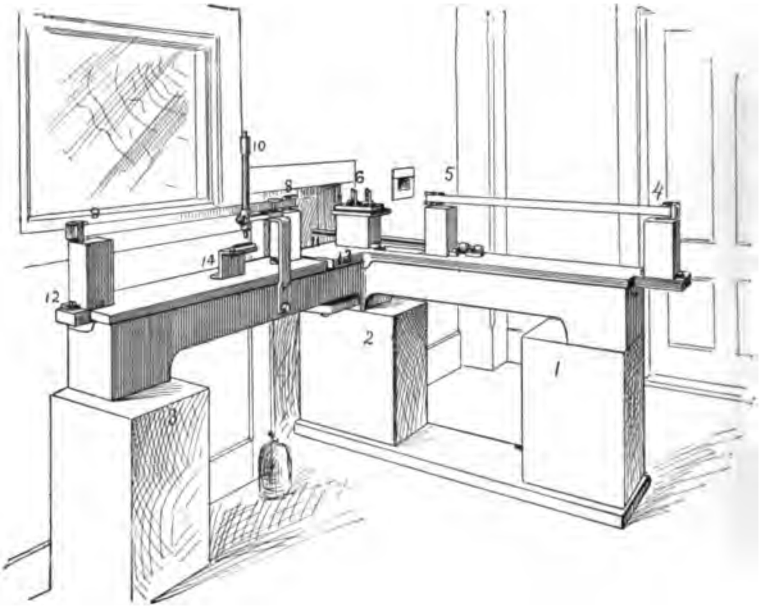

Fig. 1. First Form of Refractometer.

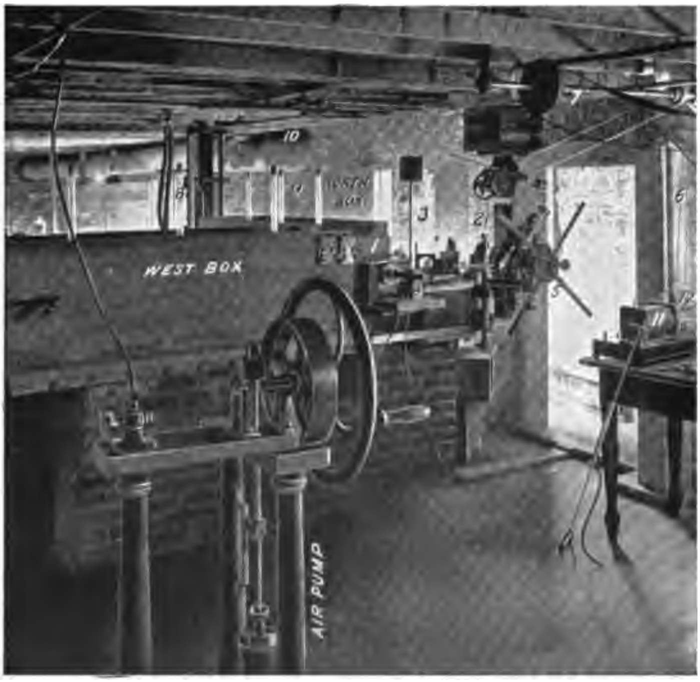

Fig. 2. Vacuum Refractometer (Front View).

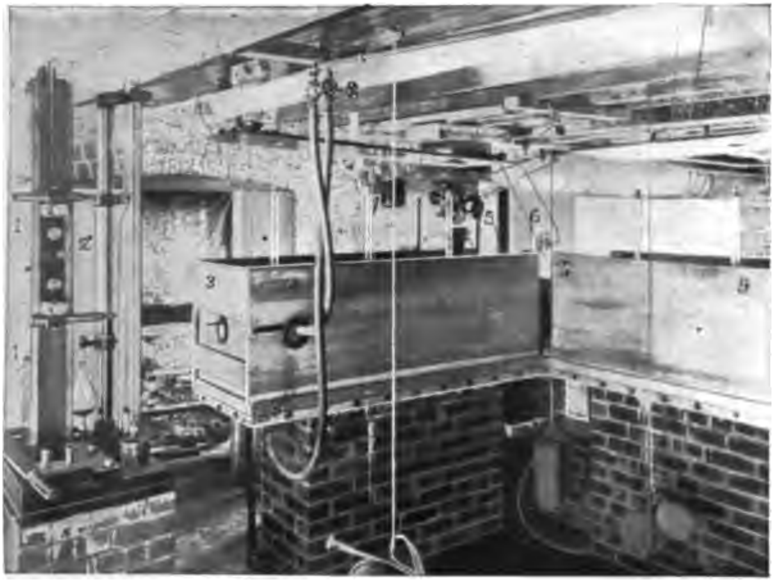

Fig. 3. Vacuum Refractometer (Rear View).

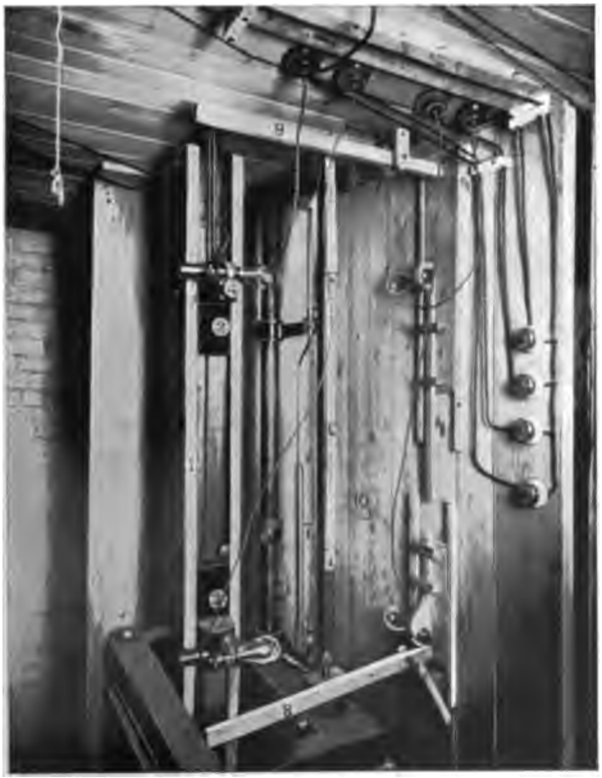

Fig. 4. Air Thermometer.

FFICIENTS OF EXPANSION.

EXPLANATION OF FIGURES SHOWN IN PLATE.

Fig. 1. First Form of Refractometer.

(1)(2)(3) Supports of the instrument.

(4)(5) Bronze bar } with mirrors attached.
(8)(9) Steel Bar }

(6) Diagonal mirrors.

(7) Opening for the passage of the light from a monochromatic flame.

(10) Reading microscope.

(11) Adjusting rod which controls the motion of the plate upon which the bar (4)(5) is mounted.

(12) Pulley and weight which hold the plate supporting (8) (9) in contact with the wedge-shaped plate (13) moved by a precision screw mounted on the outside of the comparing room in the direction of (11).

(14) Holder for plate upon which is mounted a graduated scale.

Fig. 2. Vacuum Refractometer (Front View).

(1) Front end of west box which contains the bronze bar, mirrors, adjusting rods, etc.

(2) Index of precision screw by which motion is communicated to (1), and apparatus for counting fringes.

(3) Front end of north box which contains the steel bar, mirrors, adjusting rods, etc.

(4) Extension of the vacuum boxes within which the diagonal mirrors are mounted.

(5) Fringe counting apparatus.

(6) Connections with Ruhmkorff coil, and mercury vacuum tube, etc.

(7) Shafting of apparatus for driving the precision screw with uniform motion.

(8)(9) Vacuum tubes for thermometers.

(10) Air thermometer.

(11) Automatic rotary air pump.

Fig. 3. Vacuum Refractometer (Rear View).

(1)(2) Guides to the Cathetometer.

(3) Connection of air thermometer with vacuum box.

(4)(6)(7)(9) Vacuum boxes.

(5) Diagonal mirrors.

(8) Connections for steam and water with north box.

Fig. 4. Air Thermometer.

(1) Cathetometer frame.

(2)(3) Movable plate-holders balanced by weights attached to pulleys near (9).

(4)(5) Reading microscopes.

(6) Mercury reservoir.

(7) Mercury tubes.

(8)(9) Slow motion apparatus for making contact of the jolly-point with the meniscus of the mercury column.

(10) Tube connecting the air thermometer with the vacuum boxes.

AN EXPERIMENTAL STUDY OF INDUCTION PHENOMENA IN ALTERNATING CURRENT CIRCUITS.

BY F. E. MILLIS.

II. *Circuits containing Resistance, Self-induction, and Capacity.*

1. CHARGE AND DISCHARGE OF CONDENSERS.

IN an article which appeared in a recent number of this journal, the author gave an account of some experiments upon induction phenomena in alternating current circuits containing resistance and self-induction.[1] It is the object of the present paper to present the results of some experiments made upon circuits containing resistance, self-induction, and capacity.

Since the Leyden jar was first constructed in 1745, the condenser has been a source of interesting and fruitful study. The early experiments with the Leyden jar were all made in efforts to determine the nature of electricity and its effects, and it was not until the year 1824 that the account of an attempt to study the *character* of the electrical discharge of the Leyden jar was given.

In that year Savary first observed that fine steel needles, when taken out of magnetizing helices through which these jars had been discharged, were not always magnetized in the proper direction. After many repetitions of these experiments and much study of the phenomena, Joseph Henry was led to say in 1842: "The phenomenon requires us to admit the existence of a principal discharge in one direction, and then several reflex actions backward and forward, each more feeble than the preceding, until equilibrium is established."[2]

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[1] PHYSICAL REVIEW, Vol. III., 1896, p. 351.

[2] Scientific Writings of Henry, in Smithsonian Institute Publications, 1886.

In 1847 Helmholtz said: "We assume that the discharge (of a jar) is not a simple motion of the electricity in one direction, but a backward and forward motion between the coatings, in oscillation, which becomes continually smaller until the entire *vis viva* is destroyed by the sum of the resistances." He also says: "The notion that the discharge consists of alternately opposed currents is also favored by the phenomenon, observed by Wollaston while attempting to decompose water by electric shocks, that both kinds of gas are evolved at both electrodes."[1]

In 1853 Sir William Thomson published a theoretical discussion of the different characters of condenser discharges obtained by placing in the discharge circuit different amounts of resistance and self-induction.[2]

He showed that if the discharging circuit contains a resistance and no self-induction, the discharge will be continuous, and that if times be plotted as abscissas, and the quantity of the charge in the condenser at the various times be plotted as ordinates, the curve of discharge will be a logarithmic curve, falling rapidly at first, then more and more slowly, becoming horizontal after an infinite time, though practically so after the small fraction of a second. By increasing the self-induction the discharge curve is made to fall less and less rapidly, being no longer a simple logarithmic curve, until finally when the self-induction exceeds a certain value the curve no longer falls continuously, but vibrates back and forth on each side of its zero value with ever decreasing amplitude.

If instead of *quantity* curves, the *current* at successive instants be plotted as ordinates, the current curves will have the same general characteristics as the quantity curves, except when there is an appreciable self-induction in the circuit, yet not enough to cause the oscillatory discharge. In that case the discharge current will rise rapidly at first, after which it will die down more slowly to zero.

It was the strong probability of oscillations in condenser discharges that seems to have created an interest in these phenomena. Though Henry and Helmholtz had expressed a belief that such oscillations exist, it was not until after Thomson had shown mathematically that oscillations should be expected, that any attempt was made to demonstrate their existence experimentally.

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[1] Erhaltung der Kraft (Berlin, 1874).

[2] Phil. Mag., IV., Vol. 5, 1853, p. 393.

Feddersen made public some preliminary results in 1857, and three or four years later published a complete account of his experiments.[1] His condenser consisted of Leyden jars, the discharge of which produced a spark across an air-gap. With a stationary concave mirror and a rapidly revolving mirror he projected a real image of the spark on a ground glass screen, where he viewed it with a low power telescope, or projected the image on a sensitive plate to be photographed.

Because of the small capacity of the Leyden jar and the high potential to which it may be charged, it has always been studied by means of its spark phenomena, produced either in a vacuum tube or across an air-gap. The attractiveness and simplicity of these experiments, and the theoretical interest of the Leyden jar, have led to its discharge being studied under almost all conceivable conditions.

Plate or slab condensers of large capacity have been on the market only a few years, and as they are not constructed to stand the high potentials to which Leyden jars are charged, and since their discharge occupies but a small fraction of a second, their phenomena have usually been studied by the employment of some device which enables the experimenter to charge the condenser for varying short intervals of time, then to measure these charges by ballistic methods, such as by allowing the condenser to discharge itself through a galvanometer.

The curve of charge is then obtained by plotting times of charging as abscissas, and the corresponding charges as ordinates. The discharge curves are then assumed to be the same as the charge curves, since mathematical considerations show them to be so.

Considerable work has been done on the discharge of plate condensers during the last six years, and so far as I have been able to find, there has been but a single exception to this general method of procedure.[1] Fairly uniform results have been obtained by the above method, and yet one can never feel perfectly satisfied with a curve built up in this manner. The curve is not secured by one discharge, but by as many discharges as there are points observed on the curve, and one cannot be certain that sufficiently uniform conditions can be secured to insure that every charge shall be just like every other. The difficulty of measuring accurately the exceedingly short intervals of time during which the condenser is receiving its charges, and the impossibility of varying these intervals by sufficiently small amounts to bring the consecutive points on the curve close together, must also always leave a doubt as to whether the exact curve is obtained by such readings. And even if the points were sufficiently near one another, the experimenter is at a loss whether to attribute slight variations from a smooth curve to errors of measurement, or to some peculiarity in the behavior of the condenser. One would feel much more sure of his conclusions if he were certain that he had an instrument which would trace accurately either the quantity curve or the current curve as it is actually formed.

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[1] Pogg. Ann., Vols. 103, 108, 112, 113, 115, and 116. Also, Ann. de Chim. et de Phys., III., Vol. 69, 1863, p. 178.

In the year 1889[2] Frölich attempted to obtain such a trace of current curves by attaching a small mirror at a point between the center and the circumference of a telephone diaphragm, and then discharging the condenser through the coil of the telephone. Light from an arc lamp was reflected from this mirror to a moving photographic plate, or to a revolving mirror, whence it was reflected to a screen to be observed.

Frölich argued that though there is a considerable mass to move in the telephone diaphragm, yet it must follow quite accurately minute current changes in the telephone coil, else the characteristics of the human voice would be more greatly altered when speaking through the instrument than experience shows them to be. He found, however, that the current curve as recorded by the telephone differed considerably from the known form of the current which he sent through the instrument, because of the distortions due to the natural vibrations of the diaphragm. He proposed to determine the proper correction for a given instrument, once for all, and then to apply this correction to whatever curve he might be studying. But such a procedure is not all that could be desired. The telephone also possesses a considerable self-induction, and seems to be sensitive to outside disturbances.

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[1] Frölich's experiment, described below.

[2] Elektrotech. Zeit., 1889, p. 345. Also, Electrician (Lond.), Vol. 28, 1891, p. 59.

The curves presented with this paper were obtained with a slightly modified form of the galvanometer described in a previous article by H. J. Hotchkiss and the writer.[1]

The apparatus as described in the article referred to was found to record satisfactorily alternate and induced current curves.[2] It also gave good results in tracing continuous discharge curves of the condensers when there was high resistance in the discharge circuit, but in the oscillatory discharges the oscillations of the current were so rapid that it was necessary to use a needle of higher natural period.

Mr. Hotchkiss has rendered the apparatus portable, more convenient, and easily adjustable, by substituting a permanent compound horseshoe magnet for the electromagnet. To compensate for the weaker field thus produced, the pole-pieces are brought much nearer the needle. Two coils are used instead of one, and the thin pole-pieces project in between these coils, almost touching the support of the needle. This support is a brass wire $\frac{1}{8}$ of an inch in diameter, and has a groove an inch long, $\frac{1}{16}$ of an inch wide, and $\frac{1}{16}$ of an inch deep, cut lengthwise in one side. At the bottom of this groove in the middle, a hole $\frac{3}{16}$ of an inch long, and of the same width as the groove, is drilled entirely through the wire.

The two ends of the quartz fiber, to which the needle and mirror have previously been fastened, are then securely attached with shellac to the shoulders thus formed at the bottom of the groove, so that the needle and mirror are in the center of the wire, with their longer axis, their axis of rotation, in the axis of the wire.

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[1] PHYSICAL REVIEW, Vol. 3, 1895, p. 49.

[2] *Ibid.*, 1896, p. 351.

The two coils, the thin pole-pieces, and the needle are supported in their proper positions by a block of wood. Mr. Hotchkiss has given a fuller description of this portion of the apparatus in a *thesis* which will soon be published in this journal.

The needle used was 1.5 mm. long — its vertical dimension — 0.8 mm. wide, and 0.075 mm. thick. The mirror was made of the thinnest procurable microscope cover-glass silvered, and was of about $\frac{3}{8}$ the dimensions of the needle. The length of the free portion of the quartz fiber was $\frac{3}{16}$ of an inch.

To measure the natural period of the needle, a piece of microscope cover-glass mirror, about the size of the one attached to the needle, was shellacked to the end of one prong of a heavy tuning-fork known to make 1228.8 single vibrations per second. The fork was mounted just above the needle, so that the light from the arc lamp would fall upon its mirror. The needle was then deflected by a storage battery, the fork was bowed, and the plate dropped, breaking the circuit as it passed the slit. While the needle was coming to rest it vibrated almost exactly 13.85 times as rapidly as the fork.

To measure these vibrations, the plate was placed in a lantern, and the curves thrown upon a paper screen, magnified about eighteen times; the extremities of the amplitudes were then carefully marked on the screen. With a T square, a pair of dividers, and a diagonal scale the divisions were measured and counted. 41.55 periods of the needle were found to correspond to 3 periods of the fork; hence the needle made 17,018 single vibrations per second.

A hand-feed arc lamp burning $\frac{1}{2}$-inch cored carbons was constructed and so mounted that the carbons made an angle of 30° with the vertical. A very strong steady light was obtained from this lamp by using carbons of high grade, and by adjusting the carbon points as close together as was possible without causing the arc to hiss with a strong current.

A narrow vertical slit placed three inches in front of the arc allowed the light to fall upon the mirror, from which it was reflected to a horizontal slit. Behind this second slit a photographic plate was dropped with a velocity of nearly ten feet per second.

The vertical slide down which the plate holder fell was nearly three feet high, and was constructed so as to let the plate fall freely, and yet allow as little displacement sidewise as possible.

A plate-holder was constructed to carry a plate 8 × 2½ inches, thus allowing 5 × 8 plates to be used when cut lengthwise. The apparatus was set up in a basement room, partially darkened to prevent fogging of the plate.

In Sir William Thomson's discussion above referred to, he arrived at the following general equation expressing the current of discharge of a condenser when it is short-circuited by a circuit containing resistance, self-induction, and capacity:

$$i = \frac{Q}{2CLa}\epsilon^{-\frac{R}{2L}t}\{\epsilon^{at} - \epsilon^{-at}\} \tag{1}$$

where R is the resistance and L the self-induction of the discharge circuit, C the capacity and Q the maximum charge of the condenser, and a is equal to $\left\{\frac{R^2}{4L^2} - \frac{1}{CL}\right\}^{\frac{1}{2}}$.

Since the curves here given are all current curves, Thomson's equations for the quantity of electricity and his discussions of them are not given.

CONTINUOUS CHARGE AND DISCHARGE.

Equation (1) represents two distinct characters of discharge, depending upon whether a is real or imaginary. If $\frac{R^2}{4L^2} > \frac{1}{CL}$; or, what amounts to the same thing, if $R^2C > 4L$, the discharge is continuous. The simplest case is when $L = 0$. Equation (1) then reduces to

$$i = \frac{Q}{CR}\epsilon^{-\frac{t}{CR}} \tag{2}$$

which shows that when self-induction is absent the current begins instantly with its maximum strength, and decreases according to a logarithmic curve as the charge gradually and permanently leaves the condenser.

The self-induction of the coils surrounding the needle of the galvanometer was too small to be measured by direct methods.

Indirect methods showed that it certainly was not more than a few ten-thousandths of one henry, consequently the self-induction of the instrument may be treated as practically equal to zero.

If the instrument is simply connected in series with a non-inductive resistance in the discharge circuit, the sudden rush of current upon closing the circuit throws the needle so violently as to set it vibrating and prevent its tracing the proper curve. Consequently the connections shown in Fig. 1 were used. In this diagram C represents the condenser; B, a storage battery; R, a bank of incandescent lamps; G, the galvanometer; and K the key, which is opened by the falling plate.

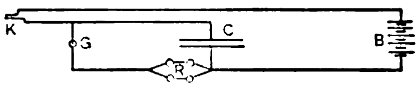


Fig. 1.

It will be seen that when K is closed, the needle is deflected by a steady current, equal to the initial value of the discharge current which flows from the condenser through the galvanometer when the *battery* circuit is broken at K by the falling plate. For the steady current is $I = \frac{E}{R}$, where E equals the difference in potential to which the condenser plates are constantly charged while K is closed. And by equation (2), the initial value of the discharge current is $i_0 = \frac{Q}{CR}$. But $Q = EC$, therefore, $I = i_0$. Under these conditions the needle traces accurately the rapidly decreasing current. Curve No. 1 (see Plate I. opposite page 142) is a discharge curve obtained with these connections.

For curve No. 1, $L = 0$; $C = 8.31$ microfarads; $V = 280$ volts. The condensers were always charged by storage cells in an adjoining building. The resistance consisted of four high voltage incandescent lamps grouped two in series, and these groups in parallel. The resistance of this group was 445 ohms, cold, and 280 ohms when subjected to a potential of 220 volts. Their resistance was therefore probably not far from 240 ohms when subjected to a potential of 280 volts. Taking the resistance of the circuit as 240 ohms, the *time constant* of the curve, $T = CR = 8.31 \times 10^{-15} \times 240 \times 10^{6} = 8.31 \times 10^{-6} \times 240 = .001994$ seconds. The curve was thrown on the screen and enlarged. This was carefully traced, and measurements made from the tracing, from which T was determined to be .00197 seconds. The difference between this and the computed value might easily be accounted for by an error in the assumed resistance of the lamps.

By assuming the initial conditions for charging the condenser in determining the constants of integration, the general equation for charging

$$i = -\frac{Q}{2CLa}\epsilon^{-\frac{R}{2L}t}\{\epsilon^{at} - \epsilon^{-at}\} \tag{3}$$

reduces to

$$i = -\frac{Q}{CR}\epsilon^{-\frac{t}{CR}} \tag{4}$$

when $L = 0$.

It should be noticed that equation (4) is the same as equation (2) except in sign, indicating that the current flows in the opposite direction, but that the curves are perfectly similar in shape. As in curve 1, the needle must be permanently deflected before the plate is dropped. This was accomplished by the connections shown in Fig. 2, where the lettered parts are the same as in Fig. 1.

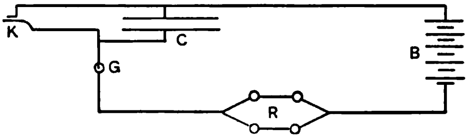


Fig. 2.

When K is closed, the current flows through the galvanometer, and since the two terminals of the condenser include no resistance they are at the same potential; but when K is opened by the falling plate the condenser will become charged, the needle following the current as before. This curve is not shown because it is perfectly similar to curve No. 1.

If we differentiate equation (1) with respect to the time, and equate $\frac{di}{dt}$ to zero, we see that the current reaches its maximum value at a time given by

$$t = \frac{1}{2a}\log\frac{\frac{R}{2L}+a}{\frac{R}{2L}-a},$$

after which it gradually diminishes to zero. We thus see that when there is considerable self-induction in either the charge or the discharge circuit, R^2C still remaining greater than $4L$, the charge and discharge will be continuous, but the current will increase slowly enough so that the needle will be able to follow it from zero. Since equation (1) differs from equation (2) only in sign, for every discharge curve there is an exactly similar charge curve for similar conditions of circuit.

To obtain the current curve of charge when the circuit contains considerable self-induction, it is only necessary that the galvanometer be placed in series with the charging battery, the condenser, the self-induction, and the resistance. For all the curves showing the charging current under the above-mentioned conditions the galvanometer was so placed.

For the corresponding discharge curves, the galvanometer should be in series with the condenser, the self-induction, and the resistance, but *not* with the battery.

In these experiments the key operated by the falling plate had three terminals, one of which could be connected to either of the other two. For the discharge curves, with self-induction in the circuit, the movable terminal was connected to one of the others so as to charge the condenser without disturbing the needle. The falling plate broke that connection, and closed the discharge circuit through the galvanometer and the induction coil.

As no compensation was made in this circuit for the resistance of the battery, the resistance of the discharge circuit was always less than that of the charge circuit. The effect of this difference in resistance will be discussed in some of the curves.

Curve No. 2 is a discharge curve for which $C = 8.31$ microfarads, $L = 0.036$ henrys, $V = 280$ volts, and $R = 400$ ohms, when the lamps are cold. As measured on the curve, $t = 0.00046$ seconds. By computation, $t = 0.00034$ seconds, for $R = 400$ ohms; and $t = 0.00041$ seconds, computed for $R = 280$ ohms, the resistance of the lamps hot. The current heats the lamps considerably even though it flows for so short a time, and it is impossible to tell what the true resistance should be. It must be remembered also that the resistance is not constant.

OSCILLATORY CHARGE AND DISCHARGE.

When $R^2C < 4L$, the right-hand member of equation (1) becomes imaginary in form. But, if we employ the exponential values of the sine and cosine, the equation becomes

$$i = \frac{Q}{CL\alpha'}\epsilon^{-\frac{R}{2L}t}\sin(\alpha' t) \tag{5}$$

where $\alpha' = \left(\frac{1}{CL} - \frac{R^2}{4L^2}\right)^{\frac{1}{2}}$.

Equation (5) may be written in the more convenient form

$$i = \frac{2Q}{\sqrt{4CL - R^2C^2}}\epsilon^{-\frac{R}{2L}t}\sin\left\{\frac{\sqrt{4CL - R^2C^2}}{2CL}t\right\}. \tag{6}$$

This equation represents an oscillatory current with a period

$$T = \frac{2\pi}{\frac{\sqrt{4CL - R^2C^2}}{2CL}}.$$

If R is so small as to be neglected this expression reduces to

$$T = 2\pi\sqrt{CL}.$$

In the case of the oscillatory curves which follow, the result is affected by less than $\frac{1}{10}$ of 1 per cent by neglecting R.

The oscillations die down according to the logarithmic curve

$$i = \frac{2Q}{\sqrt{4CL - R^2C^2}}\epsilon^{-\frac{R}{2L}t}. \quad \text{See (6).}$$

The *time constant* $\frac{2L}{R}$ of this curve will be designated by τ.

Curve No. 3 is a curve of discharge for which $C = 8.31$ microfarads, $L = 0.036$ henrys, $R = 5.3$ ohms, and $V = 210$ volts.

$$Q = CV = 0.00000831 \times 210 = 0.001745 \text{ coulombs.}$$

$$\frac{2Q}{\sqrt{4LC - R^2C^2}} = 3.2, \quad \frac{R}{2L} = 73.6.$$

and
$$\frac{\sqrt{4CL - R^2C^2}}{2CL} = 1826.85.$$

Hence the equation for curve No. 3 is

$$i = 3.2\, \epsilon^{-73.6t} \sin \{1826.85\, t\}. \tag{7}$$

The initial value of the logarithmic curve tangent to the oscillatory curve is 3.2; and

$$\tau = \frac{0.072 \times 10^6}{5.3 \times 10^6} = 0.0136 \text{ second.}$$

From measurements on the curve τ is found to be 0.011 second, a discrepancy of about 20 per cent. For the period we have

$$T = 2\,\pi\sqrt{CL} = 2\,\pi\sqrt{0.00000831 \times 0.036} = 0.00344 \text{ sec.}$$

Measurements of the curve give $T = 0.00326$ second, a difference of more than 5 per cent. It is to be noted that the computed time constant is more liable to error than is the period; for the value of the former will be greatly influenced by the presence of Foucault currents in neighboring conductors. Conducting substances were removed so far as was practicable; but it can scarcely be doubted that enough metallic particles were left to account for a considerable increase in the rate at which the oscillations were damped. Cases where the effect of eddy currents is very strikingly in evidence will be referred to later.

The following table gives the constants for five different curves:

OSCILLATORY CURVES.

	No. 3. Discharge.	**No. 4. Charge.**	**No. 5. Discharge.**	**No. 6. Discharge.**	**No. 7. Discharge.**
C, microfarads	8.31	8.31	8.31	2.66	1.00
L, henrys . .	0.036	0.036	0.036	0.036	0.036
R, ohms . .	5.3	5.3 ?	5.3	5.3	5.3
V, volts . .	210.0	280.0	210.0	210.0	280.0
i_o, computed .	3.2 amp.	4.26 amp.	3.2 amp.		
T, computed .	0.00344 sec.	0.00344 sec.	0.00344 sec.	0.00194 sec.	0.00119 sec.
T, measured .	0.00326 sec.	0.00328 sec.	0.00333 sec.	0.00190 sec.	0.00118 sec.
τ, computed .	0.0136 sec.	0.0136 sec.	0.0136 sec.		
τ, measured .	0.0110 sec.	0.0062 sec.	0.0111 sec.		
Difference in T	5 %	5 %	3 %	2.5 %	1 + %
Difference in τ	20 %	50 % +	20 %		

Curve No. 7 was obtained with a new standard mica condenser. All the others were with paraffin slab condensers which had been used by students in the general laboratory for two or three years. It will be noticed that the computed value of T is larger than the measured value in every case. This was true in all the other curves measured, and would seem to indicate that either the self-induction of the circuit or the capacity of the condensers was smaller than it was considered to be in the computations, for I think all these measurements on the curves could certainly be made to within 2 per cent.

Notice also that the observed time constant for the discharge curves is 20 per cent smaller than the computed value. This would indicate either that the resistance of the circuit is considerably more for a current making 300 oscillations per second, or that the self-induction was taken too large in the computations, or that Foucault currents were somewhere set up by the oscillating current. The latter explanation seems the most probable.

The resistance of the whole discharge circuit, key and all, was carefully measured on a Wheatstone bridge. The self-induction was measured by the three-voltmeter method, on a circuit of 117 periods per second, and the readings were made with the ordinary Whitney instruments.

The discrepancies in both T and τ would be diminished by taking the value of L smaller in the computations.

The time constant for the charge curves is always much smaller than for the discharge curves. Compare curves 4 and 3 or 5. This is in part accounted for by the fact that the charge circuit contained a few hundred feet of No. 10 connecting wire, the clips at three switchboards, and 140 storage cells, some of them quite small, in addition to the resistance of the discharge circuit.

Compare curve No. 7 with No. 3, or No. 5. The ratio of the square root of their capacities is 1.77 –, and the ratio of their periods is 1.74 +, a difference of about 2 per cent, showing that the periods are proportional to the square roots of the capacities, as the formula indicates, for small resistances. These curves, with many others obtained, show that the period is independent of the potential to which the condenser is charged.

The self-induction coil used in making the above curves consisted of 40 pounds of No. 10 copper wire, with no other metal about the coil. The same coil was used in the curves that follow, but in the case of curve No. 8 the coil contained an iron core 2.5 inches in diameter, the internal diameter of the coil being 3 inches.

In curve No. 9 the solid core was replaced by an iron tube $\frac{3}{16}$ inch in thickness and filling the opening in the coil.

In curve No. 10 the core was of nearly solid brass, and in curve No. 11 the core was a brass tube, $\frac{1}{8}$ of an inch thick. In all four of the curves $C = 8.31$ mf., $L = 0.036$ henrys, $R = 5.3$ ohms, and $V = 280$ volts.

It should be noticed that curves No. 8 and No. 9 are alike. That is, the iron tube has practically the same effect as the solid core.

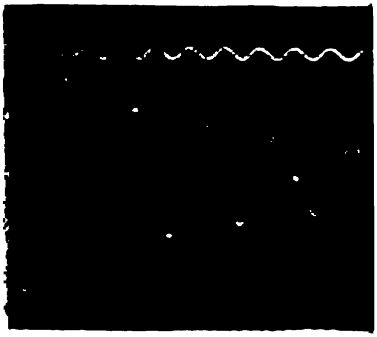
The period is just double what it is without the iron, showing that the self-induction has been increased about fourfold. Instead of getting sixteen or eighteen oscillations, as in the former discharge curves, we get only two, that is, the time constant is greatly diminished. In the two curves when the coil contained brass instead of iron the period is unchanged, but the damping is greatly increased. It is to be remembered that Thomson obtained $\frac{2L}{R}$ as a time constant on the assumption that there are no circuits near by in which eddy currents can be developed. This condition is far from fulfilled when the coils contain metallic cores. Therefore $\frac{2L}{R}$ does not represent the time constant; the denominator should contain in addition to the ohmic resistance, an important term due to the absorption of energy by the Foucault currents set up in the metallic cores. With the rapidity of oscillations here obtained these eddy currents do not penetrate far into the metal, and the tubular cores are as effectual in damping the oscillations as are the solid cores.

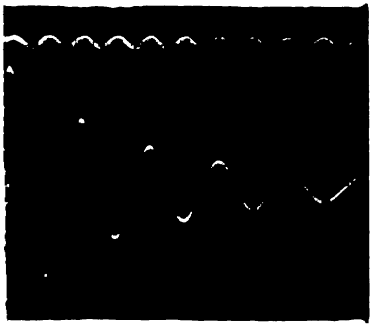
The disturbances at the end of the fourth oscillation in curve No. 5, in the sixth oscillation of No. 6, in the tenth oscillation in No. 7, in the fourth oscillation of No. 10, and in No. 2, are evidently due to some peculiarity in the discharge of the condenser. It is seen to a greater or less extent in nearly all of the curves, whether of charge or discharge. I have one negative of a charge curve in which the curve is displaced 0.0938 inch, making that particular period 0.00449 instead of 0.00333 second. In another negative of discharge one period is 0.00417 second. In both curves the periods on both sides of the disturbance are of the normal length, and on neither plate is there the slightest trace of a disturbance in the tuning-fork curve.

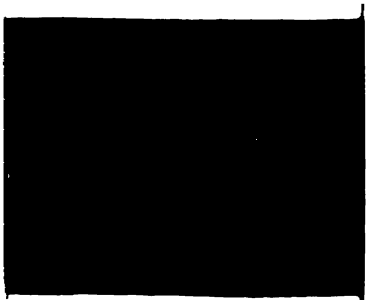
This disturbance cannot be due to any sudden motion of the plate, for the tuning-fork curve would then show the same disturbance. Neither can it be due to any disturbance of the galvanometer needle, because in all cases the particular period in which it occurs is lengthened, displacing the whole remaining portion of the curve. The disturbance is as marked in the discharge of the standard mica condenser as in that of the paraffin condensers. It is as marked when the condenser is charged to 90 volts as when charged to 280 volts. It usually occurs near the axis of the curve, which corresponds, of course, to the maximum amplitude of the electromotive force curve; but it occurs as often when the condenser is nearly discharged as when its voltage is higher. The disturbance is certainly a very puzzling one, for which I have not yet been able to find a satisfactory explanation.

(*To be continued.*)

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
June, 1896.







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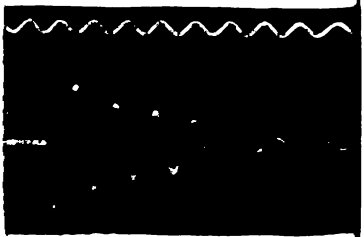

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MINOR CONTRIBUTIONS.

Admittance and Impedance Loci.[1]

By Frederick Bedell.

THE quantities chiefly considered in the discussion of alternating currents are electromotive forces and currents, the values of these being determined for different conditions. Electromotive forces and currents are commonly represented by vector diagrams; and the change in these diagrams, as some one quantity is varied, is shown by the loci of the vectors which are altered thereby. What may be termed electromotive force and current loci are thus determined. The numerical values for which these are constructed necessarily depend upon some condition involving an assumed value either of the current or of the electromotive force: thus, we may assume a certain current to be constant (as the primary current of a transformer), and construct an electromotive force diagram with loci showing the changes in the various electromotive forces as some part of the circuit is changed; or, we may assume the impressed electromotive force constant, and ascertain current loci for the same variation. Let us limit ourselves to the transformer. In the first case above, it will be found that the constant assumed primary current, I_1, is a factor in the value of every line representing the components of the primary electromotive force, E_1. By factoring out I_1, we have an impedance diagram similar to the electromotive force diagram, and without any assumption as to the value of the current or electromotive force. Similarly, each line in a current diagram, constructed for a constant impressed electromotive force E_1, represents a current which is a multiple of an admittance (the reciprocal of an impedance) and the factor E_1. By factoring out E_1, an admittance diagram is consequently obtained, similar to the current diagram but with no assumption as to the current or electromotive force. Admittance and impedance diagrams accordingly correspond to current and electromotive force diagrams, respectively, differing from them only by a factor.

Impedance and admittance loci, or electromotive force and current loci, for the primary of a transformer, will in general be arcs of circles for changes in any one of the constants of the primary or the secondary circuit.

Some interesting relations arise from the reciprocal nature of admittance and impedance. Let us note the following relations between reciprocal vectors:

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[1] Read before the Physical Society of London, June 26, 1896.

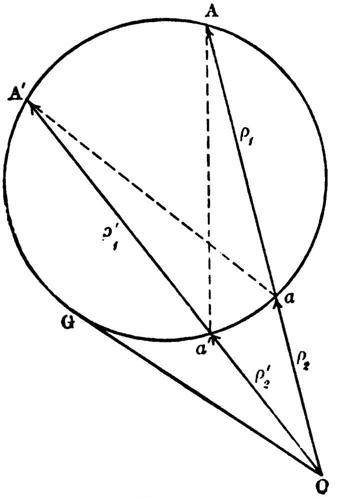

Fig. 1.
Reciprocal Vectors, ρ_1 and ρ_2.

If any vector has an arc of a circle for its locus, a vector proportional to its reciprocal will have an arc of a circle for a locus. In Fig. 1, let ρ_1 be any vector from the origin O, having its locus as shown upon the arc of a circle. The vector ρ_2, drawn in the direction of ρ_1, and proportional to its reciprocal, will have its locus upon an arc of a circle, which may be shown as follows. Let ρ_1 and ρ_1' represent the vector in any two positions, OA and OA′. The intercepts O*a* and O*a*′ will represent the reciprocal vectors ρ_2 and ρ_2'; for, in the similar triangles OA′*a* and OA*a*′,

$$\rho_1 : \rho_1' :: \rho_2' : \rho_2.$$

Hence $\rho_1'\rho_2' = \rho_1\rho_2 =$ a constant.

The value of this constant product of ρ_1 and ρ_2 is $\overline{OG}^2$.

By a suitable selection of scale, the loci of ρ_1 and its reciprocal ρ_2 may be represented as arcs of the same circle, as in Fig. 1; or they may be represented by different circles, as in Fig. 2. In the latter case,

$$\rho_1\rho_2 = OG_1 \cdot OG_2 = \text{a constant}.$$

As the origin O approaches the circle which represents the locus of ρ_1, the center of the reciprocal circle becomes more distant, and its radius becomes greater. When the origin O is a point in the circumference of the first circle, the center of the reciprocal circle is at an infinite distance; that is, the reciprocal locus is a straight line.

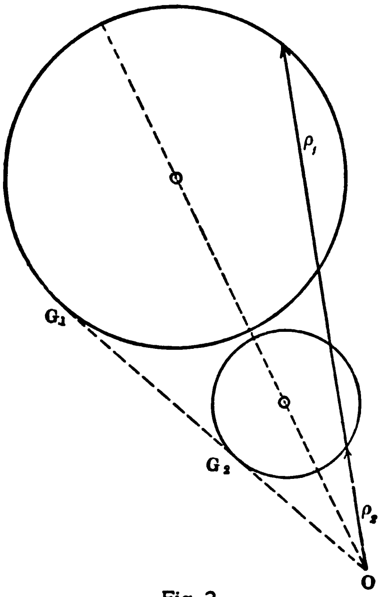

Fig. 2.
Reciprocal Vectors.

Let us apply these principles to the transformer diagrams. The locus of the primary impedance, as some particular quantity is varied, is a portion of a circle. For example, this may be shown to be the case when the secondary resistance is varied. Since the admittance of the primary is the reciprocal of its impedance, the admittance may be represented by the vector ρ_2 in the above construction, if the impedance is represented by ρ_1. These loci may be drawn to scale for actual values.

In a constant current transformer, the primary electromotive force varies directly as the primary impedance. In a constant potential transformer the primary current varies directly as the primary admittance. But the admittance is the reciprocal of the impedance; hence if we have an arc of a circle for the locus of the primary electromotive force when the primary current is maintained constant, we may employ the above method to obtain the arc of a circle which will be the locus for the primary current when the transformer is supplied with a constant electromotive force. The converse operation may likewise be performed.

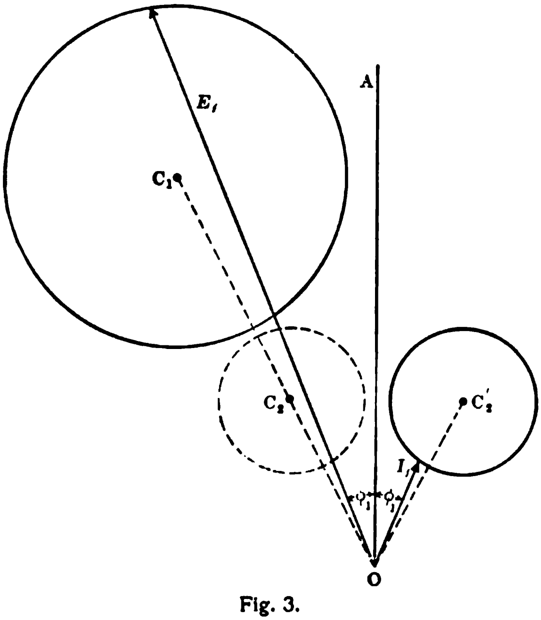

Fig. 3.

In Fig. 3, let the circle C_1 represent the locus of the primary electromotive force E_1 during some particular change of condition, the primary current meanwhile being maintained constant and in the direction OA. The difference in phase between the current and electromotive force is the angle ϕ_1. The locus of the primary current under the same change of conditions, if the primary electromotive force is maintained constant, is the dotted circle C_2, which is reciprocal to C_1. If the constant electromotive force is drawn in the direction OA, the locus of the primary current is the circle C_2', drawn so that the angles AOC_1 and AOC_2' are equal.

An application of the method of reciprocal vectors is shown in Fig. 4. Positive rotation is counter-clockwise. The semicircle JKN represents the locus of the primary electromotive force of a transformer, when the primary current is constant and is in the direction of OA, and the secondary resistance is varied.

The electromotive force has the position OJ on open circuit, and ON on short circuit. OH is the power electromotive force on open circuit, and includes the effects of primary resistance and the losses due to hysteresis and eddy currents; HJ is the electromotive force to overcome the primary self-induction. These are in the direction of the primary current, and at right angles to it, respectively. A line from J to K, at right angles to the secondary current, would show the reaction of the secondary upon the primary. It is to be noted that the line NH represents the effects due to magnetic leakage. It is desired to find the locus for the primary current when the primary electromotive force has a constant value, and is drawn in the direction OA. The angle of lag θ_1, between the primary electromotive force and current on open circuit, is JOH. Accordingly, with a constant electromotive force in the direction OA, the open circuit current I_0 is laid off lagging behind the electromotive force at an angle of $AOj' = \theta_1 = JOH$. The open circuit current I_0 may be laid off to any convenient scale. To construct the locus for the primary current, proceed as follows: Lay off the line OC_2 so that the angles AOC_1 and AOC_2 are equal. The point C_2

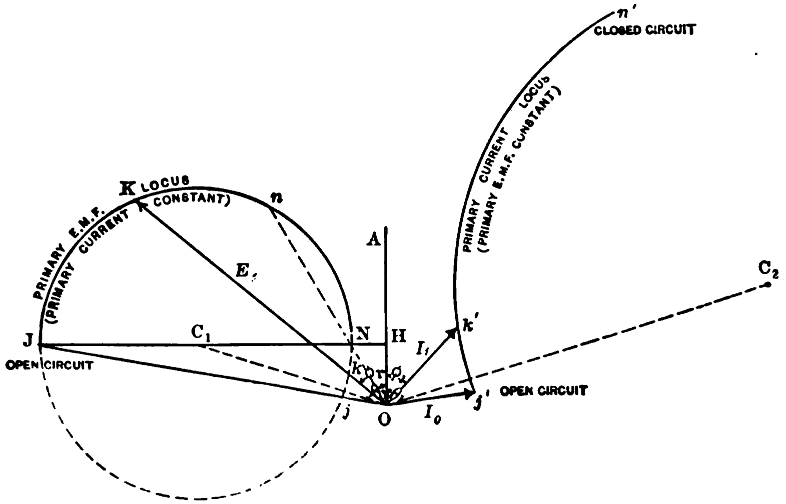

Fig. 4.
Method of obtaining Primary Current Locus by the Principle of Reciprocal Vectors.

is located so that $OC_2 : OC_1 :: Oj' : Oj$. The primary current locus is then drawn as the arc of a circle with C_2 as a center, passing through j'.

The limits of the primary electromotive force locus are the points J and N. The corresponding limits of the primary current locus are the points j' and n'. It will be noted that these points correspond to the points j and n on the circle C_1, which are reciprocal to the points J and N.

In the absence of magnetic leakage, the points N and H coincide. The point n' would then lie in the line OA. The deviation of the primary current locus from the line OA is produced by magnetic leakage.

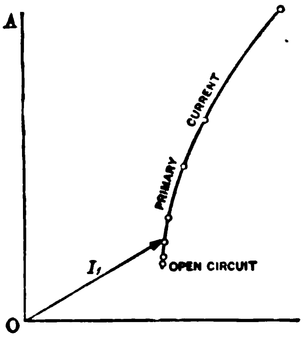

Fig. 5.

Primary Current Locus for Constant Potential Transformer; Determined Experimentally.

An experimental curve showing the primary current locus for a constant potential transformer, as affected by magnetic leakage, is shown in Fig. 5.

The reciprocal relation between admittance and impedance vectors gives a simple method for determining the conditions for consonance and resonance in transformer circuits.[1]

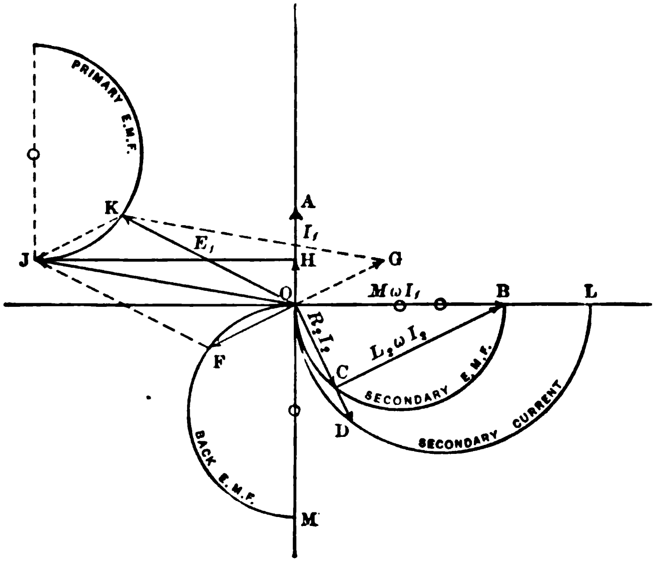

Fig. 6.

Effect of the Variation of the External Secondary Self-Induction in a Constant Current Transformer.

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[1] Resonance in Transformer Circuits, by Bedell and Crehore, the PHYSICAL REVIEW, Vol. II., p. 442.

Figure 6 is given as a particular instance in illustration of the statement given above that loci produced by the variation of any one constant are usually arcs of circles. The primary loci are always arcs of circles. The diagram shows the changes due to a variation in the secondary self-induction.

Figure 7 shows the effect of magnetic leakage. The curves shown are loci for primary electromotive force, when the primary current is I_1. The primary electromotive force is composed of the components OH, to overcome ohmic resistance and supply open circuit losses, HJ to overcome self-induction, and (with absence of magnetic leakage) JK_0 to overcome the back electromotive force of mutual induction.

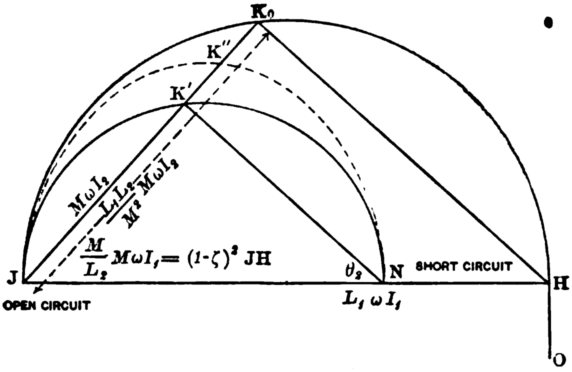

Fig. 7.

Effect of Magnetic Leakage.

The semicircle $\mathrm{JK}_0\mathrm{H}$ is the locus for the primary electromotive force in the absence of magnetic leakage. The semicircle JK'N is the locus for the primary electromotive force when the coefficient of magnetic leakage ζ is constant from open circuit to short circuit. In this case we have the relations

$$\frac{\mathrm{JK}'}{\mathrm{JK}_0} = \frac{\mathrm{JN}}{\mathrm{JH}} = \frac{M^2}{L_1L_2} = (1-\zeta)^2;$$

$$1-\zeta = \sqrt{\frac{\mathrm{JK}'}{\mathrm{JK}_0}} = \sqrt{\frac{\mathrm{JN}}{\mathrm{JH}}};$$

$$\zeta = 1 - \sqrt{\frac{\mathrm{JK}'}{\mathrm{JK}_0}} = 1 - \sqrt{\frac{\mathrm{JN}}{\mathrm{JH}}}.$$

In an actual transformer the magnetic leakage is not constant, but varies with the load. The locus represented by the dotted curve JK''N is for such a case in which the magnetic leakage is zero on open circuit and increases to the maximum at short circuit. Where the magnetic leakage is variable, it is determined for any point as K″ by the ratio of JK″ to JK_0.

$$\zeta = 1 - \sqrt{\frac{JK''}{JK_0}}.$$

Thus let us suppose that the back electromotive force JK″, actually induced in the primary by the secondary current I_2, is eighty-one volts; and that JK_0, which would be the back electromotive force in the absence of magnetic leakage, is 100 volts. We then have the equation

$$1 - \zeta = \sqrt{\frac{81}{100}} = \frac{9}{10} = 0.9,$$

which indicates that the mutual induction is nine-tenths of the value it would have in the case of no magnetic leakage. The coefficient of magnetic leakage is accordingly found to be 10 per cent, thus:

$$\zeta = 1 - 0.9 = 0.10.$$

Figure 7 is drawn to represent the values of the various electromotive forces in the primary circuit of a transformer, for a given value of the primary current and for different values of the secondary resistance. If the magnitude of each line is divided by the primary current I_1, Fig. 7 represents the values for the primary impedance (without any assumption as to constant current or electromotive force) for different values of the secondary resistance. The effect of magnetic leakage upon the primary impedance of any transformer is thus shown for different values of the secondary resistance.

The above construction affords a simple method of studying the conditions for a decrease of the primary impedance of a transformer when the secondary circuit is closed.[1]

Curves corresponding to the dotted curve in Fig. 7 have been experimentally determined[2] by the writer.

Cornell University, May, 1896.

Visible Electric Waves.

By B. E. Moore.

SOON after Hertz detected electric waves by means of a spark gap oscillator,[3] other methods were devised. A Geissler tube was substituted for the spark-gap by Dragoumis.[4] The bolometer has furnished the most elegant and best adopted method for accurate determinations of wave-lengths in wires.[1] Having no bolometer the author concluded he would place a Crookes tube upon the free end of the wires, to show the waves to a class. A large induction coil was tried for the purpose of exciting the primary vibrations, but found not to be in working condition. Little hope was entertained of obtaining any results from a Toepler-Holtz machine, but the venture was made.

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[1] Discussed at length before the Physical Society by Mr. E. C. Rimington in his paper on Air-core Transformers, Philosophical Magazine, Vol. 37, p. 394.

[2] Proceedings of the International Electrical Congress, Chicago, 1893, p. 234.

[3] Wiedemann's Annalen, Vol. 31, p. 421, 1887, and subsequent volumes, or Hertz Electric Waves, translated by D. E. Jones.

[4] Nature, Vol. 39, p. 548.

The apparatus was arranged as shown in the diagram.[2] *a* and *b* are the terminals of a quadruple-plate Toepler-Holtz machine. *c* and *c'* are each three-quart Leyden jars, carefully insulated. The outside coatings of these jars were connected to the terminals of the machine. The inside coatings of the jars were connected to a Crookes tube, *t*, by means of the wires *E* and *E'*. The latter were 20 cm. apart and each 7.4 m. long. The Crookes tube, *t*, which gave the brightest flash upon discharge contained some phosphorescent material. The vacuum tube was 4 cm. in diameter, and the distance between its terminals was 26 cm. The vacuum must have been a high one as the condensers would discharge through a 5 cm. air gap in preference to discharging through the vacuum.

With the apparatus arranged as above described and a powerful induction coil substituted for the Toepler-Holtz machine, one can locate the nodes of the electric waves by simply sliding a rider, *r*, along the wires. As was rather expected, the rider placed at any point whatever along the wire obliterated the discharge through the Crookes tube. The room was darkened, with the hope that a slight glow could be obtained when the rider rested upon the node points. This led to an unexpected result. When the discharge between *a* and *b* occurred, the wires were aglow with the electric waves. (There was no rider across *E* and *E'*.)

The observer working the machine sees, at every discharge of the primary, the wires momentarily illuminated. But all parts of the wire are not illuminated alike. At some points of the negative wire, the circle of illumination is little larger than the wire itself. Halfway between two such points, the circle of illumination is often 4 cm. in diameter. The whole presents the appearance of a solid sine wave — not unlike the appearance of circularly vibrating strings.

Looking at these waves from the side instead of from the end, the phenomena are different. The nodes and crests are distinct as before, but the illumination no longer appears continuous. In a plane normal to the negative wire, a fine illuminated rectilinear discharge, in the form of rays or pencils, occurs. These rays are often branched. They radiate from the wire as a center, and form a circle about the same, often 4 cm. in diameter at the crest of the wave. At a distance of 6 mm. to 10 mm. on either side of the crest appears another such illuminated circle. These circles diminish in size as the node is approached. The circumferences of these circles form the solid sine curve.

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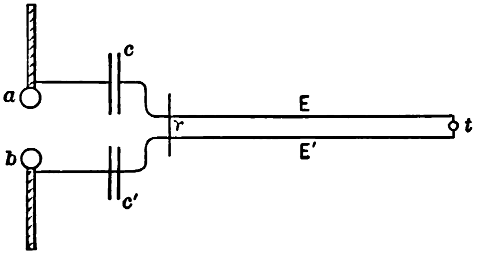
[1] Rubens and Ritter, Wiedemann's Annalen, Vol. 40, p. 55, and subsequent volumes.

[2] See p. 152.

The positive discharge was entirely different. The crests and nodes upon the positive wire were only apparent to a close observer, and were not distinguishable at all until the room was well darkened and a wood screen so placed as to cut off the light radiated by the Crookes tube. The circles of illumination, so apparent upon the negative wire, were entirely absent. The illumination of the wire was continuous, and confined almost to the surface of the wire. The light emitted by the crest of the wave seemed only more intense than that emitted by the node. Only those who had some experience in photometric measurements could easily detect a difference. The waves along the negative wire can be made visible to an entire audience by cutting off the light of the Crookes tube. The eyes should be protected from the spark of the primary discharge. However, shielding the wire from the light of the discharge is detrimental.

The negative wire was used for determination of wave-lengths. With a spark-gap of 47 mm. there was one node at the middle of the wire. On diminishing the striking distance no nodes appeared until a distance of 30 mm. was reached. There were now two nodes at a distance of 4 m. from each other, and nearly symmetrically located as to the ends of the wire. At a striking distance of 23 mm. the nodes appeared again. The effect was now more beautiful. A fundamental wave, whose half-wave-length varied from 180 cm. to 185 cm., was obtained, upon which were superimposed at times two other waves, corresponding to the first and second octave. Measurement showed their half-wave-lengths to be 87 cm. to 92 cm., and 40 cm. to 45 cm., in length respectively. These harmonics would die out and in their place would appear a half-wave varying in length from 58 cm. to 62 cm. Measurements at this striking distance were easily made when the waves were once established. However, there seemed to be something necessary more than obtaining a spark-gap of 23 mm. Only after long-continued effort could they be duplicated. The fundamental wave was easily obtained. At the longer striking distances the nodes shifted frequently 20 cm. Coincident with this change occurred a decided difference in the tone of the primary discharge. A little observation showed that the spark of the primary discharge was longer when the report was less decisive. When the spark-gap was diminished to 17 mm., half-wave-lengths varying from 45 cm. to 53 cm. were obtained. The discharges of the primary followed each other now very rapidly, and the phenomenon became very pleasing to spectators. The node points varied with every change in the sound of the primary discharge. Yet this change was not so distinct that all or even a greater part of the observers could

see it when their attention was called to it. When the spark-gap was reduced to 10 mm. or 12 mm., a shower of sparks occurred. It then became impossible to locate nodes. Over half of the wire was not illuminated. However, in the neighborhood of the crests the discharge was brilliant and nearly as large as in the longer waves. The successive rings died out so rapidly that one could have located the top of the crests within one mm., had they not shifted so much along the wire with the ever varying length of the spark between electrodes. The machine used consisted of two parts, upon one of which the driving belt slipped considerably. This enabled one to obtain a very beautiful effect. The sparks would follow each other at about the rate of five to a second, and then cease for about the same period. Every time the sparks ceased there would occur a splash at the crest of the wave.



The polarity of the machine would change nearly every time it was stopped. This action was attributed to the condensers. By the arrangements of the circuit they would charge the machine every time it was set in motion. As a result the negative discharge was continually changing wires.

One wire was now removed. The condenser to which this wire was attached and the corresponding side of the Crookes tube were grounded. The half-wave-lengths were again determined and found to differ in no respect from those already mentioned. One inconvenience arises, however, with this arrangement: since it is desirable to have the wire with which one is working negative, it becomes generally necessary, after stopping the machine, to start it, then to stop and to start it again, before one can get the wire charged the same as at the beginning.

This method of showing electric waves is particularly serviceable for lecture purposes. A general audience can more highly appreciate it, as no process of inductive reasoning is involved. The waves stand out vividly and are no abstractions.

UNIVERSITY OF ILLINOIS, June, 1896.

NOTE ON THE REFRACTIVE INDEX OF WATER AND ALCOHOL FOR ELECTRICAL WAVES.

By John F. Mohler.

In reviewing an article by A. D. Cole on the "Refractive Index and Reflecting Power of Water and Alcohol for Electrical Waves,"[1] Professor Rowland called my attention to the fact that in the first part of the paper the author had ignored an important factor introduced by the absorption of the medium; a factor which in the latter part of the article he shows to be very prominent.

As one object of the experiment is to prove or disprove Maxwell's electro-magnetic theory of light, I will begin with his equations for a plane wave:

$$K\mu\frac{d^2F}{dt^2} + 4\pi\mu C\frac{dF}{dt} = \nabla^2 F, \tag{1}$$

$$K\mu\frac{d^2G}{dt^2} + 4\pi\mu C\frac{dG}{dt} = \nabla^2 G, \tag{2}$$

$$K\mu\frac{d^2H}{dt^2} + 4\pi\mu C\frac{dH}{dt} = \nabla^2 H, \tag{3}$$

where K is the specific inductive capacity, μ the magnetic permeability, F, G, and H the displacement in the direction of the three axes, t is the time, and C the conductivity.

If we consider a plane wave propagated in the direction of the axis of x, F becomes zero, and a solution of (2) is

$$G = Ae^{-hx+ib(vt-x)}, \tag{4}$$

with a similar solution for H. In this equation $i=\sqrt{-1}$, $b=\frac{2\pi}{\lambda}$, and b is the dampened velocity of the disturbance in the medium.

Substituting (4) in (2) and equating the real and imaginary parts, we have

$$-K\mu b^2v^2 - h^2 + b^2 = 0, \tag{5}$$

$$4\pi\mu Cbv - 2hb = 0, \tag{6}$$

or

$$h = 2\pi\mu Cv, \tag{7}$$

and

$$v^2 = \frac{b^2}{b^2K\mu + 4\pi^2\mu^2C^2}. \tag{8}$$

If $C=0$, *i.e.* if the medium is non-conducting, $v^2 = \frac{1}{K\mu} = V^2$, the velocity in the medium if there is no absorption.

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[1] Physical Review, Vol. IV., p. 50, 1896; Wied. Ann., 57, p. 290, 1896.

However, C is *not* zero, and from equation (5) we have

$$v^2 = \left(1 - \frac{h^2}{b^2}\right) V^2. \tag{9}$$

Now to find the meaning and value of the constant h, we can write equation (4) in the form

$$G = Ae^{-hx} \cos b(vt - x), \tag{10}$$

Which shows that e^{-hx} is the damping factor. Mr. Cole's experiments show that the damping for both water and alcohol was very great for short waves. For 0.4 cm. of water the absorption was 33 per cent of the incident radiation, therefore $0.67 = e^{-\frac{2}{5}h}$, which gives for h the value 1.014.

$$b = \frac{2\pi}{\lambda}, \text{ and in his exponent } \lambda = 5 \text{ cm.}$$

$$\therefore \qquad b = 1.2566;$$

$$\text{then} \qquad \frac{h^2}{b^2} = \left(\frac{1.014}{1.2566}\right)^2 = 0.6511,$$

$$v = \sqrt{1 - \frac{h^2}{b^2}}\, V = 0.59\, V.$$

This shows that if the first method of experiment had been used with waves 5 cm. long, the measured distance between the bridges would have been shortened 41 per cent *by absorption*, and the value of the index of refraction would have been 1.61 times what it ought to have been. He did not use this method, however, for finding the index of refraction for short waves, but the experiment shows that there was also quite strong absorption for the long waves. He could find only four bridge positions, three half-wave-lengths for water, and for alcohol only *one* half-wave-length could be measured.

His results by the first method, then, would give values for the index of refraction that are too large, the error depending on the absorption of the energy by the medium. His results, therefore, which give a refractive index of 5.24 for alcohol for long waves and 3.2 for short waves, do *not* prove that the refractive index of alcohol is greater for long than for short oscillations. Neither does his work prove that the index of refraction of water is the same for long and short waves. An experiment to determine the absorption of water and alcohol for the long waves used will be necessary in order to find out the factor with which to correct the values given in the first part of the paper.

DICKINSON COLLEGE, CARLISLE, PA.

NEW BOOKS.

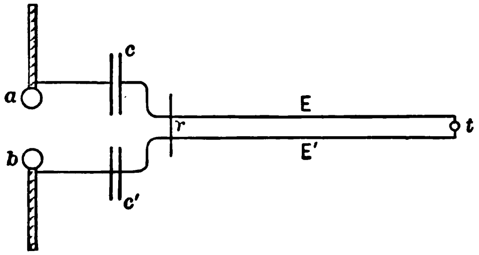
Theoretical Chemistry from the Standpoint of Avogadro's Rule and Thermodynamics. By W. Nernst. Translated by C. S. Palmer. Macmillan, 1895.

To all who are interested in physical chemistry this work, in one of its German forms, must already be familiar. It first appeared as an introductory section in the first volume of Dammer's *Anorganische Chemie*, the preface bearing the date May, 1891. Two years later it was brought out as a separate volume, after having been somewhat changed and enlarged. The preface to this edition is dated April, 1893. The translation before us appeared after nearly two years more, Professor Palmer dating his preface December, 1894. The latest work finds a place in an appendix which consists of extracts from Professor Nernst's articles on "Physical Chemistry" in Meyer's *Jahrbuch der Chemie.*

Professor Nernst's book meets, partially, the need of a text-book of physical chemistry for students who have passed the stage of Ostwald's *Grundriss*, and who are familiar with the elements of calculus and of thermodynamics. It does not aim at the completeness of the second edition of Ostwald's large *Lehrbuch der Allgemeinen Chemie*, and it thereby avoids some of Ostwald's diffuseness. Looked at by itself, the book seems to treat its subjects with very unequal fulness, but when we consider the time of its appearance, we understand the arrangement better. The great need was for a treatment of chemical mechanics, since Ostwald's volume on that subject was not to be expected immediately (indeed it has not yet appeared), while his first edition was out of date. The fullest parts of Professor Nernst's work are, whether for this reason or not, those devoted to chemical mechanics. It might have been expected that electro-chemistry, upon which no one is better qualified to write than Professor Nernst, would be treated in detail. We find, however, this section hardly more than outlined in a single chapter of only twenty-six pages. This may, possibly, have been due to the fact that Ostwald's volume on electro-chemistry was soon to appear. Ostwald's work, when it did come out, was so carelessly and unsatisfactorily done that it is much to be regretted that Professor Nernst limited his treatment of electro-chemistry to so small a compass.

The work begins with an introductory chapter devoted to the statement of a few general laws and principles, including the two laws of thermody-

see it when their attention was called to it. When the spark-gap was reduced to 10 mm. or 12 mm., a shower of sparks occurred. It then became impossible to locate nodes. Over half of the wire was not illuminated. However, in the neighborhood of the crests the discharge was brilliant and nearly as large as in the longer waves. The successive rings died out so rapidly that one could have located the top of the crests within one mm., had they not shifted so much along the wire with the ever varying length of the spark between electrodes. The machine used consisted of two parts, upon one of which the driving belt slipped considerably. This enabled one to obtain a very beautiful effect. The sparks would follow each other at about the rate of five to a second, and then cease for about the same period. Every time the sparks ceased there would occur a splash at the crest of the wave.



The polarity of the machine would change nearly every time it was stopped. This action was attributed to the condensers. By the arrangements of the circuit they would charge the machine every time it was set in motion. As a result the negative discharge was continually changing wires.

One wire was now removed. The condenser to which this wire was attached and the corresponding side of the Crookes tube were grounded. The half-wave-lengths were again determined and found to differ in no respect from those already mentioned. One inconvenience arises, however, with this arrangement: since it is desirable to have the wire with which one is working negative, it becomes generally necessary, after stopping the machine, to start it, then to stop and to start it again, before one can get the wire charged the same as at the beginning.

This method of showing electric waves is particularly serviceable for lecture purposes. A general audience can more highly appreciate it, as no process of inductive reasoning is involved. The waves stand out vividly and are no abstractions.

University of Illinois, June, 1896.

NOTE ON THE REFRACTIVE INDEX OF WATER AND ALCOHOL FOR ELECTRICAL WAVES.

By John F. Mohler.

IN reviewing an article by A. D. Cole on the "Refractive Index and Reflecting Power of Water and Alcohol for Electrical Waves,"[1] Professor Rowland called my attention to the fact that in the first part of the paper the author had ignored an important factor introduced by the absorption of the medium; a factor which in the latter part of the article he shows to be very prominent.

As one object of the experiment is to prove or disprove Maxwell's electro-magnetic theory of light, I will begin with his equations for a plane wave:

$$K\mu\frac{d^2F}{dt^2}+4\pi\mu C\frac{dF}{dt}=\nabla^2F, \tag{1}$$

$$K\mu\frac{d^2G}{dt^2}+4\pi\mu C\frac{dG}{dt}=\nabla^2G, \tag{2}$$

$$K\mu\frac{d^2H}{dt^2}+4\pi\mu C\frac{dH}{dt}=\nabla^2H, \tag{3}$$

where K is the specific inductive capacity, μ the magnetic permeability, F, G, and H the displacement in the direction of the three axes, t is the time, and C the conductivity.

If we consider a plane wave propagated in the direction of the axis of x, F becomes zero, and a solution of (2) is

$$G=Ae^{-hx+ib(vt-x)}, \tag{4}$$

with a similar solution for H. In this equation $i=\sqrt{-1}$, $b=\frac{2\pi}{\lambda}$, and b is the dampened velocity of the disturbance in the medium.

Substituting (4) in (2) and equating the real and imaginary parts, we have

$$-K\mu b^2v^2-h^2+b^2=0, \tag{5}$$

$$4\pi\mu Cbv-2hb=0, \tag{6}$$

or

$$h=2\pi\mu Cv, \tag{7}$$

and

$$v^2=\frac{b^2}{b^2K\mu+4\pi^2\mu^2C^2}. \tag{8}$$

If $C=0$, *i.e.* if the medium is non-conducting, $v^2=\frac{1}{K\mu}=V^2$, the velocity in the medium if there is no absorption.

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[1] Physical Review, Vol. IV., p. 50, 1896; Wied. Ann., 57, p. 290, 1896.

chung *unterbleiben*," the sense of the translation being opposite to that of the original. At p. 250, line 31, the German "in*tra*molekularen" is rendered by "in*ter*molecular." This mistake, which occurs in other places, does not speak well for the translator's familiarity with either Chemistry or Latin. At p. 473, line 28, there is a curious admission of inability to translate "tagelang," which means "for days." At p. 532, line 35, we have "was *fortunately* discovered by Roozeboom in an investigation," etc. The German being "wurde von Roozeboom *gelegentlich* einer Untersuchung . . . aufgedeckt." We can, on demand, cite other cases as bad as these, but more would be superfluous here. To appreciate the force of our remark on the English one must read the translation. The continued misuse of shall and will is, to our ears, one of the most unpleasant faults.

We are sorry to have to say this of a translation which we awaited so impatiently; for, in spite of its defects, we use it constantly as a text-book for students who are not familiar with German. We must, however, recommend the original of 1893 (F. Enke, Stuttgart) to all who can read it.

EDGAR BUCKINGHAM.

BRYN MAWR.

Die Spectralanalyse. Dr. JOHN LANDAUER. pp. vii + 174. Braunschweig, Vieweg, 1896.

This little treatise is a reprint of a cyclopedia article. Its usefulness to the physicist will nevertheless be found to be considerable on account of the very full biographical references, and because we have here brought together into compact form tabular data concerning the spectra of nearly all the elements. These tables are made up from nearly all the available sources, including such of Rowland's data as were obtainable at the time that the book was compiled. It was the author's misfortune to have completed his work just before the publication of Rowland's later table of wave-lengths, which would have added greatly to the value of the volume.

Wave-lengths are denoted in accordance with the system generally adopted by spectroscopists in America, which places the decimal point so as to express wave-lengths in ten-millionths of a millimeter. The wisdom of the adoption of this system over others which have been in vogue is not altogether clear; but uniformity of practice is the first consideration, and it is gratifying, therefore, to find this work in conformity with the system which a majority of writers have agreed to use. The author is not entirely consistent in this matter, however. He uses occasionally the system suggested by Kayser, which pushes the decimal point one place further to the left. Thus on p. 46 we find the wave-lengths of the visible spectrum spoken of as lying between $400\mu\mu$ and $760\mu\mu$. The diagrams of absorption spectra on pp. 51 and 130 are also marked in accordance with that system. Fortunately no confusion is likely to arise. The range of values occupied by the wave-lengths with which the spectroscopist has to deal is not sufficient to make it possible to mistake one region for another. It is to be regretted that the system of designating wave-lengths in all works should not conform to the plan of using *millionths of a centimeter* as a basis, rather than some other multiple.

The illustrations used in this little work are for the most part old and familiar to the eyes of all readers. The historical sketch with which the volume opens is, however, a very satisfactory one, and the descriptions of the development of the science are terse and for the most part accurate.

Dr. Landauer is to be congratulated upon having brought so much that is serviceable together in so small a space.

E. L. Nichols.

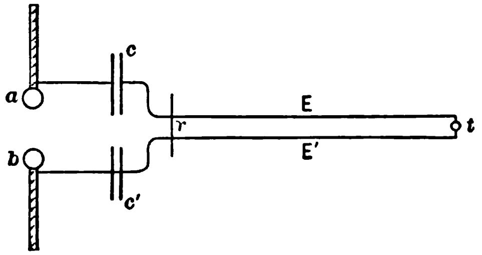
The Principles of Physics. By A. P. Gage. 8vo. pp. ix, 634. Boston, Ginn & Co., 1895.

Although this work is an outgrowth of Gage's *Elements of Physics*, so long and favorably known as an elementary text-book, it has been so thoroughly revised, and so greatly enlarged, as to constitute practically a new book. The first aim of the author has been to present the subject in a form suitable for the students of our best high schools and academies; but sufficient additional matter has been provided, in small type, to make the book available for the non-technical students of many colleges.

Such a work must of necessity be largely descriptive. Scientific education in this country has not yet reached such general development that we may expect to find the equipment of all our high schools adequate to a course of experimental lectures or laboratory work. To provide, as far as possible, for the case of schools whose equipment is insufficient, the author has introduced a considerable amount of purely descriptive matter, as well as numerous illustrative experiments. These additions account for the fact that the volume before us is noticeably larger than its predecessor. The phenomena and experiments described have been chosen, however, with great care, and are presented in direct and logical connection with the principles which they are to illustrate. While the book contains sufficient suggestive matter to convince the student that there is something to be learned besides what is contained in his text-book, it is by no means open to the objection of being bulky or diffuse. There are several text-books in use at the present time which are not free from these faults.

It is hardly to be expected that all teachers should agree as to exactly

see it when their attention was called to it. When the spark-gap was reduced to 10 mm. or 12 mm., a shower of sparks occurred. It then became impossible to locate nodes. Over half of the wire was not illuminated. However, in the neighborhood of the crests the discharge was brilliant and nearly as large as in the longer waves. The successive rings died out so rapidly that one could have located the top of the crests within one mm., had they not shifted so much along the wire with the ever varying length of the spark between electrodes. The machine used consisted of two parts, upon one of which the driving belt slipped considerably. This enabled one to obtain a very beautiful effect. The sparks would follow each other at about the rate of five to a second, and then cease for about the same period. Every time the sparks ceased there would occur a splash at the crest of the wave.



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[1] PHYSICAL REVIEW, Vol. IV., p. 50, 1896; Wied. Ann., 57, p. 290, 1896.

gators to secure materials much more nearly pure than those formerly available, and, in this respect at least, recent work should carry more weight than that of former investigators. In addition to this uncertainty as to the purity of the metals used, attention should be called to the condition and previous treatment of the metals. We note determinations made upon metals powdered, in small fragments, crystalline, cast into small or large masses, drawn into rods or wires, or rolled into sheets. It is well known that molecular structural changes produced by mechanical operations produce a very perceptible change in the specific heat of a metal. Results obtained from metals which have been subjected to such diverse treatment are, therefore, not properly comparable. It is also probable that determinations made by use of powdered metal are somewhat higher than they should be, owing to the effect of the heated air carried into the calorimeter cup among the small fragments.

II.

The Determination of Temperatures.

In this we have a source of error, common to all calorimetric methods, which is undoubtedly the cause of many of the discrepancies observed in the results obtained. All the difficulties involved in the exact use of mercurial thermometers are concerned in this. It would appear that undue reliance has been placed upon the degree of accuracy obtainable by the correction of the readings of the thermometers which have generally been used. This holds true especially in the case of thermometers used for determining temperatures somewhat above or below room temperature when a correction for stem exposure has been applied. The methods employed for standardizing thermometers have also often given untrustworthy results, thus introducing constant errors.

III.

The Methods Employed.

Of the many calorimetric methods that have been devised, those most worthy of consideration are based upon the heat equivalent of fusion of ice, the heat equivalent of vaporization of water, and the change in temperature produced in a liquid by the immersion of a body of different temperature.

The first of these methods, commonly known as the method of melting ice, has been best applied by use of the well-known form of Bunsen's ice calorimeter. It is unfortunate that this calorimeter, which appears theoretically so satisfactory, is found to be deficient in accuracy in operation. The sources of error, exclusive of errors of thermometry common to all methods, are chiefly those arising from the dependence of the results upon the calibration of the instrument by use of previous determinations of the specific gravity of ice, and those arising from uncertain "rate" of motion of the mercury in the capillary tube. As to the former, it is obvious that the differing results obtained by different investigators, varying from 0.905 to 0.950, suggest doubt as to the accuracy of results based upon a quantity the value of which appears so uncertain. As to the latter, the effect of slight impurities in the snow or ice surrounding the calorimeter is to cause such a constant deposition of ice on the ice cylinder as to cause the mercury thread to move many scale divisions in a minute. This motion, even under favorable conditions, is not exactly proportional to the time, and often occurs in a very irregular manner, thus rendering uncertain the confidence which may be placed in the indications of the mercury thread when corrected for "rate" of motion. This calorimeter is also subject to error in failure to exclude absorbed air from the instrument when first filled. This can be done if the operation is conducted with great care, but it is doubtful whether operators have usually been successful in this respect. The fact that the above sources of error have rendered the results obtained by use of this calorimeter somewhat doubtful is rendered evident by a review of the best results obtained. We note that the results obtained by different investigators are not concordant, that they do not agree well with results obtained by other methods, and that even the results obtained by the same person using the same material on different occasions are not concordant. Attention will be called to this later. At present it may be sufficient to cite the results obtained by Bunsen for calcium, — 0.1722 and 0.1688, —

showing a difference of 2 per cent in the values as determined by one presumably thoroughly conversant with the best methods of procedure with this calorimeter. It should, however, be borne in mind that this method was designed especially for the determination of the specific heats of the rare elements, of which but a small quantity may be obtained in a pure state, and for this purpose it has been most frequently applied.

Upon the heat equivalent of vaporization of water has been based the method of condensation as applied in the gravimetric calorimeter devised by Dr. J. Joly. The principle applied is simple. If a given mass of a substance at known temperature be plunged into steam, it experiences a rise in temperature and absorbs a number of calories of heat from the steam which is condensed upon its surface. The heat equivalent of vaporization of water at the given temperature being known, we have to determine the temperatures and the weights of the given body, before and after the condensation of steam upon its surface, to obtain the necessary data. The most important conditions of operation are thus: Sudden replacement of the air surrounding the substance by the vapor to be condensed; perfect mechanical conditions permitting the evaluation of the weight condensed while the substance is still surrounded by the vapor. In practice, however, several corrections must be applied. The weight of steam condensed upon the bucket which holds the given substance must be determined from preliminary experiments for the given temperature range and subtracted from the total weight of steam deposited. Correction to the weight of the body when immersed in steam must be made, owing to the different densities of steam and air. Exclusive of errors of thermometry this method is, therefore, subject to the following sources of error: Errors arising from the application of the correction factors; loss of water condensed upon the bucket which may drop from time to time; variations produced by deposition of water upon the suspension wire at the point at which it enters the steam chamber; a constant dependence upon the accuracy of the values previously determined for the heat equivalent of vaporization of water at the given temperature. It is evident, therefore, that although results obtained by this method may agree quite closely among themselves, there may be constant errors in a series of observations which may render the results less worthy of confidence than the method might at first promise.

The third method, commonly known as the method of mixtures, was first successfully applied by Victor Regnault, and, as described in his classic researches, it deserves especial attention. This method, as applied by Regnault and his followers, together with its modifications, has yielded the best results thus far obtained in all cases in which a considerable mass of pure metal might be obtained. This fact is readily attested by a comparison of the greater uniformity of the results obtained by different observers employing this method at various times as compared with results obtained by other methods. The facilities available for his work, and the extraordinary ability in overcoming experimental difficulties which he possessed, render Regnault's work admirable in many respects. Without entering into details as to the familiar apparatus used and its operation, it is perhaps sufficient to say that probably the greatest source of error in the application of this method is found in the calculation of the radiation correction. This correction factor has been applied by Regnault's followers in practically the same manner as he applied it. For the purpose of avoiding the necessity for the determination of this factor, and consequently any errors which may arise from it, Prof. M. N. Hesehus[1] has suggested that the calorimeter cup be introduced into the bulb of an air thermometer and maintained at a constant temperature by the introduction of a sufficient amount of cold water, of known temperature, immediately after the introduction of the heated solid. By this means the correction for radiation and that due to the "water equivalent" of the calorimeter cup are avoided. The heat received by the cold water being equal to that given out by the heated body, the following simple equations may be used: —

$$MS(T-\theta) = ms(\theta - t),$$

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[1] Journal de la Société Physico-chemique Russe, Nov. 1887; Journal de Physique, tom. vii., p. 489, 1888.

in which

M = mass of substance, the specific heat of which is to be determined.
m = mass of cold water introduced into the calorimeter cup.
T = temperature of the heated body.
θ = initial temperature of the calorimeter cup.
t = temperature of cold water introduced into the calorimeter cup.
s = mean specific heat of water at temperature used.
S = specific heat sought.

From this equation we have

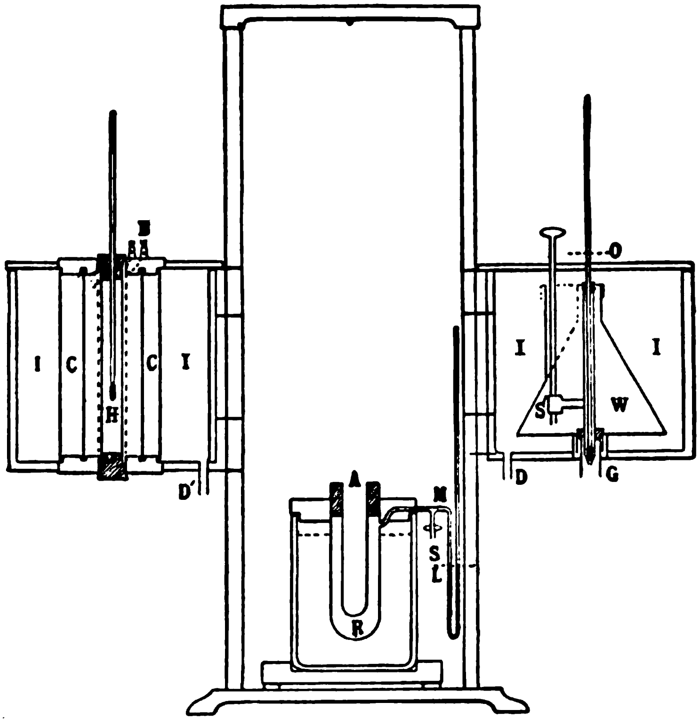
$$S = \frac{ms(\theta - t)}{M(T - \theta)},$$

the working equation used.

The results obtained by this method, however, as submitted by Professor Hesehus, do not appear tc be as accurate as those obtained by the method of mixtures as practiced by Regnault. The ten determinations given of the specific heat of brass varied from 0.0821 to 0.0969, or, omitting the result, 0.0821, which was apparently erroneous, nine determinations varied from 0.0908 to 0.0969, the probable error of the mean of the whole being much higher than is usual in the case of a similar series of results obtained by Regnault's method. As the method suggested by Professor Hesehus appears to be a desirable one, I have endeavored to perfect a calorimeter by means of which it might be applied with greater accuracy. The form of apparatus which I have devised has proved very satisfactory, after repeated trials, as is shown by the results submitted. It consists essentially of the following parts: —

A glass jar which supports, by a wooden cover, the bulb R of a glass air thermometer, the manometric tube M of which, in the form of a capillary U-tube, is supported in a vertical position. The manometric tube is supplied with a stopcock S. The glass jar is kept nearly full of water for the purpose of maintaining the thermometer bulb constantly at room temperature and preventing any sudden variation of temperature due to air currents. For the purpose of increasing the sensitiveness of the thermometer, the liquid selected was one of low density, and kerosene, of specific gravity approximately 0.8, was taken. The air thermometer thus constructed proved very sensitive, and a change in the temperature of the calorimeter cup of 0°.01 C. was clearly shown by an elevation of the manometric column.

The calorimeter cup *A* is made in the form of a tube of silver, 0.1 millimeter in thickness, of the general form of a test tube, cemented to a rubber stopper which may be inserted air-tight into the thermometer bulb. This device was first used, but the calorimeter as now made has an air-tight connection at this part, similar to that used in the calorimeter for the method of cooling as usually applied. All errors due to leakage of air at stopper, etc., which might arise at this point, are thus avoided. Silver was selected as the material used for the cup on account of its high conductivity for heat, which causes the temperature of the cup to quickly



become that of its contents. The lower half of the outer surface of the tube is coated with a thin layer of lampblack mixed with turpentine and shellac for the purpose of increasing its radiating power.

The water cooler and dropper is supported upon a vertical rod in such a manner that it may be quickly turned about the rod as an axis, and may deliver water directly to the calorimeter cup. The cooler consists of a copper receiver, covered with heavy felt, within which the water receiver *W* is placed. A drainage tube *D*, provided with a rubber tube connection, conducts away the water from the melting ice *I* with which the receiver is filled. The water receiver *W* is made in the form of a cone of tin-plated copper. This form has been found most satisfactory for maintaining the water at constant temperature, as the ice, resting upon the sides of the receiver, cannot melt away from them, leaving an intervening air space. The water dropper consists of a siphon, within the longer arm of which a thermometer is introduced, the bulb of which is held very near the small orifice from which the water drops when the stopcock *S'* is opened. Thus no barrier is interposed at any time between the thermometer bulb and the point of delivery of the cold water. The zero point of the scale is just above the cover of the ice receiver, the stem being immersed in cold water nearly up to that point.

An electric heater is supported upon a second vertical rod, and may be turned about the rod as an axis until the heater *H* is directly over the calorimeter cup, allowing the heated body to be transferred directly to the cup. The heater, which is somewhat similar in form to that devised by Prof. Henry Crew,[1] consists of a copper tube *H*, about which is wound a heating coil of iron wire, insulated from the tube by narrow strips of asbestos paper. This is inclosed within a larger tube, and all is inserted within an ice receiver *I*, the space *C* being packed with cotton to prevent convection currents in the inclosed air. Water from the melting ice of the ice receiver escapes from the drainage tube *D'* by means of a rubber tube connection. The temperature of the outer wall remaining constantly that of the melting ice, the fall of temperature remains constant between the inner and outer walls as long as heat is uniformly supplied to the inner chamber. This is readily maintained by a constant electric current supplied to the heating coil, through the binding screws *B*, from a storage battery of sufficient capacity having a variable resistance in its circuit. The battery which has been used maintained a normal electromotive force of 100 volts. The temperature of the heater is readily maintained constant within 0°.1 C. for any time required, at any temperature desired for the conditions of the work in hand, from 0.2 to 0.6 ampere of current being required according to the temperature desired. The thermometer for the determination of the temperature of the heated body is supported by a cork inserted in the top of the tube *H*. The bottom of the tube is provided with a sliding cover which remains closed while the body is heating, and automatically opens as the heater is turned around into position to deliver the heated body to the calorimeter cup.

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[1] Philosophical Magazine (5), 33, 1892.

The method of making a determination of the specific heat of a metal by means of this calorimeter is as follows: —

The metal may be suspended in the heater in small fragments, contained in a basket of fine wire gauze of known specific heat, as employed by Regnault; or, if in the form of sheet metal or fine wire, may be suspended in contact with the thermometer bulb by a thread of known weight. In the case of metals of low melting-points I have cast thin cylinders, slightly smaller in diameter than the calorimeter cup, which were so suspended in the heater that the bulb of the thermometer was within the cylinder while heating. The metal is then maintained at the desired temperature from one-half to three-quarters of an hour, according to its conductivity. The silver tube, which forms the calorimeter cup, is then withdrawn from the bulb of the air thermometer, pure water at room temperature is introduced in sufficient quantity to cover the metal used, and the whole is weighed. The cup is then placed in position in the bulb of the air thermometer, the stopcock *S* being opened while the rubber stopper is pressed firmly into place to insure an air-tight joint. Any "crawling" of the rubber stopper is easily detected by closing the stopcock *S*, and observing whether the manometric column remains stationary. A few moments are allowed for this observation in each experiment. Any variation in the temperature of the cup is also readily detected in this observation. In each observation the manometric column has been found to remain stationary after a very short time. The initial temperature of the calorimeter cup, which remains at room temperature, is then taken by means of a thermometer suspended within the cup, having its bulb in contact with the side of the cup but not touching the water. This is found to remain practically constant throughout a series of observations, agreeing with the temperature of the water in the jar. Water is allowed to drop from the water dropper and its thermometer read as soon as its temperature becomes constant. This occurs after a few drops have fallen. The manometric column is then brought to the same level *L* in each arm of the U-tube by opening the stopcock *S*. A strip of white cardboard attached to the U-tube, with a horizontal ink-line at the back of the level surface of the manometric liquid, renders the position of the liquid column plainly visible. The stopcock is then closed, the heater turned around into position, and the heated metal quickly introduced into the cup. The heater is then turned back and the water dropper quickly turned into position, and water is allowed to drop into the cup, at first rapidly, then more slowly, until the manometric column remains at its original position, indicating that the initial temperature of the cup has been maintained. For several moments the manometric column is then observed, in order to make sure that the heated body has acquired the exact temperature of the cup. In the case of a body of low conductivity the cold water is more slowly introduced, as the heat from the body is more slowly transmitted to the water. The specific heats of substances of poor conductivity may thus be obtained with great exactness; the mixture being practically at room temperature, radiation loss is inappreciable during the time spent in observing the manometric column, and in adding water just sufficient in amount to maintain the initial temperature of the cup.

A little practice in the manipulation of the calorimeter enables the operator to keep the cup practically at its initial temperature throughout each determination, thus obviating the use of radiation correction and "water equivalent" of the cup. The falling of the cold water from the water dropper into the tube-shaped cup also serves to agitate the water in the cup, and keeps its temperature uniform without the use of a stirrer. The cup and its contents are then removed from the bulb of the air thermometer and weighed. The weight thus obtained, minus the initial weight of the cup, water contained, and weight of metal used, gives the weight of cold water added. The loss in weight due to the evaporation of the water in the cup, during the time required for the necessary observations, is practically inappreciable, as the average loss was found to be less than 0.3 milligram. If desired, a correction, based upon preliminary observations, may be introduced to compensate for this loss.

The necessary operations, briefly stated, are thus: —

The heating and the determination of the temperature of a body of known weight.

Weighing of the cup containing a suitable amount of water.

Determination of the initial temperature of the cup.

Introduction of the heated body into the cup, quickly followed by the introduction of an amount of cold water sufficient to maintain the initial temperature of the cup; the temperature of the cold water being observed during this operation.

Final weight of the cup and its contents determined, and the weight of cold water added determined from data obtained.

Calculation of results by use of the equation $S = \frac{ms(\theta - t)}{M(T - \theta)}$.

This method may readily be extended over a considerable range of calorimetric work. The electric heater permits the heating of the body under investigation to any temperature ordinarily desired. The body may be cooled by a suitable freezing mixture, and the initial temperature of the cup maintained by the introduction of warm water of known temperature, thus extending the range of temperatures throughout which an investigation may be carried. The specific heat of a liquid may be determined by inclosing it in a suitable vessel, the constants of which have previously been determined. Some liquid other than water may be found preferable for special work and may readily be used in the cup and cooler.

Aniline, the use of which is advocated by Mr. E. H. Griffiths,[1] has a smaller capacity for heat and a higher boiling-point than water. It would, therefore, increase both the sensitiveness of the calorimeter in measuring small quantities of heat, and the range of temperatures throughout which an investigation might be carried.

IV.

Results Obtained.

Though subject to the errors which may arise in the application of the above three calorimetric methods, the following tabulated results give, I believe, the best determinations that have yet been made of the specific heats of the metals. They are suggestive as affording a basis of comparison of the best calorimetric methods, so far as such a comparison is possible while doubt exists as to the purity of the metals employed and the accuracy of the thermometry. I have excluded results obtained by the method of cooling, together with those obtained by various other methods which have been employed to a greater or less extent, but have been shown to be unreliable. The method of cooling has retained some place in calorimetry, presumably owing to its historic interest in connection with the work of Dulong and Petit; but since its examination and rejection by Regnault, it has been generally admitted to be erroneous both in theory and practice. I have also excluded some results, notably those of Kopp, because of the fact that approximate results only were secured, and have presented only those results which have been secured by the most accurate work possible under the given conditions. No satisfactory determinations of the specific heats of the following metals could be found: Barium, cæsium, chromium, erbium, niobium, rubidium, scandium, strontium, tantalum, terbium, vanadium, ytterbium, and yttrium. The selection of the most probable values of the mean specific heats between 0° and 100° C., which are given below, has been made after a careful consideration of the determinations that have been made and their relative values in view of the nature of the metal used, the thermometry, and the accuracy of the method as applied. The extent and accuracy of Regnault's work have naturally suggested that it should be given a leading place.

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[1] Philosophical Magazine (5), 39, p. 47, 1895.

References to original sources of information are given by numbers following the names of investigators, and referring to an appended list of important papers.

Of the above results the determinations which I have made of the specific heats of bismuth and tin are probably very nearly correct values for their means for the given temperature range. These metals were supplied to me by Mr. G. A. Hulett in a chemically pure state. The bismuth was prepared by the method employed by Prof. A. Classen.[1] The tin was prepared in the following manner. Pure Malacca tin was taken. Analysis revealed traces of copper, iron, and lead. It was oxidized with nitric acid and digested for several days with strong nitric acid, washed by decantation and again digested with strong nitric acid. It was then frequently washed by decantation for a month. The oxide was then reduced with chemically pure potassium cyanide. It was used for my determinations in the form of a cast disk of metal. The aluminium used was furnished by the Pittsburg Reduction Company. By analysis it proved to be 99.9 per cent pure, the impurity being chiefly silicon. It was prepared by the Hall process.[2] The copper used was from Lake Superior, and was about 99.98 per cent pure. Not so much reliance may be placed upon the values found for gold and zinc, as the metals were not so pure as those above mentioned. The thermometers used for these determinations were three in number. That for the heater was graduated to fifths of a degree, that for the cup to twentieths of a degree, and that for the water cooler to tenths of a degree. Hundredths of a degree could readily be estimated on each in reading temperatures. All were made by Mr. H. J. Green. They were twice standardized; once by myself by comparison with Yale Observatory Standard No. 59, made by Tonnelot of Paris; again at Johns Hopkins University, under the direction of Prof. J. S. Ames, by comparison with a certified thermometer made by Gerhardt. The latter comparison was made by use of the apparatus

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[1] Berichte der Deutschen Chemischen Gesellschaft, Vol. 23, 938, 1890.

[2] Aluminium, by Prof. J. W. Richards, 2d edition, p. 288, 1890.

	Observer.	Method.	Mass used and condition.	Temperature range.	Results.	Mean.
Aluminium	Regnault[1]	Mixtures	134.810^{g} 2 % iron and traces of silicon 124.595^{g}	97°.0– 19°.0	0.21236 0.21212	0.21224
	Tomlinson[2]	Mixtures		100°.0– 0°.0		0.21852
	Joly[3]	Condensation	5.376^{g} cast; pure metal prepared by Professor Reynolds	100°.0– 16°.0	0.2247 0.2236 0.2242 0.2243 0.2236 0.2215	0.2236
	Naccari[4]	Mixtures	37.25^{g}	100°.0– 0°.0	0.2156 0.2164 0.2168 0.2166	0.2164
	Mallet[5]	Melting Ice	Absolutely pure metal	100°.0– 0°.0		0.2253
	Richards[27]	Mixtures	$100–200^{g}$ prepared by Hall process, by analysis: Silicon 0.07 Iron trace Aluminium 99.93	100°.0– 0°.0		0.2270
	Waterman[6]	Mixtures	6.8504^{g} cast, prepared by Hall process: 99.9 % pure, 0.1 % silicon	100°.0– 22°.0	0.21947 0.21943 0.21948	0.21946

Antimony	Regnault[7]	Mixtures	362.3^{g} commercial metal purified	97°.0– 16°.0	0.05113 0.05037 0.05084 0.05076	0.05078
	R. Bunsen[8]	Melting Ice	3.857^{g} cast	99°.8– 0°.0		0.0495
	Schüz[9]	Mixtures		100°.0– 20°.0	0.05077 0.05070 0.05078 0.05032 0.05041	0.05059
	Naccari[4]	Mixtures	76.6^{g}	100°.0– 0°.0	0.05041 0.04976 0.04994	0.05031
Arsenic	Regnault[7]	Mixtures	216.18^{g} purified by sublimation once ;		0.08163 0.08204 0.08114	0.08140
			228.02^{g} purified by sublimation twice		0.08082 0.08136 0.08117	
	Bettendorff and Wüllner[10]	Mixtures	6.14^{g} crystals	68°.0– 21°.0	Mean of nine determinations	0.0830
Beryllium	Humpidge[11]	Mixtures	0.66^{g} in form of a compressed disk. Mean corrected for impurities	45°.0– 50°.0	0.4326 0.4264 0.4357	0.4453

	Observer.	Method.	Mass used and condition.		Temperature range.	Results.	Mean.
Bismuth	Regnault[7]	Mixtures	521.56^{g} purified with nitre		98°.0– 15°.0	0.03082	0.03084
						0.03089	
						0.03082	
			555.26^{g} purified again		99°.0– 11°.0	0.03084	
						0.03093	
	Bède[12]	Mixtures	Metal purified by twice melting with nitrate of potassium and cast	135.00^{g}	102°.0– 9°.0	0.02927	0.02979
				101.00		0.02981	
				92.25		0.02997	
				100 07		0.03010	
	Schüz[9]	Mixtures			100°.0– 20°.0	0.03119	0.03137
						0.03155	
	Waterman[6]	Mixtures	4.891^{g} cast		100°.0– 22°.0	0.03033	0.03035
						0.03032	
						0.03039	
Cadmium	Regnault[7]	Mixtures	439.19^{g}		98°.0– 12°.0	0.05695	0.05669
						0.05673	
						0.05639	
	R. Bunsen[8]	Melting Ice	1.867^{g} cast		99°.8– 0°.0		0.0548
	Schüz[9]	Mixtures			100°.0– 20°.0	0.05638	0.05669
						0.05698	
						0.05661	
						0.05596	
						0.05705	
						0.05720	

Cadmium	Naccari[4]	Mixtures	101.65^{g}	100°.0– 21°.0	0.05618 0.05576 0.05611 0.05595	0.0560
Calcium	R. Bunsen[8]	Melting Ice	0.2823^{g} electrolytically prepared from pure chloride of calcium	99°.78– 0°.0	0.1722 0.1686	0.1704
Cerium	Hillebrand[18]	Melting Ice	2.0935^{g}	100°.0– 0°.0	0.04613 0.04554	0.04583
Cobalt	Regnault[7]	Mixtures	87.87^{g}	99°.0– 11°.0	0.10629 0.10784 0.10685	0.10710
			164.65^{g}	97°.0– 8°.0	0.10707 0.10772 0.10682	
Copper	Regnault[7]	Mixtures	314.77^{g}	98°.0– 15°.0	0.09537 0.09546 0.09497 0.09480	0.09515
	Bède[12]	Mixtures	72.28^{g} fragments	100°.0– 15°.0	0.09430 0.09200 0.09364	0.09331
	Tomlinson[2]	Mixtures		100°.0– 0°.0		0.09332
	Schüz[9]	Mixtures		100°.0– 20°.0	0.09307 0.09278 0.09334	0.09306

	Observer.	Method.	Mass used and condition.	Temperature range.	Results.	Mean.
Copper	Waterman[6]	Mixtures	99.98 % pure; thin bar of drawn metal { 10.9322^g 23.6625	100°.0– 23°.0	0.09475 0.09470 0.09467 0.09474	0.09471
	Naccari[4]	Mixtures	131.7^g	100°.0– 17°.0	0.09341 0.09400 0.09340	0.09360
Didymium	Hillebrand[13]	Melting Ice	1.844^g	100°.0– 0°.0	0.04649 0.04557	0.04603
Gallium	Berthelot[14]	Mixtures	About 34^g liquid	119°.0– 13°.0 106°.0– 12°.5		0.0802
Gold	Regnault[7]	Mixtures	471.19^g granulated, refined	98°.0– 12°.0	0.03250 0.03238	0.03244
	Violle[15]	Mixtures	Pure	100°.0– 0°.0		0.03160
	Waterman[6]	Mixtures	Nearly pure; in a thin bar, 17.8292^g	100°.0– 24°.0	0.03066 0.03065 0.03074	0.03068
Indium	R. Bunsen[8]	Melting Ice	1.1514^g	99°.82– 0°.0	0.0574 0.0565	0.05695
Iridium	Regnault[16]	Mixtures	262.78^g	99°.0– 18°.0	0.03246 0.03273	0.03259
	Violle[15]	Mixtures		100°.0– 0°.0		0.0323

Iron	Regnault[7]	Mixtures	In the form of soft wire, 316.66g	98°.0– 19°.0	0.11362	0.11379
					0.11322	
					0.11373	
			Tempered wire, 293.69		0.11284	
					0.11398	
	Bède[12]	Mixtures	77.56g fragments	99°.2– 15°.5	0.11334	0.11230
					0.11120	
					0.11236	
	Nichol[18]	Mixtures		100°.0– 15°.0	0.1127	0.11520
					0.1154	
					0.1158	
					0.1168	
	Tomlinson[2]	Mixtures		100°.0– 0°.0		0.11302
	Naccari[4]	Mixtures	116.67g	100°.0– 15°.0	0.1114	0.1119
					0.1119	
					0.1122	
					0.1119	
Lanthanum	Hillebrand[13]	Melting Ice	0.8911g	100°.0– 0°.0	0.04582	0.04637
			1.682		0.04692	
Lead	Regnault[7]	Mixtures	526.33g assayed	98°.0– 13°.0	0.03134	0.03140
					0.03177	
					0.03109	
					0.03137	
			560.52 Spanish, very pure		0.03145	
					0.03129	
					0.03150	
					0.03133	

	Observer.	Method.	Mass used and condition.	Temperature range.	Results.	Mean.
Lead	Bède[12]	Mixtures	138.49g fragments	108°.0– 14°.0	0.03108 0.03154 0.03046	0.03103
	Tomlinson[2]	Mixtures		100°.0– 0°.0		0.03151
	Joly[3]	Condensation	89.508g cast	16°.0–100°.0	0.03091 0.03093	0.03092
	Schüz[9]	Mixtures	Chemically pure	100°.0– 20°.0	0.03177 0.03166	0.03171
	Naccari[4]	Mixtures	160.2g 103.2	100°.0– 15°.0	0.03082 0.03058 0.03084	0.03075
Lithium	Regnault[16]	Mixtures	0.945g	99°.0– 26°.0	0.9412 0.9405 0.9407	0.94080
Magnesium	Regnault[16]	Mixtures	91.87g 92.22	98°.0– 23°.0	0.2466 0.2533	0.24990
Manganese	Regnault[7]	Mixtures	54.98g, mean corrected for impurities	97°.0– 14°.0	0.1207 0.1227	0.1140
Mercury	Regnault[7]	Mixtures	Purified by distillation, 480.01g In glass tube, 558.58	97°.0– 12°.0	0.03318 0.03336 0.03343	0.03332
	Winkelmann[19]	Mixtures		20°.0– 50°.0		0.03330

Molybdenum	Regnault[7]	Mixtures	Agglomerated mass, 65.83g 64.23	98°.0– 12°.0	0.07254 0.07182	0.07218
Nickel	Regnault[7]	Mixtures	132.68g	99°.0– 14°.0	0.10874 0.10837 0.10878	0.10863
Osmium	Regnault[16]	Mixtures	204.59g	98°.0– 19°.0		0.03113
Palladium	Regnault[16]	Mixtures	293.37g, plate 3 mm. thick, containing slight traces of gold	98°.0– 14°.0	0.05974 0.05893 0.05916	0.05928
	Violle[20]	Mixtures		100°.0– 0°.0		0.0592
Platinum	Regnault[7]	Mixtures	569.01g pure	99°.0– 12°.0	0.03279 0.03246 0.03228 0.03223 0.03238 0.03268 0.03263	0.03243
	Violle[21]	Mixtures	Pure metal from Sainte-Clare Deville	100°.0– 0°.0		0.0323
	R. Bunsen[8]	Melting Ice		100°.0– 0°.0		0.03234
	Tomlinson[2]	Mixtures		100°.0– 0°.0		0.03261
	Joly[8]	Condensation	34.764g rolled	100°.0– 14°.0	0.0326 0.0328 0.0327 0.0328 0.0331 0.0328 0.0330	0.0328

	Observer.	Method.	Mass used and condition.	Temperature range.	Results.	Mean.
Platinum	Schüz[9]	Mixtures		100°.0– 20°.0	0.03298 0.03292	0.03295
Potassium	Schüz[9]	Mixtures		−78°.5– 23°.0		0.1662
Rhodium	Regnault[16]	Mixtures	47.107^{g}	97°.0– 10°.0	0.05742 0.05865	0.05803
Ruthenium	R. Bunsen[8]	Melting Ice	1.7927^{g}	99°.6– 0°.0		0.0611
Silicon	Regnault[16]	Mixtures	78.7^{g} cast	99°.0– 22°.0	0.1732 0.1767	0.1750
	H. F. Weber[22]	Melting Ice	1.123^{g} small brilliant gray crystals	101°.0– 0°.0		0.1795
Silver	Regnault[7]	Mixtures	345.66^{g} 496.72	99°.0– 13°.0	0.05739 0.05691 0.05685 0.05679 0.05712	0.05701
	R. Bunsen[8]	Melting Ice	3.63^{g} cast	100°.0– 0°.0		0.0559
	Naccari[4]	Mixtures	91.35^{g}	99°.0– 23°.0	0.05581 0.05631 0.05641	0.05618
	Humpidge[11]	Mixtures			0.05677 0.05568 0.05553	0.05600

Silver	Tomlinson[2]	Mixtures		100°.0– 0°.0		0.05684
	Joly[8]	Condensation	4.027g pure rolled, prepared by Professor Reynolds	79°.0– 12°.0	0.0583 0.0571 0.0569	0.0574
Sodium	Regnault	Mixtures		−79°.5– 17°.0		0.2830
	Schüz[9]	Mixtures		−28°.0– 6°.0		0.2934
Thallium	Regnault[16]	Mixtures		100°.0– 17°.0		0.03355
Thorium	Nilson[28]	Melting Ice		100°.0– 0°.0		0.02757
Tin	Regnault[7]	Mixtures	460.65g pure 462.74	99°.0– 13°.0	0.05624 0.05652 0.05619 0.05622 0.05601	0.05623
	Bède[12]	Mixtures	51.01g cast	99°.0– 15°.0	0.05414 0.05475	0.05445
	R. Bunsen[8]	Melting Ice	3.0017g	99°.6– 0°.0		0.0559
	Tomlinson[2]	Mixtures		100°.0– 0°.0		0.05592
	Schüz[9]	Mixtures	Chemically pure metal	100°.0– 20°.0	0.05573 0.05554	0.05563
	Waterman[6]	Mixtures	8.3556g chemically pure, cast	100°.0– 24°.0	0.05453 0.05455 0.05451	0.05453
Titanium	Nilson and Pettersson[24]	Melting Ice	0.4468g	100°.0– 0°.0	0.1127 0.1126 0.1121	0.11250

	Observer.	Method.	Mass used and condition.	Temperature range.	Results.	Mean.
Tungsten	Regnault[7]	Mixtures	420.8^g crystalline	98°.0– 12°.0	0.03358 0.03326	0.03342
Uranium	Blümcke[25]	Melting Ice	1.887^g to 3.7999^g	98°.0– 0°.0		0.0280
Zinc	Regnault[7]	Mixtures	247.61^g distilled 284.69 293.65	99°.0– 14°.0	0.09589 0.09528 0.09548	0.09555
	Bède[12]	Mixtures	70 7^g fragments	101°.0– 16°.0	0.09137 0.09059 0.09068	0.09088
	R. Bunsen[8]	Melting Ice	2.51^g cast	100°.0– 0°.0		0.09350
	Tomlinson[2]	Mixtures		100°.0– 0°.0		0.09383
	Joly[3]	Condensation	48.3^g cast	14°.0–100°.0	0.09434 0.09421 0.09434 0.09453 0.09443	0.09437
	Schüz[9]	Mixtures		100°.0– 20°.0	0.09676 0.09595	0.09635
	Waterman[6]	Mixtures	20.3805^g, a thin cast bar, nearly pure	100°.0– 23°.0	0.09547 0.09544 0.09549	0.09547
Zirconium	Mixter and Dana[26]	Melting Ice		100°.0– 0°.0		0.0660

constructed by Professor Rowland for testing the thermometers used in his determinations of the mechanical equivalent of heat.[1] Their correction factors were thus accurately determined and the thermometers proved very satisfactory in all respects. The thermometer used for the cup remaining constantly at room temperature, and that used for the water cooler constantly at the temperature of the water, up to the point read, no variation due to stem exposure could occur. In the case of the thermometer used for the heater especial care was taken to secure accurate determinations of temperatures. As the usual correction for stem exposure was applied ($R' = R + 0.000156\, n\, (R - s)$), it was desirable to check these values in order to avoid any uncertainty as to the reliability of such a correction under the given conditions. For this purpose a platinum thermometer, consisting of a spiral of platinum wire 0.2 mm. in diameter, mounted upon strips of mica, was introduced into the heater. The metal used and the mercurial thermometer could thus be introduced into the heater as before, as the platinum spiral left the central portion of the tube unoccupied. Temperatures obtained by the mercurial thermometer, corrected for stem exposure, could thus be compared with the temperatures obtained by the use of the platinum thermometer in the manner described by Professors Callendar and Griffiths.[2] By this method it was found that the mercurial thermometer used in the heater, when corrected for stem exposure, gave temperature readings sufficiently accurate and within the limits of probable error from other sources. Weighings were made upon a Sartorius balance sensitive to one-tenth milligram under the given load. In the calculation of results, s, the specific heat of the pure water used, was taken as unity for the given temperature range. An examination of the results obtained by use of my calorimeter will demonstrate the low probable error of their mean. The results obtained are as follows: —

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[1] Proceedings of the American Academy of Arts and Sciences, Vol. 7, p. 90, 1879–80.

[2] L. E. and D. Philosophical Magazine (5), Vol. 32, p. 104, 1891.

	M	t	θ	T	m	S
	grams	degrees	degrees	degrees	grams	
Bismuth .	4.891	0.20	22.32	99.71	0.519	0.03033
		0.20	22.65	100.15	0.512	0.03032
		0.25	22.45	100.10	0.520	0.03039
					Mean	0.03035
Tin . . .	8.3556	0.20	22.75	100.06	1.562	0.05453
		0.20	24.20	100.16	1.4425	0.05455
		0.20	24.15	99.96	1.4416	0.05451
					Mean	0.05453
Aluminium	6.8504	0.20	22.04	100.02	5.367	0.21943
		0.20	22.58	99.86	5.194	0.21947
		0.20	21.88	100.07	5.423	0.21948
					Mean	0.21946
Copper . .	10.9322	0.19	23.39	100.00	3.4204	0.09475
		0.30	22.42	100.20	3.6403	0.09470
	23.6625	0.25	23.62	99.80	7.3020	0.09467
		0.30	21.51	100.00	8.2960	0.09474
					Mean	0.09471
Gold. . .	17.8292	0.20	24.00	100.00	1.7456	0.03066
		0.35	24.41	100.00	1.7101	0.03065
		0.35	23.45	99.95	1.8153	0.03074
					Mean	0.03068
Zinc . . .	20.3805	0.20	23.40	99.90	6.4160	0.09547
		0.20	23.50	100.00	6.3860	0.09544
		0.27	23.85	100.20	6.3017	0.09549
					Mean	0.09547

The selection of the probable values of the mean specific heats of the metals between 0° and 100°, from the tabulated results, is difficult in view of the considerations which have been stated. In the case of those metals of which but one presumably creditable determination has been made, that one has been given provisionally. In the case of the metals which have been subject to several creditable determinations, the attempt has been made to select the values best sustained. It will be observed that in this selection greater weight has been given to results obtained by the method of mixtures than to those obtained by the method of condensation, or the method of melting ice. This preference is due partly to the sources of error, which may exist in the latter methods, and partly to the fact that, judging by results, those obtained by use of the method of mixtures seem to be best sustained. We may judge of the accuracy of the method of condensation by the work of Dr. Joly, but it has not been generally used and we are unable to determine what concordance there might be in the work of various investigators employing this method. We have more data for the examination of the results obtained by the method of melting ice, however, and such an examination may be desirable. A comparison may readily be made by reference to the determinations made of the specific heats of platinum, silver, and zinc, for which all three methods have been employed. It is seen that, although some variations exist in the values found by the method of mixtures as employed by different investigators, they are very evidently higher than those obtained by the method of melting ice as applied by use of Bunsen's ice calorimeter. The values found by use of the method of condensation by Dr. Joly are also higher than those obtained by the Bunsen calorimeter. It is improbable that the results obtained by so many skillful investigators, applying the method of mixtures with considerable variations at different times, their results sustained by an entirely different method, that of condensation, should be incorrect, while those obtained by the few investigators who have used the Bunsen ice calorimeter should be correct. Exception might be taken to this statement by citing the case of the results obtained for the specific heat of aluminium, among which Professor Mallet's determination, made by use of the Bunsen calorimeter, has a high value. This is readily explained, however, as nearly all of the aluminium used has contained considerable traces of silicon, which has lowered the value, while Professor Mallet's determination was made by use of chemically pure aluminium, prepared for the determination of its atomic weight.

Considerable weight should be given to a low probable error of the mean of a series of determinations. Other things being equal, it naturally gives greater confidence in the result. In the case of results obtained by use of the Bunsen calorimeter, it is difficult to find material from which to determine the probable error of the mean values given, as most investigators who have used this method seem to be unwilling to publish more than their final values.

The fact that the specific heats of most metals increase as their temperatures rise leads us to another consideration; namely, whether determinations made at temperatures varying greatly as to their proximity to the melting-points of the various metals, are properly comparable. This can be made clear only by the work of those investigators who have examined the variations in the specific heats of the metals up to their melting-points. For this reason I shall not enter into the matter in the following comparison.

It will be observed that if we select a few of the specific heats most probably accurate, namely, those of aluminium, 0.2236; bismuth, 0.0303; copper, 0.0947; gold, 0.0316; iron, 0.1130; lead, 0.0310; platinum, 0.0326; silver, 0.0574; and tin, 0.0545, and multiply by the atomic weights corresponding, the mean of the atomic heats thus formed is 6.2444.

The mean atomic heat of the whole series is slightly lower than this value.

Metals.	Temperature range.	Specific heats.	Atomic weights.	Atomic heats.
	degrees			
Aluminium	100.00–16.0	0.2236	27.1	6.0595
Antimony	100.00– 0.0	0.0505	120.0	6.0600
Arsenic	68.00–21.0	0.0830	75.0	6.2250
Barium				
Beryllium	45.00–50.0	0.4453	9.0	4.0077
Bismuth	100.00–22.0	0.0303	208.0	6.3024
Cadmium	100.00–00.0	0.0565	112.0	6.3280
Cæsium				
Calcium	99.78–00.0	0.1704	40.0	6.1860
Cerium	100.00–00.0	0.0458	140.0	6.4120
Chromium				
Cobalt	99.00–11.0	0.1071	58.5	6.2653
Copper	100.00–23.0	0.0947	63.6	6.0429
Didymium	100.00–00.0	0.0460	146.6	6.7436
Erbium				

Metals.	**Temperature range.**	**Specific Heats.**	**Atomic weights.**	**Atomic heats.**
	degrees			
Gallium	106.00–12.5	0.0802	70.0	5 6140
Gold	100.00–00.0	0.0316	198.0	6.2468
Indium	99.82-00.0	0.0569	113.0	6.4297
Iridium	100.00–00.0	0.0324	193.0	6.2532
Iron	100.00–00.0	0.1130	56 0	6.3280
Lanthanum	100.00-00 0	0.0463	138.0	6.3894
Lead	100.00–00.0	0.0310	206.0	6.2006
Lithium	99.00–26.0	0.9408	7.0	6.5856
Magnesium	98.00–23.0	0.2499	24.4	6.0975
Manganese	97.00–14.0	0.1140	55.0	6.2700
Mercury	97.00–12.0	0.0333	200.0	6.6600
Molybdenum	98.00–12.0	0.0721	96.0	6.9216
Nickel	99.00–14.0	0.1086	58.5	6.3531
Niobium				
Osmium	98 00–19.0	0.0311	191.0	5.9301
Palladium	100.00–00.0	0.0592	106.0	6.2752
Platinum	100.00–00.0	0.0326	196.0	6.3896
Potassium	−78.50–23.0	0.1662	39.0	6.4818
Rhodium	97.00–10.0	0.0580	104.0	6.0320
Rubidium				
Ruthenium	99.60–0.00	0.0611	103.0	6.2933
Scandium				
Silicon	101.00–0.00	0.1795	28.4	5.0978
Silver	79.00–12.0	0.0574	108.0	6.1992
Sodium	−79.50–17.0	0.2830	23.0	6.5090
Strontium				
Tantalum				
Terbium				
Thallium	100.00–17.0	0.0335	204.0	6.8340
Thorium				
Tin	100.00–24.0	0.0545	118.0	6.4310
Titanium	100.00–00.0	0.1125	48.0	5.4000
Tungsten	98.00–12.0	0.0334	184.0	6.1456
Uranium	98.00–00.0	0.0280	240.0	6.7200
Vanadium				
Ytterbium				
Yttrium				
Zinc	99.00–14.0	0.0955	65.0	6.2075
Zirconium	100.00–00.0	0.0660	90.0	5.9400

REFERENCES.

1. Regnault. — Annales de Chemie et de Physique (3), Vol. 46, 1856.
2. Tomlinson. — Proceedings of the Royal Society of London, Vol. 37, 1884.
3. Joly. — Proceedings of the Royal Society of London, Vol. 41, 1886.

4. NACCARI. — Atti della R. Accademia delle Scienze de Torino, Vol. 33, 1887.
5. MALLET. — Chemical News, Vol. 46, 1882.
6. WATERMAN. — L. E. and D. Philosophical Magazine (5), Vol. 40, 1895.
7. REGNAULT. — Annales de Chemie et de Physique (2), Vol. 73, 1840.
8. BUNSEN. — L. E. and D. Philosophical Magazine (4), Vol. 41, 1871.
9. SCHÜZ. — Wiedemann's Annalen der Physik und Chemie, Vol. 46, 1892.
10. BETTENDORFF and WÜLLNER. — Poggendorff's Annalen der Physik und Chemie, Vol. 133, 1868.
11. HUMPIDGE. — Proceedings of the Royal Society of London, Vol. 35, 1883.
12. BÈDE. — Mémoires Couronnés par L'Academie Royale de Belgique, Vol. 27, 1856.
13. HILLEBRAND. — Poggendorff's Annalen der Physik und Chemie, Vol. 158, 1876.
14. BERTHELOT. — Annales de Chemie et de Physique (5), Vol. 12, 1877; (5), Vol. 15, 1878.
15. VIOLLE. — Comptes Rendus, Vol. 89, 1879.
16. REGNAULT. — Annales de Chemie et de Physique (3), Vol. 63, 1861.
17. LORENZ. — Wiedemann's Annalen der Physik und Chemie, Vol. 13, 1881.
18. NICHOL. — L. E. and D. Philosophical Magazine (5), Vol. 12, 1881.
19. WINKELMANN. — Poggendorff's Annalen der Physik und Chemie, Vol. 149, 1873; Vol. 159, 1876.
20. VIOLLE. — Comptes Rendus, Vol. 87, 1878.
21. VIOLLE. — L. E. and D. Philosophical Magazine (5), Vol. 4, 1877.
22. WEBER. — L. E. and D. Philosophical Magazine (4), Vol. 49, 1875.
23. NILSON. — Berichte der Deutschen Chemischen Gesellschaft, Vol. 16, 1883.
24. NILSON and PETTERSSON. — Zeitschrift für Physikalische Chemie, Vol. 1, 1887.
25. BLÜMCKE. — Wiedemann's Annalen der Physik und Chemie, Vol. 24, 1885.
26. MIXTER and DANA. — Liebig's Annalen der Chemie und Pharmacie, 169, 1873.
27. RICHARDS. — Journal of the Franklin Institute, February, 1892.

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EXPERIMENTAL DETERMINATIONS OF THE TEMPERATURE IN GEISSLER TUBES.

By R. W. Wood.

§ 1. The opinion that the temperature of the gas in the positive part of the discharge in Geissler tubes lay far below the red heat was first expressed by E. Wiedemann,[1] and Hittorf[2] was led to the same conclusion from simple experiments. The theoretical calculations of the temperature in the unstratified anode light by Warburg[3] yielded similar results. Hittorf[4] showed, further, that the temperature in the negative light lay at least below the melting point of platinum, for at this temperature the luminosity of the gas disappeared entirely.

Calorimetric observations have been made by Paalzow and Neesen[5] and others, which give some idea of the total heat production, but as far as I know no accurate measurements have been made of the temperature of the gas in different parts of the discharge. It is clear that such measurements must be made in the discharge of a constant battery, and not in the intermittent discharge of an induction coil.

The prime object of this investigation has been to obtain the relative temperatures in different parts of the discharge, with especial reference to the stratifications. Owing to the narrow space limits between which it was necessary to work in the stratified discharge, a bolometer, consisting of a loop or spiral of very fine platino-iridium wire, appeared to be the most suitable measuring instrument.

§ 2. The constant current for producing the discharge in the vacuum tube was furnished by a high potential accumulator battery of 600 cells, the electromotive force being about 1250 volts. A Hittorf resistance tube, with a movable electrode filled with a solution of cadmium iodide for regulating the current strength, and a Siemens & Halske direct-reading torsion galvanometer for measuring the current, were introduced into the circuit.

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[1] E. Wiedemann, Wied. Annalen, 6, p. 298, 1879.

[2] W. Hittorf, Wied. Annalen, 7, p. 577, 1879.

[3] E. Warburg, Wied. Annalen, 54, p. 265, 1895.

[4] W. Hittorf, Wied. Annalen, 21, p. 123, 1884.

[5] H. Paalzow and F. Neesen, Wied. Annalen, 56, p. 276, 1895.

§ 3. Before commencing work with the bolometer, a preliminary investigation was made on the rise of pressure within the tube due to the discharge, and from this pressure increment the corresponding temperature increment was calculated. Hittorf speaks of using one of his tubes as an air thermometer, and of the very slight expansion of the gas due to the heating, but he gives no numerical results. The change of pressure being too slight to be accurately measured by means of a mercurial manometer, sulphuric acid was used in place of the heavier fluid. The apparatus is shown in Fig. 1, and consists of a small glass flask *A*, into the neck of which is fused a vertical tube, which passes to the bottom and communicates above with the discharge tube *B*. The flask contains a little sulphuric acid, and the space above the liquid is also in communication with the discharge tube through the lateral tube, which is furnished with a stopcock. By this arrangement the pressure above the liquid can be made just sufficient to support a short column of acid in the vertical tube, the cock being opened during the process of exhaustion and closed a few minutes before the requisite vacuum is reached, the further exhaustion causing a rise of the fluid. On closing the cock between the tube and the pump, and turning on the current, a fall of the column is immediately noticed, and by measuring this the increase of pressure and consequently of temperature can be determined from the formula

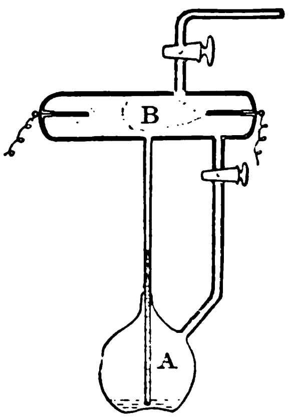

Fig. 1.

$$t - t_0 = \frac{p - p_0}{p_0}(272.5 + t_0).$$

The temperature at the start was 20° and the pressure 2 mm. The passage of a current of 0.001 ampere caused a depression of

1 mm. in the H_2SO_4 barometer, corresponding to 0.135 mm. of mercury. By the formula we find that the rise in temperature, or $t - t_0$,

$$= \frac{0.135}{2}(272.5 + 20) = 19.7.$$

The change of volume in the two vessels due to the sinking of the column is vanishingly small in comparison to their volumes, and can be neglected. This method, of course, shows only the average temperature rise, and the result cannot be strictly compared with any of those found with the bolometer, which gives local temperatures.

§ 4. Warburg has shown by his calculations that in a gas at low pressure, heated by the discharge, a stationary temperature is established within a fraction of a second, so rapid is the conduction of heat. In other words, a gas is heated to its maximum, or cools to its original temperature within a small fraction of a second, on the commencement or cessation of the discharge. With an interrupted current, providing the interruptions are not too rapid, we should then expect the temperature to rise and fall exactly in time with the interrupter. Corresponding to this rise and fall of temperature there should be a rise and fall of pressure, and this I find to be the case.

The inertia of the column of acid in the apparatus last described (Fig. 1) is too great to allow the fluid to respond to these rapid changes of pressure, but I find that with the device shown in Fig. 2 they can be shown very well. This consists of a discharge tube with a rectangular U manometer, in which are two drops of sulphuric acid. The tube is connected with the pump by means of a short piece of rubber hose, and during the process of exhaustion the acid is collected into a single drop. When the desired vacuum has been reached, the drop can be separated into two by carefully tipping the tube back and forth, forming a bubble between, in which the gas is at the same pressure as that in the discharge tube. On connecting the electrodes with a Ruhmkorff coil in action, the

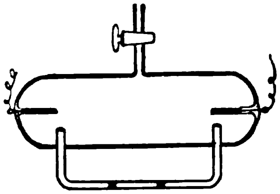

Fig. 2.

drops, which are about 3 cm. apart, alternately approach and recede, vibrating exactly in unison with the interrupter. The pressure at which this is shown to the best advantage is about 5 mm. When it is too low, the amplitude of the vibrations is too small to be observed.

§ 5. Of the measurements made with the bolometer I shall first take up those which show the relation of the rise in temperature to the current strength. In this investigation the bolometer wire was fixed in position, while in experiments to be described further on it was so arranged that it could be moved up and down within the discharge tube.

The arrangement of the apparatus is shown in Fig. 3.

In the axis of the discharge tube is mounted a spiral of very fine platino-iridium wire. The length of the wire was 2 cm., its diameter 0.035 mm., and the distance between the ends of the large wires which supported the spiral was 0.5 cm. The internal diameter of the tube was 1.5 cm., being the same as used by A. Herz in his research on the fall of potential in the anode light, which will be alluded to later, and on which Warburg based his calculations. The high potential battery *A*, the resistance tube *B*, and the

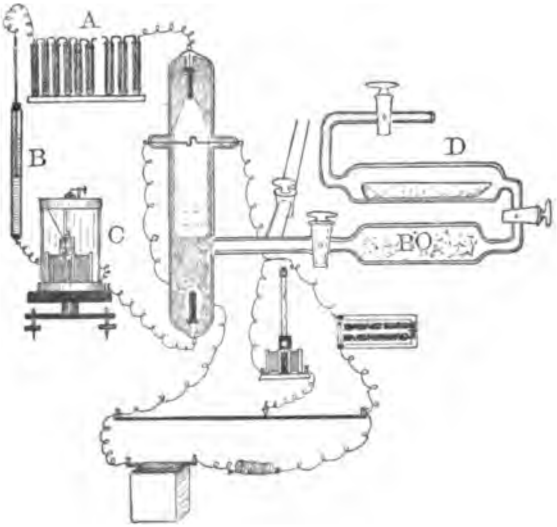

Fig. 3.

torsion galvanometer *C*, I have already alluded to. The bolometer was connected with a mirror galvanometer, resistance box, and Wheatstone bridge in the usual manner, as shown in the diagram, the current being furnished by a single accumulator, with a 20-ohm resistance coil in the circuit.

§ 6. Nitrogen made by passing air over heated phosphorus contained in a porcelain boat in the tube *D*, and then dried by means of phosphoric acid, was used in most of the experiments, the current in air not being steady enough to admit of accurate measurements being made.

§ 7. The apparatus was so arranged that within the temperature limits that were observed, the rise in temperature was proportional to the deflections of the galvanometer, as observed with a scale and telescope. The temperature increment corresponding to a deflection of one scale division was found by immersing the bolometer wire in warm oil with a normal thermometer reading to tenths of a degree, and allowing the oil to slowly cool. One degree was found to correspond to 4 mm. scale deflection.

§ 8. It is of course a question how nearly the wire takes the temperature of the gas. On account of radiation and conduction it must lag behind the gas, and to give absolutely correct values should be of infinite thinness. Some idea of the size of the error can be formed by working with wires of different thickness. The one used was 0.035 mm. in diameter, and since finer wire is hard to procure and difficult to manage, platinum leaf less than 0.001 mm. in thickness was used, which was furnished rolled together with silver. A strip 1 mm. wide and 1.5 cm. long was mounted in the tube in place of the spiral, the silver backing giving the necessary rigidity. The soldered joints were protected with asphalt varnish, and the silver removed with dilute nitric acid. Owing to the extreme tenuity of the remaining strip, great care was necessary in washing and drying out the tube. The resistance of the strip was found to be 2.8 ohms, which is almost identical with the value calculated for a strip of its size having a thickness of 0.001 mm. The tube was exhausted to a vacuum of 2 mm. and a current of 0.0012 ampere sent through it, the strip of platinum leaf being in the anode light. The scale deflection was 40 cm., about ten times as large as with the wire. This was due principally to the larger temperature coefficient of the strip, which was of pure platinum, while the wire was an alloy of platinum and iridium. The strip was now immersed in oil at 25°, and the temperature was raised until a galvanometer deflection of 40 cm. scale was produced, the temperature of the oil being found to be 43°, showing a rise of 18°. The temperature rise of the gas in the anode light, under similar conditions, as measured with the wire, was 19°, as will be seen by reference to Table IV. The platinum foil took the temperature of the gas more rapidly than the wire, and would consequently have been better adapted to the experiments, were it not for the influence that the wide strip had on the uniformity of the discharge. With the fine wire this deforming action was noticeable to a certain degree, especially in working with stratified discharges with the movable bolometer, the strata appearing as though elastic, and only yielding to the wire after slight deformation. This effect will be alluded to subsequently. The similarity of results obtained with the wire and the foil justify the assumption that the wire takes the temperature of the gas nearly enough for our purpose.

§ 9. It is also necessary to make sure that the discharge within the tube can only affect the galvanometer by changing the resistance of the wire. The current was shut off from the bolometer circuit and a discharge passed through the tube, which caused no deflection, showing the absence of thermo-currents, or currents switched from the high potential discharge.

§ 10. The first sets of measurements were made in the unstratified anode light, at different pressures and with varying current strength. The cock communicating with the air pump was left open, to eliminate as much as possible the rise of pressure due to the warming of the gas. The values are given in the following tables: i being the current strength, w the temperature rise, and $\frac{w}{i}$ the ratio between these values. To avoid complicating the tables I have omitted the galvanometer deflections and given the corresponding temperatures.

TABLE I.

PRESSURE, 0.3 mm.

i	w	$\frac{w}{i}$	i	w	$\frac{w}{i}$
0.0015	13°.0	87	0.0022	18°.0	82
0.0017	15°.0	88	0.0025	19°.5	78
0.0018	15°.7	87	0.0032	22°.5	73
0.0021	17°.7	84	0.0036	25°.7	71

TABLE II.

PRESSURE, 1.8 mm.

i	w	$\frac{w}{i}$	i	w	$\frac{w}{i}$
0.0007	11°.5	164	0.0019	26°.2	138
0.0009	14°.0	155	0.0025	34°.0	136
0.0012	17°.0	142	0.0032	42°.2	132
0.0015	21°.7	145			

TABLE III.

PRESSURE, 2 mm.

i	w	w calculated by Warburg.	$\frac{w}{i}$
0.0012	19°.0	22°.0	158
0.0020	30°.0	—	150
0.0032	45°.0	40°.6	143

TABLE IV.

PRESSURE, 3 mm.

i	w	w calculated.	$\frac{w}{i}$	i	w	w calculated.	$\frac{w}{i}$
0.0010	23°.0		230	0.0022	44°.0		200
0.0012	27°.0	30°.0 Warburg	225	0.0025	49°.7		198
0.0015	31°.0		207	0.0027	50°.7		187
0.0018	37°.0		205	0.0032	59°.5	62.7 Warburg	186
0.0020	40°.0		200	0.0046	73°.0		160

It will be seen from the values w, which show the rise in degrees above the room temperature, 25°, that the views held by Wiedemann and Hittorf regarding the comparatively low temperature in the Geissler tube are corroborated. The theoretical calculations by Warburg, to which I have previously alluded, which are based on the supposition that the electrical energy consumed in a portion of the tube, and represented by the product of the current strength and the potential fall, is practically all transformed into heat, give results which agree well with the values which I have found. These calculated temperatures for the axis of the anode light are given with the observed ones in Tables III. and IV.

The bolometer wire, not coinciding with the axis of the discharge, but occupying a space 0.5 cm. wide, gives the average temperature in this space, which is found by calculation to be about $\frac{17}{18}$ of the value in the axis.

The ratio $\frac{w}{i}$ is, according to the theory of Warburg, proportional to the potential gradient. The decrease in the values, with increasing current strength, is most probably due to the fact that, the gas being at constant pressure, its density, and therefore the gradient, diminish with rising temperature. The stop-cock leading to the pump was next closed, thus keeping the gas in the tube at a constant volume and density. In this case it was found that $\frac{w}{i}$ was nearly a constant, as shown in the following tables. According to the work of Herz on potential fall, we should expect $\frac{w}{i}$ to decrease in this case also, but this did not seem to be the case.

TABLE V.

PRESSURE, 0.3 mm.

i	w	$\frac{w}{i}$	i	w	$\frac{w}{i}$
0.0015	13°.0	87	0.0026	21°.9	84
0.0017	15°.2	89	0.0029	24°.2	83
0.0020	17°.4	87	0.0034	28°.5	84

TABLE VI.

PRESSURE, 1 mm.

i	w	$\frac{w}{i}$	i	w	$\frac{w}{i}$
0.0015	19°.0	126	0.0028	34°.0	121
0.0018	21°.0	117	0.0039	48°.0	123
0.0022	26°.0	118			

§ 11. By reversing the direction of the current the bolometer was brought into the dark space which separates the positive and negative light, and a set of values obtained in this portion of the discharge with currents of varying strength. The temperature here was found to be much lower than in the anode light, and the quotient $\frac{w}{i}$ was nearly constant, indicating that in this case the potential fall was independent of the current strength. Table VII. gives the values found in the dark space.

TABLE VII.

PRESSURE, 2 mm.

i	w	$\frac{w}{i}$	i	w	$\frac{w}{i}$
0.0009	08°.7	97	0.0038	35°.7	94
0.0010	10°.2	93	0.0040	37°.5	94
0.0012	11°.5	96	0.0048	45°.0	93
0.0021	19°.5	93			

§ 12. The measurements described thus far have been for a fixed part of the discharge. The complete investigation of the temperature fluctuations in passing along the tube from anode to cathode was next undertaken.

The chief difficulty lay in devising a method of easily and rapidly moving the bolometer back and forth in the discharge without impairing the vacuum. The apparatus is shown in Fig. 4. A vertical tube *A*, 1 cm. in diameter and 80 cm. long (shortened in diagram), carries on its upper end the discharge tube with its laterally placed electrodes. Through *A* passes a small tube *C*, bent in a U at the bottom and carrying on its upper end the loop *D* of platino-iridium wire which serves as the bolometer wire. Within *C* are the two insulated copper conducting wires, terminating in platinum points which are fused through the glass, and to which are soldered the terminals of the bolometer loop, which is bent into a horizontal plane by manipulation through the side tube *E*, subsequently joined to the pump.

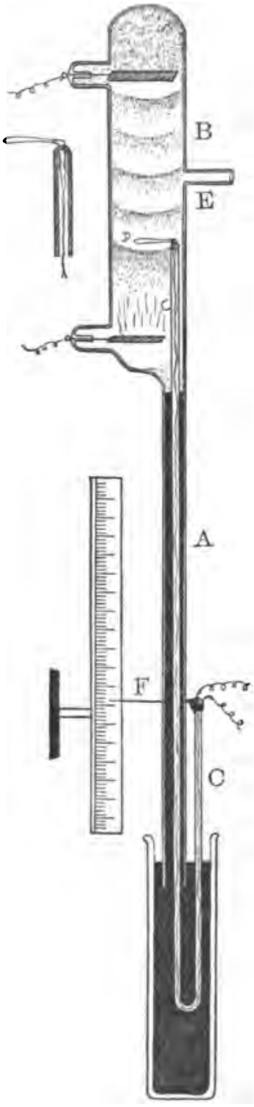

Fig. 4.

An enlarged view of the upper end of the tube carrying the bolometer is shown to the left of the discharge tube. The lower end of *A* is now introduced into a deep jar of mercury, and on exhausting the apparatus the fluid rises, forming a barometer column, through which the smaller tube, carrying the bolometer on its end, slides up and down. The arm of the U which projects from the mercury jar carries a pointer *F*, which moves along a millimeter scale and indicates the exact position of the wire loop in the discharge. All the connections were carefully soldered, otherwise changes of resistance occurred when the bolometer tube was moved. With this device it was possible to explore the entire tube from the cathode to the anode and make a complete map of the temperature changes.

§ 13. In the unstratified anode light the temperature was sometimes constant for the greater part of the column, and sometimes rose to a maximum near the middle, falling away again as the dark space was approached. This maximum was always found when the light was on the point of stratifying, and sometimes at higher pressures. The exact conditions that determined whether the temperature had a maximum in the middle of the column or was constant for the greater part, could not be determined, but the extent of the anode light appeared to have something to do with it. Under exactly the same conditions of pressure and current strength, so far as can be determined, the light will sometimes extend much further down the tube than at others, and I could find no way of remedying this trouble, which has been observed by other experimenters.

There was always a decrease of temperature on nearing the dark space. On leaving the anode light, the temperature drops very suddenly, reaching a minimum near the middle of the dark space, and then rises again very rapidly on entering the blue negative light. The results obtained with the movable bolometer are clearer when shown by curves than when expressed in tabular form. The condition where a maximum exists in the anode light is shown in Fig. 5, the abscissæ being distances from the anode, as shown by the pointer on the scale, and the ordinates temperatures as calculated from the galvanometer deflections. The room temperature (26°) is shown by the horizontal line, and the character of the discharge and its position on the scale indicated by the drawing beneath. The pressure was about 1.5 mm. and the current strength 0.001 ampere. A series showing constant temperature in the anode light is shown in the upper curve in Fig. 6, the position of the boundary between the positive light and the dark space being indicated.

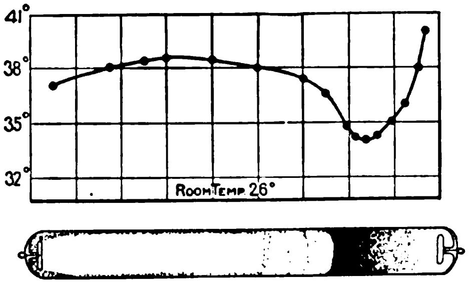

Fig. 5.

§ 14. When the pressure in the tube is sufficiently reduced to cause the appearance of stratifications in the anode light, a maximum in the middle of the column is always to be found, the temperature rising as we recede from the anode, and falling again after the middle of the column is passed. In addition to this there is a periodic rise and fall, the light disks being warmer than the dark spaces between them, although one often finds a point where there is no change of temperature on passing from a light space to a dark. The cause of this is at once apparent when the results are plotted on coördinate paper, the decrease due to the passage from light to dark being compensated by the increase due to approach to the maximum point (Fig. 7).

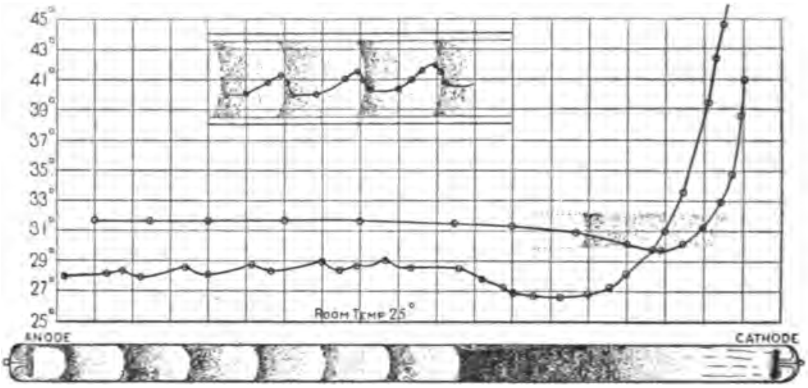

Fig. 6.

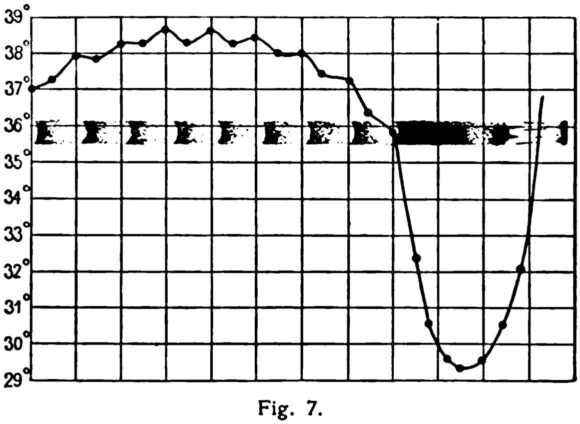

Fig. 7.

Lehmann[1] came to the conclusion that in the stratified discharge the temperature of the light spaces was higher than that of the dark. By mixing oil vapor with the gas, he observed that the carbon particles set free glowed in the light spaces, but were invisible in the dark. If this luminosity of the carbon was true incandescence caused by the high temperature of the gas, the conditions of current strength and pressure must have been very different from any that I have investigated, since in no case have I found the temperature to be over 100°.

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[1] O. Lehmann, Zeitschrift für Electrochemie, 21 and 22, 1896.

The curve showing the temperature fluctuations in the stratified discharge at low pressure (0.1 mm.) is shown in Fig. 6 (lower curve), the straight line showing the room temperature, and the drawing beneath indicating the character and position of the discharge with reference to the curve. The maximum in the anode light is not as marked as it is at higher pressure, owing to the smaller changes of temperature. In Fig. 7, which is for a pressure of 0.5 mm., it is very evident, as is the existence of two spots, one on each side of the maximum, where there is no change on going from a light to a dark space.

A careful set of observations was made of the periodic change in the stratified anode light, five points being taken for each cylinder of light.

The results are shown for three consecutive disks in Fig. 6 in the small drawing above the curves. The conditions are the same as in the tube represented below, only the ordinates are taken on a larger scale, and more points are taken.

The temperature is steady for a certain distance, then rises gradually to a maximum, situated in the brightest part of the disk, turns and drops suddenly as we pass out of the sharply defined edge of the disk. The difference in temperature between the

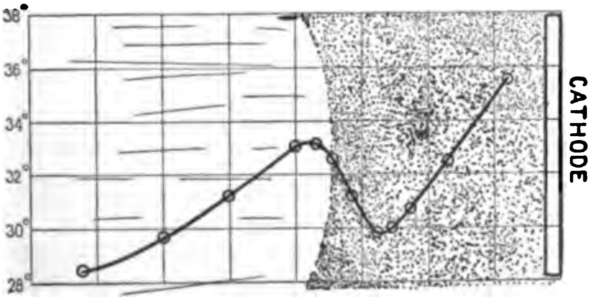

Fig. 8.

light and dark spaces varies from 0°.5 to about 1°.5, depending on the degree of exhaustion and current strength.

§ 15. Assuming that the electrical energy is wholly converted into heat, these temperature curves *should* resemble closely the curves representing the potential fall in different parts of the discharge. At the present time no very accurate maps have been made of the potential fall between the anode and cathode. What is already known agrees with the results found in this investigation, which may be expressed thus:

Positive light . . .	Medium potential fall . . .	Medium temperature.
Dark space	Small potential fall	Low temperature.
Negative light . . .	Large potential fall	High temperature.

§ 16. It may not be out of place to mention here one or two phenomena that were observed in working with stratified discharges, though they have no direct bearing on the subject. Though in general the bolometer wire passes through the strata cleanly without moving them or altering their position, at certain pressures I find that the wire drags the stratum through which it is passing (moving from + to − electrode) into the one immediately below it, the two dissolving into one. At this moment, however, a new stratum springs off the anode, so that as we work down the tube dragging each stratum in succession into its neighbor, the total number remains always the same.

Another effect to which I have alluded before, is the apparent resistance which the sharply defined edges of the strata offer to the penetration of the wire. The edge acts as if it had an elastic skin, or a sort of surface tension, bending in as the wire pushes against it, and finally snapping back to its original position, leaving the wire well within the luminous disk. Too little is known about the strata to make any discussion of these two facts worth while, and I merely record them without comment.

§ 17. A few experiments were made with hydrogen, but owing to the difficulty of maintaining a steady current in this gas, they were not carried very far. The fact was established, however, that under similar conditions of pressure and current strength the heating was only about 11 per cent of that found in the case of nitrogen, this being due to the fact that the potential fall is less, and also to the fact that hydrogen is a much better conductor of heat than nitrogen. With hydrogen at 1.8 mm. pressure, with a current of 0.0015 ampere, a rise of temperature of only 2°.5 was observed. Referring to Table II., the rise obtained for nitrogen under similar conditions was 21°.7, about nine times as great. Warburg calculated the temperature rise for hydrogen at 5 mm. with a current strength of 0.001 to be 4°.1, and for nitrogen under nearly similar conditions 41°.3, or ten times as great.

§ 18. While working with hydrogen at very low pressures, I found evidence of a fall of temperature on passing from the negative light into the Crookes dark space, and have found mention made of this by Wiedemann,[1] who worked with a thermo-element applied to the outside of the tube. It is only to be found when the length of the dark space is considerable; at higher pressures, if it exists at all, it is completely masked by the very rapid rise in temperature on approaching the cathode. This was also noticed by Wiedemann. This same temperature minimum was found in nitrogen by subsequent trials.

It is not always to be found, however, even with hydrogen. One tube with comparatively small rectangular electrodes, showed it very distinctly (Fig. 8), while another, with large aluminium plates, gave no indication of it. In the latter tube the Crookes dark space was more or less filled with blue negative light, which may have accounted for the absence of the minimum. It will be interesting to see whether future measurements of the potential fall in this part of the discharge show a corresponding minimum.

§ 19. Commenting in general on the results found in this investigation, I consider the curves obtained with the movable bolometer to indicate the relative temperatures in different parts of the discharge with considerable accuracy. As to the results found with the stationary wire and variable current strength, the irregularity of the series $\frac{W}{i}$ shows that there is a source of error somewhere. This, I think, may be the instability of the conditions in the tube, such as the varying length of the anode light, to which I have already alluded. The values may, however, be considered correct to within two or three degrees, I think. They agree well with theory, though a too strict comparison cannot be made on account of the probable convection currents in the tube and the radiations from the always more or less warm cathode.

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[1] E. Wiedemann, Annalen, 20, p. 775, 1883.

I have, in concluding, to thank Professor Warburg for the kind interest that he has taken in the progress of the work.

Berlin, Physical Institute of the University.

THE VISCOSITY OF MERCURY VAPOR.[1]

By A. A. Noyes and H. M. Goodwin.

THE uncertainty which attaches to the specific heat ratio of gases as a means of distinguishing between monatomic and polyatomic molecules has been recently made evident by the extended discussions of the significance of that property in connection with the atomic weights of argon and helium. It is therefore of great interest to investigate other properties which may be expected to be related to the atomicity of the molecule. Of such properties those dependent on the volume or cross-section of the molecules seem most promising. We have therefore undertaken the investigation of one of these, the viscosity or internal friction, in order to determine whether a marked difference in its value exists in the case of gases with monatomic and those with polyatomic molecules. To this end we have made comparative measurements of the viscosity of hydrogen, carbon dioxide, and mercury vapor at the boiling temperature of the last-named substance.

According to the kinetic theory of gases, the viscosity coefficient has the theoretical significance expressed by the following equation[2]:

$$\eta = \frac{1}{\pi} NmLc,$$

in which N is the number of molecules in the unit of volume, m the mass of a single molecule, L the mean free path, and c the mean velocity. Moreover, the free path L is dependent solely on the number of molecules N and the mean cross-section q of a single molecule, or of its sphere of action[3]:

$$L = \frac{1}{4\sqrt{2}\,Nq};$$

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[1] Read at the Buffalo meeting of the American Association for the Advancement of Science, August 25, 1896, and at the meeting of the American Academy of Arts and Sciences, October 14, 1896.

[2] O. E. Meyer, Kinetische Theorie der Gase, 1st ed., 130, 139.

[3] O. E. Meyer, Kinetische Theorie der Gase, 206, 218. The symbol Q used by the author represents the total cross-section of all the molecules in the unit of volume, and is therefore evidently equal to Nq.

whence it follows that

$$\frac{1}{4\pi\sqrt{2}} = \frac{mc}{q};$$

or, for any two different gases,

$$\eta_1 : \eta_2 = \frac{m_1c_1}{q_1} : \frac{m_2c_2}{q_2}.$$

But, since for any two gases at constant temperature,

$$m_1c_1^2 = m_2c_2^2,$$

the above proportion may be simplified to the following equation:

$$\frac{q_1}{q_2} = \frac{\eta_2}{\eta_1}\sqrt{\frac{m_1}{m_2}}, \qquad (1)$$

from which it is evident that the relative mean cross-sections of the molecules of the two gases are readily calculated from their molecular weights and viscosity coefficients. It was thought by us that monatomic molecules might prove to be much smaller than polyatomic ones, since it seems *a priori* not improbable that the spaces between the atoms of the latter are large in comparison with the dimensions of the atoms themselves. The experiments to be here described show, however, that no marked distinction exists between monatomic and polyatomic gases in this respect.

Experiments on the viscosity of mercury vapor, and especially on the effect of temperature upon it, have been made already by S. Koch,[1] who calculated that at 300° the volume of the mercury molecule is 4.4 as great as that of the hydrogen molecule. As this calculation was not based on direct comparative experiments made by passing the two gases through the same capillary, but was an indirect one involving the measurements of different experimenters and the dimensions of the capillaries used by them, it seemed desirable to subject the matter to further investigation in the direct manner indicated. Moreover, the author does not discuss the significance of his result in its bearing on the relative magnitude of atoms and molecules.

[1] Wied. Ann. 19, 857, 1883.

The method used by us in determining the relative viscosity consisted in measuring the quantities of the different gases which, under a constant difference of pressure, passed in a given time through the same capillary kept at a definite constant temperature. O. E. Meyer[1] has derived the following formula for calculating the viscosity coefficient of a gas from its rate of flow through a capillary tube:

$$\eta = \frac{\pi}{16} \frac{R^4}{\lambda} \frac{(p_1^2 - p_2^2)t}{p_1 V_1},$$

where λ is the length and R the radius of the tube, t the time, p_1 the pressure at which the gas enters, and p_2 that at which it leaves the tube, and V_1 the volume of the transpired gas measured at the pressure p_1. In case of comparative experiments made with the same capillary on two different gases, the following proportion holds true:

$$\eta_1 : \eta_2 = \frac{(p_1^2 - p_2^2)_1 t_1}{n_1} : \frac{(p_1^2 - p_2^2)_2 t_2}{n_2}, \qquad (2)$$

in which n_1, n_2 represent respectively the number of gram molecular weights of the two gases transpired (since n is proportional to the product pV).

The apparatus and experimental method that we employed were necessarily quite different from the usual ones, and they will therefore be briefly described. The capillary used in the most complete series of experiments consisted of a glass tube about 74 cm. in length and 0.34 mm. in internal diameter (determined by measuring the volume of a known length by means of mercury). A smaller capillary about 49 cm. in length and 0.22 mm. in diameter was used in a preliminary series. The capillary was bent in the manner shown in Fig. 1, except that as actually constructed it was made much more compact. To its ends were fused pieces of ordinary glass tube, as shown in the figure; one of these was provided at the point A with a ground glass joint. The capillary was placed in a vertical position in a heavy steel cylinder A (Fig. 2) (30 cm. high, 2.8 cm. internal diameter), having a small orifice at the side, through which the ground joint protruded for a distance of about one centimeter. The capillary was held in position in the orifice by packing with loose asbestos. Although the capillary was vertical, the influence of gravity was eliminated by reason of the fact that the ascending and descending parts were made equal in length. The top of the cylinder was closed by an iron plate screwed down with a nut *N*. The nut and the plate were perforated in the center, and into the latter was welded an open iron tube *B*, projecting upwards, 25 cm. in length and 1.5 cm. in diameter. The cylinder was completely covered, except on the bottom, with a jacket of asbestos, about 5 cm. thick, and the projecting tube *B*, which was to serve as a condenser, was wound with spirals of copper wire to increase the cooling surface. Pure mercury was placed in the cylinder and boiled vigorously by means of a number of lamps beneath. The capillary was thus kept at the boiling temperature of mercury under atmospheric pressure. No regard was paid to the variations of temperature arising from changes in barometric pressure, as their effect would evidently be entirely negligible.

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[1] Pogg. Ann. 127, 269.

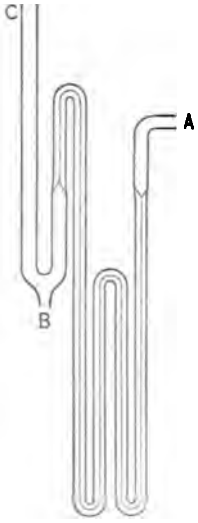

Fig. 1.

Any desired difference of pressure at the two ends of the capillary was attained by inserting a tube in the ground joint and connecting it with a large air reservoir *R*, which was itself connected through the cock *S* with a suction pump and with an open mercury manometer *M*. The gas or vapor entered at the other end of the capillary always under atmospheric pressure. The whole apparatus in the form used for measuring the rate of transpiration of the mercury vapor is shown in Fig. 2.

In making the experiments, the rate of flow of the mercury vapor was first determined in the following manner: While the cylinder was being heated, carbon dioxide was forced through the capillary to prevent the condensation in it of liquid mercury, and the formation of its oxide. After the mercury was boiling actively, and its vapor entirely enveloped the capillary, as shown by a mercurial thermometer inserted in the tube *B* (Fig. 2), it was connected with the suction pump, and mercury vapor drawn through for half an hour. The carefully ground end of a weighed bulb *W* was then inserted in the ground joint, its other end being connected, by means of a clamped rubber tube *C*, with the air reservoir, in which the desired reduction of pressure had been produced. At a definite moment the clamp *C* was opened and the time noted. As the volume of the condensing bulb *W* was very small, compared with the volume of the air reservoir, no sensible change in the pressure was thus produced. The mercury vapor was found to be completely condensed in *W* about two or three centimeters from the ground joint.

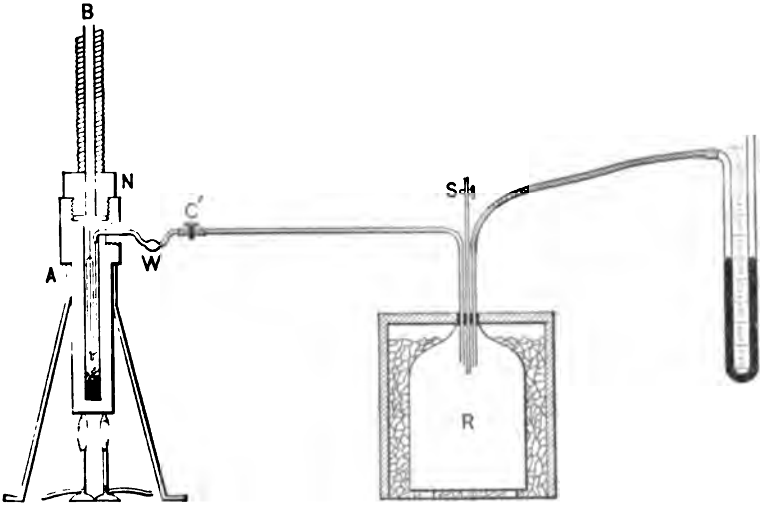

Fig. 2.

It was found that a slight and unavoidable leakage[1] through the ground joint occurred, and it was therefore necessary to readjust the pressure occasionally. It could easily be maintained constant to 0.2 or 0.3 mm., or even closer. After a sufficient time, usually sixty minutes, the clamp C^1 was closed, and at a noted instant the bulb removed and subsequently weighed. Check experiments were made in this way at each of two or three different pressures.

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[1] In the case of the mercury experiments, no error could arise from this source, as the leakage was inward. In the case of those with carbon dioxide and hydrogen, it was proved by blank experiments that the amounts of carbon dioxide and water which leaked in were less than one per cent of the total weight.

The capillary was now removed from the cylinder, and the opening *B* (Fig. 1) carefully closed by fusion. A glass tube, long enough to project beyond the upper end of *B* (Fig. 2), was also fused on to the end *C*, and the capillary was then ready for the experiments with carbon dioxide and hydrogen. It was replaced in the cylinder as before, and the glass tube projecting through *B* connected through suitable wash-bottles with the gas generator. The carbon dioxide was made in a Kipp generator by the action of dilute sulphuric acid on lumps of pure fused sodium carbonate, and was dried by passing through two Allihn gas wash-bottles containing strong sulphuric acid. The hydrogen was prepared from pure Bertha zinc and dilute sulphuric acid, as in the case of the carbon dioxide, and was purified by means of caustic soda solution and concentrated sulphuric acid. In order to maintain the gas entering the capillary at atmospheric pressure, a T-tube was inserted between the wash-bottles and the capillary, and its perpendicular arm was turned downwards, and caused to dip into sulphuric acid barely below its surface. The cock of the generator was opened sufficiently to cause the gas to bubble out steadily through the sulphuric acid.

The transpiration measurements were made as in the case of the mercury. The carbon dioxide flowing through in a definite time was determined by absorption in weighed tubes filled with lumps of soda lime. The hydrogen was burned by passing it over hot copper oxide, contained in hard glass tubes, from which the air had been previously displaced by carbon dioxide, and the water formed was collected in weighed calcium chloride tubes.

The results are presented in the following table. In the first column is given the symbol of the substance; in the second, the atmospheric pressure p_1; in the third, the difference in pressure $(p_1 - p_2)$; in the fourth, the time t, expressed in hours; in the fifth, the weight w, in grams, of the substance weighed; in the sixth, the mean weight transpired in one hour as computed from the separate check experiments; and in the last, the quotient obtained by dividing this weight by the molecular weight m of the substance, the time, and the pressure function

Series I.

SMALLER CAPILLARY.

	p_1	$p_1 - p_2$	t	w	$w : t$	$\frac{w \times 10^{10}}{tm(p_1^2 - p_2^2)}$
Hg	753	200	1½	0.738	—	—
	753	—	1½	0.740	—	—
	756	—	1½	0.745	0.494	93.6
	755	300	1	0.685	—	—
	755	—	1	0.686	0.686	93 6
	754	400	1	0.834	—	—
	754	—	1	0.831	0.833	92.9
CO_2	752	200	1	0.237	—	—
	752	—	1	0.239	—	—
	752	—	1½	0.357	0.238	205.0
	759	300	1	0.329	—	—
	759	—	1	0.327	—	—
	766	—	1	0.324	0.327	203.0
	766	400	1	0.400	—	—
	766	—	1	0.396	—	—
	759	—	1	0.396	0.397	202.0

Series II.

LARGER CAPILLARY.

	p_1	$p_1 - p_2$	t	w	$w : t$	$\frac{w \times 10^{10}}{tm(p_1^2 - p_2^2)}$
Hg	769	150	1	1.548	—	—
	769	—	1	1.548	1.548	374
	769	300	1	2.763	—	—
	769	—	1	2.764	—	—
	769	—	1	2.739	2.755	373
CO_2	765	150	1	0.704	—	—
	765	—	1	0.706	0.705	774
	765	300	1	1.267	—	—
	765	—	1	1.264	1.265	779
H_2	766	150	1	0.557	—	—
	766	—	1	0.557	0.557	1517
	766	300	1	1.009	—	—
	766	—	1	1.006	1.008	1496
Hg	766	300	1	2.728	—	—
	766	—	1	2.746	—	—
	764	—	1	2.743	2.739	371

$(p_1^2 - p_2^2)$.[1] (See preceding expression (2), p. 3.) As the headings indicate, the first of these two series of experiments was made with the smaller and the second with the larger capillary. It will be seen that in the second series two sets of determinations are given for mercury vapor. The first one was made before, and the second one after, the experiments with carbon dioxide and hydrogen, and their agreement shows that the capillary had undergone no change by stoppage or otherwise during the course of the experiments with it.

Attention may be first called to the agreement of the values of the last column, in the case of the transpiration of the same substance under different differences of pressure, thus proving that the effect of pressure is in close accordance with that required by the formula, and consequently that the capillaries are of sufficient length and small enough bore to give the true values of the viscosity coefficients.

Of the two series of experiments the first one made with the smaller capillary is to be regarded as less reliable by reason of the fact that, owing to an accident to the capillary, check experiments with mercury vapor after completion of those with carbon dioxide could not be made as in the former case, in consequence of which it is not certain that a stoppage did not occur in the course of the series. While there was nothing to indicate that such was the case, yet, on account of its extremely small bore, it was much more liable to stoppage than the larger capillary. We consider the experiments with the latter to be certainly accurate within two per cent.

Nevertheless, the agreement between the results of the first and the second series is perhaps as close as could be expected with capillaries so different from each other in character. The relative viscosity coefficients of the different gases were calculated from the values of the last column in the table by means of formula (2). The results are as follows:

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[1] In the calculation of this quantity, the same mean value of p_1 was used in all the experiments of each series; namely, 760 for those with the smaller capillary, 765 for those with the larger.

FIRST SERIES.

$$\frac{\eta_{Hg}}{\eta_{CO_2}} = 2.17.$$

SECOND SERIES.

$$\frac{\eta_{Hg}}{\eta_{CO_2}} = 2.08. \quad \frac{\eta_{Hg}}{\eta_{H_2}} = 4.04. \quad \frac{\eta_{CO_2}}{\eta_{H_2}} = 1.94.$$

The relative values for mercury and carbon dioxide agree within about four per cent. That of the second series is, however, more reliable, for the reason above stated, and will be used in the subsequent calculations. It may be noted that the value of the ratio $\eta_{CO_2} : \eta_{H_2}$ at ordinary temperature is 1.72,[1] somewhat smaller than that found by us at 357°, a result which is in accordance with the greater temperature coefficient of carbon dioxide established by several investigators.

The corresponding values of the relative mean cross-sections as calculated by formula (1) are

$$\frac{q_{Hg}}{q_{CO_2}} = 1.02\,; \quad \frac{q_{Hg}}{q_{H_2}} = 2.48.$$

That is to say, the average cross-section of the mercury molecule or atom is very nearly the same as that of the carbon dioxide molecule, and is about 2½ times as large as that of the hydrogen molecule. This last result does not differ very greatly from that (2.68) corresponding to the relative molecular volumes of mercury and hydrogen as calculated for 300° by Koch.

These results indicate that *atoms and molecules are of the same order of magnitude*, and that the spaces between the atoms within the molecule, if any exist, are not large in comparison with those occupied by the atoms themselves. And, consequently, the viscosity of gases or any other property which, like it, is dependent only on the size or form of the molecules is not adapted for distinguishing between monatomic and polyatomic molecules.

In considering the significance of the above values of the cross-section, the different masses of the various molecules, to be sure, ought not to be entirely disregarded. The mercury and carbon dioxide have, as we have seen, the same cross-section, and, therefore, assuming both to be of the same general form, they occupy the same volume. The mass of the former is, however, 4.55 times as great as that of the latter. The density of the mercury molecule is consequently greater in this same proportion. But this difference is not marked enough to make it necessary to attribute it to free space within the carbon dioxide molecule. For it is not improbable that the inherent density of massive atoms, like those of mercury, may be considerably greater than that of light atoms, like those of carbon and oxygen.

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[1] O. E. Meyer, Kinetische Theorie der Gase, 142.

In closing we desire to point out that the principle here established that atoms and molecules are of the same order of magnitude, and that no considerable free interatomic spaces exist within the molecule, is in accordance with the remarkable fact that the molecular cross-section of most comparatively simple molecules is approximately an additive property calculable from certain constant values of the atomic cross-section.[1] This fact would be unintelligible were the principle not correct; for, if considerable space existed between the atoms, it is not to be supposed that those spaces would be the same in entirely dissimilar molecules — that, for example, the space between the hydrogen and chlorine atoms in hydrochloric acid would have any relation to the spaces between the atoms in the elementary gases hydrogen and chlorine.

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[1] See O. E. Meyer, Kinetische Theorie der Gase, 209.

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MINOR CONTRIBUTIONS.

AN EXAMPLE IN THERMOMETRY.[1]

BY A. S. COLE AND E. L. DURGAN.

SINCE the publication of the methods of procedure employed at the International Bureau of Weights and Measures, at Breteuil, the ordinary method of obtaining the corrections of a thermometer by comparison with another thermometer whose errors are known, is neither defensible on the score of precision nor on the score of economy in time. The determination of the absolute corrections of a thermometer at a large number of points requires but little more time than the determination of the relative corrections at a few points.

The details of the absolute method are scattered through three volumes of the *Memoirs* of the Bureau, and, being written in French, they are not readily available to the English student. The study of thermometer Gerhardt No. 2403 is given in the following paper as an illustration of the new method of procedure.

The observed reading of a thermometer is subject to the following corrections: —

(1) A correction for calibration.

(2) Corrections for external and internal pressure.

(3) A correction for the fundamental interval between the 0° point and the 100° point. These corrections will be considered in the order in which they are named.

The Calibration of a Thermometer.

We take, as an example, the calibration of the 10° points of thermometer Gerhardt No. 2403. If the graduations upon the stem are exactly equal in value, the corrections to these values will be indicated by the lengths of a column of mercury placed successively end to end for each of the 10° points. Since it is not easy to break off a column having a definite length, we must measure the excess of the length of the column over the length of the successive 10° graduations. If we designate the excess of the length of the column over the length of the 10° spaces by λ, we have only to apply to each of the 10° graduations successively the values $\frac{\lambda}{10}, \frac{2\lambda}{10}, \cdots, \frac{10\lambda}{10}$, in order to obtain the corrected values of the 10° points of the thermometer.

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[1] Contribution from the Shannon Physical Laboratory of Colby University.

But we cannot assume that all the subdivisions of the scale are equal. (1) Let $x_1, x_2, x_3, x_4, \cdots, x_{11}$ represent the *corrections* to the 10° points of the scale. Represent the observed excess of the length of the column by $a_1, a_2, a_3, a_4, \cdots, a_{10}$; then, for any space, *e.g.* between the line 0° and line 10°, the true relation will be expressed by the equation: —

$$x_1 - x_2 + \lambda = a_1.$$

Following the notation given on p. C 37, Vol. II., we shall have a series of equations of the form: —

$$\begin{aligned} &x_1 - x_2 + \lambda_{10} = a_1, \\ &x_2 - x_3 + \lambda_{10} = a_2, \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ &x_{10} - x_{11} + \lambda_{10} = a_{10}. \end{aligned} \tag{2}$$

Breaking off successively columns having the lengths 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80°, and 90°, and measuring the excess of these columns over the corresponding spaces, we have a series of fifty-four equations to be solved by the process of least squares.

It is to be noted that the signification of λ, which was assigned to it under the supposition of equal subdivisions of the entire length of the scale, does not hold when we assume that the divisions have the corrections x_1, x_2, x_3, x_4, etc. In this case, this constant will be affected by the assumption that the scale graduations themselves require corrections.

We take, as an example of calibration, thermometer Gerhardt No. 2403. The readings were all made with a telescope of short focus and a filar micrometer, in which one division equals 0°.00066. The method of making the observations will appear from the following example: —

Let R_l = the micrometer reading of the graduated scale, *e.g.* for 0°.

R_u = the micrometer reading of the upper end of the division, *e.g.* for 10°.

R_m = the micrometer reading of the miniscus of the mercury column.

n = the number of tenths of the graduations between which the miniscus falls.

By estimation.	**Measurement with the micrometer.**	
	For 0°.	**For 10°.**
$l= 0.025$	$R_l= 82$ div.	$R_l= 44$ div.
$u=10.110$	$R_m=131$ div.	$R_m= 62$ div.
$u-l=10.085$	$R_u=261$ div.	$R_u=224$ div.
	$n=0.000$	$n=0.10$ 1 div.$=0.00066$
	$\frac{49}{179}\times 0.1=0.027$	$\frac{18}{180}\times 0.1=0.010$
	Sum$=0.027$	Sum$=0.110$
	$0.110 - 0.027 = +0.083.$	

All the measurements for this series of observations were made in the manner indicated above. We take as an example of the method of reducing the observations given in the first column of Table I.

Interval.	**Observed reading.**	**Relative errors.**	**Relative corrections.**
° °	°	°	°
0- 10	10.019	−0.001	+0.001
10- 20	10.034	+0.014	−0.014
20- 30	10.006	−0.014	+0.014
30- 40	10.030	+0.010	−0.010
40- 50	10.034	+0.014	−0.014
50- 60	10.008	−0.012	+0.012
60- 70	10.012	−0.008	+0.008
70- 80	10.020	+0.000	+0.000
80- 90	10.075	+0.055	−0.055
90-100	10.012	−0.008	+0.008
Mean	10.020		

At this point two courses are open to us. Either we may take the results obtained by subtracting 10 from each of the values, or we may obtain a set of residuals by subtracting the mean value of column (1) from each separate value giving column (2). In the former case we must follow formulæ 1 and 8 given on pp. 8 and 12, Vol. V. In the latter case, the values of s and Σ become zero, and the reduction becomes more simple. We shall in this example follow the latter method. [See a paper on the Cumulative Errors of a Graduated Scale, by W. A. Rogers, *Proceedings Mechanical Engineers*, Vol. XV. The correction is here noted that the denominations S given on p. 134 should be written in small type.]

The numerical data given in Table I. has been obtained from the mean of independent observations taken by each of us.

(3)

TABLE I.

λ_{10}		λ_9		λ_8		λ_7		λ_6	
° °	°	° °	°	° °	°	° °	°	° °	°
0- 10	+0.001	0- 20	−0.027	0- 30	+0.014	0- 40	−0.001	0- 50	−0.022
10- 20	−0.014	10- 30	−0.005	10- 40	−0.016	10- 50	−0.055	10- 60	−0.036
20- 30	+0.014	20- 40	+0.003	20- 50	−0.038	20- 60	−0.035	20- 70	+0.004
30- 40	−0.010	30- 50	−0.039	30- 60	−0.040	30- 70	−0.004	30- 80	−0 001
40- 50	−0.014	40- 60	−0.012	40- 70	+0.016	40- 80	+0.025	40- 90	+0.014
50- 60	+0.012	50- 70	+0.048	50- 80	+0.035	50- 90	+0.037	50-100	+0.041
60- 70	+0.008	60- 80	+0.036	60- 90	+0.034	60-100	+0.043		
70- 80	+0.000	70- 90	−0.009	70-100	−0.005				
80- 90	−0.005	80-100	+0.005						
90-100	+0.008								

λ_5		λ_4		λ_3		λ_2			
0- 60	−0.038	0- 70	+0.017	0- 80	+0.014	0- 90	+0.002		
10- 70	−0.002	10- 80	−0.012	10- 90	−0.018	10-100	−0.002		
20- 80	+0.002	20- 90	+0.001	20-100	+0.004				
30- 90	+0.013	30-100	−0.006						
40-100	+0.025								

Since the coefficients of the unknown quantities are all unity, we have only to take the sums of the coefficients, in order to obtain the solution by least squares. Table II. is obtained by writing the figures in vertical columns, commencing at the left in the horizontal columns. The same figures are then written below the line of division with their signs changed. (See Table II. on following page.)

FORMULÆ FOR COMPUTING THE QUANTITIES D AND R.

$$D_{11} = t_{11} - t_1 \qquad R_{10} = \tfrac{1}{11}\,(D_{10} - D_{11})$$
$$D_{10} = t_{10} - t_2 \qquad R_9 = \tfrac{1}{11}\,(D_9 - D_{10})$$
$$D_9 = t_9 - t_3 \qquad R_8 = \tfrac{1}{11}\,(D_8 - D_5)$$
$$D_8 = t_8 - t_4 \qquad R_7 = \tfrac{1}{11}\,(D_7 - D_8)$$
$$D_7 = t_7 - t_5$$

CALCULATION OF D AND R.

$t_{11} = -0.113$	$t_{10} = -0.061$	$t_9 = -0.027$	$t_8 = -0.101$	$t_7 = +0.198$
$-t_1 = +0.040$	$-t_2 = +0.151$	$-t_3 = +0.004$	$-t_4 = +0.120$	$-t_5 = -0.078$
$D_{11} = -0.073$	$D_{10} = +0.090$	$D_9 = -0.023$	$D_8 = +0.019$	$D_7 = +0.120$
	$-D_{11} = +0.073$	$-D_{10} = -0.090$	$-D_9 = +0.023$	$-D_8 = -0.019$
	$11\,R_{10} = +0.163$	$11\,R_9 = -0.113$	$11\,R_8 = +0.042$	$11\,R_7 = +0.001$
	$R_{10} = +0.014$	$R_9 = -0.010$	$R_8 = +0.004$	$R_7 = +0.000$

FORMULÆ FOR THE COMPUTATION OF THE QUANTITY Q.

$$Q_{10} = R_{10} + Q_{11} - \frac{2}{10} Q_{11} \qquad Q_8 = R_8 + Q_9 - \frac{2}{3 \times 8}(Q_{11} + Q_{10} + Q_9)$$

$$Q_9 = R_9 + Q_{10} - \frac{2}{2 \times 9}(Q_{11} + Q_{10}) \qquad Q_7 = R_7 + Q_8 - \frac{2}{4 \times 7}(Q_{11} + Q_{10} + Q_9 + Q_8) \qquad Q_{11} = 0$$

TABLE II.

COMPUTATION OF $t_1, t_2, t_3, \cdots, t_{11}$.

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
+0.001	−0.014	+0.014	−0.010	−0.014	+0.012	+0.008	+0.000	−0.005	+0.008	
−0.027	+0.005	+0.003	−0.039	−0.012	+0.048	−0.036	−0.009	+0.005	−0.002	+0.002
+0.014	−0.016	−0.038	−0.040	+0.016	+0.035	+0.034	−0.005	−0.014	+0.018	−0.004
−0.001	−0.055	−0.035	−0.004	+0.025	+0.037	+0.034	−0.017	+0.012	−0.001	+0.006
−0.022	−0.036	+0.004	−0.001	+0.014	+0.014	+0.038	+0.002	−0.002	−0.013	−0.025
−0.038	−0.002	+0.002	+0.013	+0.025	+0.022	+0.036	−0.004	+0.001	−0.014	−0.041
+0.017	−0.012	+0.001	−0.006	+0.001	+0.055	+0.035	+0.004	−0.025	−0.037	−0.043
+0.014	−0.018	+0.004	−0.014	+0.016	+0.038	+0.010	−0.016	−0.035	−0.043	+0.005
+0.002	−0.002	+0.027	−0.005	−0.003	+0.039	+0.012	−0.048	+0.036	+0.009	−0.005
	−0.001	+0.014	−0.014	+0.010	+0.014	−0.012	−0.008	−0.000	+0.005	−0.008
−0.010	−0.151	−0.004	−0.120	+0.078	+0.341	+0.198	−0.101	−0.027	−0.061	−0.113
t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	t_{11}

CHECK.

$$t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5 \quad t_6 \quad t_7 \quad t_8 \quad t_9 \quad t_{10} \quad t_{11} = 0.$$

$$-0.040 - 0.151 - 0.004 - 0.122 + 0.073 + 0.311 + 0.198 - 0.101 - 0.027 - 0.061 - 0.113 = 0.$$

FORMULÆ FOR THE COMPUTATION OF THE QUANTITY *P*.

$$
\begin{array}{lll}
S_{11} = t_{11} + t_1 & 11\,P_{11} = S_{11} + 2X & 11\,P_{11} = 0 \\
S_{10} = t_{10} + t_2 & 11\,P_{10} = S_{10} + 2X & \text{Since } P_{11} = X_{11} + X_1 \\
S_9 = t_9 + t_3 & 11\,P_9 = S_9 + 2X & \therefore 2X = -S_{11} \\
S_8 = t_8 + t_4 & 11\,P_8 = S_8 + 2X & \\
S_7 = t_7 + t_5 & 11\,P_7 = S_7 + 2X & \\
S_6 = t_6 + t_6 & 11\,P_6 = S_6 + 2X &
\end{array}
$$

CALCULATION OF *S* AND *P*.

$t_{11} = -0.113$	$t_{10} = -0.060$	$t_9 = -0.027$	$t_8 = -0.101$	$t_7 = +0.198$	$t_6 = +0.341$
$t_1 = -0.040$	$t_2 = -0.151$	$t_2 = -0.004$	$t_4 = -0.120$	$t_5 = +0.078$	$t_6 = +0.341$
$S_{11} = -0.153$	$S_{10} = -0.212$	$S_9 = -0.031$	$S_8 = -0.221$	$S_7 = +0.276$	$S_6 = +0.682$
$2X = +0.153$	$2X = +0.153$	$2X = +0.153$	$2X = +0.153$	$2X = +0.153$	$2X = +0.153$
$11\,P_{11} = 0$	$11\,P_{10} = -0.059$	$11\,P_9 = +0.122$	$11\,P_8 = -0.068$	$11\,P_7 = +0.429$	$11\,P_6 = +0.835$
$P_{11} = 0$	$P_{10} = -0.005$	$P_9 = +0.011$	$P_8 = -0.006$	$P_7 = +0.039$	$P_6 = +0.076$

CHECK.

$$
\begin{array}{l}
P_{11} + P_{10} + P_9 + P_8 + P_7 + \tfrac{1}{2}P_6 = x. \\
0 - 0.005 + 0.011 - 0.006 + 0.039 + 0.038 = +0.077.
\end{array}
$$

CALCULATION OF Q.

$Q + R - \frac{1}{p}\Sigma Q$	ΣQ	p	$\frac{1}{p}\Sigma Q$
$Q_{11} = \qquad + 0.000$		5	
$Q_{10} = \qquad + 0.014$	$+0.014$	9	-0.002
$Q_9 = - 0.010 + 0.014 + 0.002 = + 0.006$	$+0.020$	12	$+0.002$
$Q_8 = + 0.004 + 0.003 - 0.002 = + 0.007$	$+0.027$	14	$+0.002$
$Q_7 = + 0.000 + 0.006 - 0.002 = + 0.004$	$+0.031$		

FORMULÆ FOR COMPUTING $x_1, x_2, x_3, \cdots, x_{11}$.

$x_{11} = \frac{1}{2}(P_{11} + Q_{11}) \qquad x_1 = \frac{1}{2}(P_{11} - Q_{11})$
$x_{10} = \frac{1}{2}(P_{10} + Q_{10}) \qquad x_2 = \frac{1}{2}(P_{10} - Q_{10})$
$x_9 = \frac{1}{2}(P_9 + Q_9) \qquad x_3 = \frac{1}{2}(P_9 - Q_9)$
$x_8 = \frac{1}{2}(P_8 + Q_8) \qquad x_4 = \frac{1}{2}(P_8 - Q_8)$
$x_7 = \frac{1}{2}(P_7 + Q_7) \qquad x_5 = \frac{1}{2}(P_7 - Q_7)$
$x_6 = \frac{1}{2}P_6$

CALCULATION OF $x_1, x_2, x_3, \cdots, x_{11}$.

Indices.	P	Q	$P+Q$	$P-Q$		
11	$+0.000$	$+0.000$	$+0.000$	$+0.000$	$x_{11} = 0.0$	$x_1 = 0.0$
10	-0.005	$+0.014$	$+0.009$	-0.019	$x_{10} = +0.005$	$x_2 = -0.010$
9	$+0.011$	$+0.006$	$+0.017$	$+0.005$	$x_9 = +0.009$	$x_3 = +0.002$
8	-0.006	$+0.007$	$+0.001$	-0.013	$x_8 = +0.001$	$x_4 = -0.007$
7	$+0.039$	$+0.004$	$+0.043$	$+0.035$	$x_7 = +0.021$	$x_5 = +0.018$
6	$+0.076$		$+0.076$		$x_6 = +0.038$	

CHECK.

Sum of $x1 + x2 + x3 + x4 + x5 + x6 + x7 + x8 + x9 + x10 + x11 = -\frac{1}{2}S_{11}$.
$0.000 - 0.010 + 0.002 - 0.007 + 0.018 + 0.038 + 0.021 + 0.001 + 0.009 + 0.005 + 0.000 = +0.077$.

FORMULÆ FOR THE COMPUTATION OF $\lambda_2 \cdots \lambda_{10}$.

$2\lambda_2 = S_2 + Q_{11} + Q_{10}$	$\lambda_2 = + 0.007$
$3\lambda_3 = S_3 + Q_{11} + Q_{10} + Q_9$	$\lambda_3 = + 0.006$
$4\lambda_4 = S_4 + Q_{11} + Q_{10} + Q_9 + Q_8$	$\lambda_4 = + 0.007$
$5\lambda_5 = S_5 + Q_{11} + Q_{10} + Q_9 + Q_8 + Q_7$	$\lambda_5 = + 0.006$
$6\lambda_6 = S_6 + Q_{11} + Q_{10} + Q_9 + Q_8 + Q_7$	$\lambda_6 = + 0.005$
$7\lambda_7 = S_7 + Q_{11} + Q_{10} + Q_9 + Q_8$	$\lambda_7 = + 0.004$
$8\lambda_8 = S_8 + Q_{11} + Q_{10} + Q_9$	$\lambda_8 = + 0.003$
$9\lambda_9 = S_9 + Q_{11} + Q_{10}$	$\lambda_9 = + 0.002$
$10\lambda_{10} = S_{10} + Q_{11}$	$\lambda_{10} = + 0.000$

Substituting in the original equations,

$$x_1 - x_2 + \lambda_{10} = a_1, \quad x_1 - x_3 + \lambda_9 = b_1, \quad x_1 - x_4 + \lambda_8 = c_1, \text{ etc.},$$

we have

CALCULATION OF RESIDUALS.

+0.009	+0°.027	−0°.004	−0°.013	−0°.011	−0°.015	−0°.011	−0°.017	+0°.000
+0.002	+0.004	−0.009	+0.011	+0.010	−0.003	+0.000	+0.009	−0.001
−0.005	−0.017	+0.004	+0.020	+0.002	−0.003	+0.003	+0.004	
+0.015	−0.004	+0.015	+0.000	−0.010	−0.019	+0.006		
−0.006	+0.011	+0.003	−0.012	+0.004	−0.001			
+0.005	−0.009	−0.003	+0.000	+0.002				
+0.012	−0.002	−0.015	−0.018					
−0.008	+0.007	+0.009						
+0.009	+0.006							
−0.003								

As a check upon the values of the corrections thus obtained, it may be found profitable to compare these results with those found by the method in use at the Shannon Physical Laboratory of Colby University.

This method consists in the *independent* measurement of the corrections for the scale divisions and of the equal-volume points of the mercurial column for any selected length.

It is obvious that the final corrections will be found by subtracting the equal-scale corrections from the equal-volume corrections for the same points, since the graduations upon the scale must correspond in position with the position of equal-volume points.

This method seems to possess three distinct advantages.

(1) In the ordinary method only a low magnifying power can be used in the telescope or the microscope employed, on account of the necessity of observing the scale and the column at the same time. In this method each focus is obtained independently.

(2) It can only be assumed that the equal-volume points hold true for the middle point of the column. This method allows the zero point of the scale corrections to be coincident with the zero of the equal-volume corrections.

(3) Since the value of the equal volume corrections will, in a good thermometer, have a considerable degree of symmetry, it will be possible to obtain a smooth curve for single degrees from observation at a few points. Since the operation of obtaining the corrections of the single degree points of the scale involves but slight labor, it will be found feasible to obtain the calibration corrections for a large number of points by taking the corrections for the equal-volume points from the constructed curve.

These advantages will be more clearly seen in the example given below.

The data are taken by the stop method. The stops of the comparator are set a distance apart equal approximately to the length of the spaces to be compared. The microscope carriage having been brought into contact with the stop on the left, the micrometer of the microscope is brought into coincidence with, *e.g.*, the line for 0°. The carriage is then brought into contact with the stop on the right and the reading for the coincidence with line 5° is taken. The difference in these readings will give the deviation from the distance between the stops expressed in divisions of the micrometer. In the same way the deviation of the remaining spaces from the constant distance between the stops will be obtained. Column 1 of Table III. contains the values derived from the observation in the manner described. Column 2 of this table contains the results obtained after subtracting the mean of the values in the first column from each value of the screw, since in this method, an increasing reading of the micrometer screw indicates that the space measured is too short. We thus obtain the relative corrections with respect to the mean value of the corrections. The values in column 3 are obtained by the summation of the values in column 2. These values represent the derived corrections which are to be applied to the actual divisions of the scale in order to obtain the subdivision of the total length of the scale into equal parts.

In the same way the corrections for the equal volume points of the corresponding points of the tube are obtained.

It is to be noted that when this method of discussion is applied to the measured excess of the mercury column over the distance selected between the scale divisions, we shall obtain *exactly* the same results as are derived from the same data by the more complex formulæ in the method of Kohlrausch.

The next column of this table contains the determined corrections to the scale readings for the 5° points. It will be seen that these values show a close agreement with the values derived by the method of Marek which are given in the column next following.

Thus far we have assumed that the equal volume corrections hold true for every part of the space occupied by the mercurial column in each of the 10° subdivisions. It is doubtful whether any reliable evidence upon this point can be derived from an examination of these observations, but the following method of investigation may throw a little light upon the subject. In Fig. 1 the two sets of corrections are laid off as ordinates, with the value of one small square for two units in the third decimal place. The equal-space corrections are represented by the full line curve, and the equal-volume corrections by the dotted curve. The values from which the curves are derived, are taken from Table I.

TABLE III.

	Scale divisions.	Relative corrections	Summed series = Σ.	Column calibrations	Relative corrections.	Summed series = Σ.	Final corrections.	By method of Marek.	Zero of Scale at 5°.		
									From computation.	From observation.	Mean.
°	°	°	°	°	°	°	°	°	°	°	°
									−0.029	−0.019	−0.024
0– 5	5.058	+0.028	+0.028	5.076	+0.036	+0.036	+0.008		+0.000	+0.000	+0.000
5– 10	5.057	+0.027	+0.055	5.057	+0.017	+0.053	−0.002	−0.010	+0.028	+0.019	+0.024
10– 15	5.054	+0.024	+0.079	5.065	+0.025	+0.078	−0.001		+0.054	+0.042	+0.048
15– 20	5.061	+0.031	+0.110	5.080	+0.040	+0.118	+0.008	+0.002	+0.086	+0.073	+0.079
20– 25	5.044	+0.014	+0.124	5.061	+0.021	+0.139	+0.015		+0.102	+0.092	+0.097
25– 30	5.041	+0.011	+0.135	5.036	−0.004	+0.135	+0.000	−0.007	+0.114	+0.108	+0.111
30– 35	5.014	−0.016	+0.119	5.017	−0.023	+0.112	−0.007		+0.100	+0.103	+0.102
35– 40	5.024	−0.006	+0.113	5.063	+0.023	+0.135	+0.022	+0.018	+0.096	+0.098	+0.097
40– 45	5.007	−0.023	+0.090	5.017	−0.023	+0.112	+0.022		+0.074	+0.089	+0.081
45– 50	5.016	−0.014	+0.076	5.054	+0.014	+0.126	+0.050	+0.038	+0.061	+0.072	+0.066
50– 55	5.014	−0.016	+0.060	5.038	−0.002	+0.124	+0.064		+0.047	+0.059	+0.053
55– 60	5.023	−0.007	+0.053	5.013	−0.027	+0.097	+0.044	+0.021	+0.041	+0.042	+0.041
60– 65	5.017	−0.013	+0.040	5.019	−0.021	+0.076	+0.036		+0.030	+0.033	+0.032
65– 70	5.015	−0.015	+0.025	5.093	−0.047	+0.029	+0.004	+0.001	+0.016	+0.023	+0.019
70– 75	5.012	−0.018	+0.007	5.027	−0.013	+0.016	+0.009		+0.000	+0.012	+0.006
75– 80	5.024	−0.006	+0.001	5.033	−0.007	+0.009	+0.008	+0.009	+0.006	+0.008	+0.001
80– 85	5.044	+0.014	+0.015	5.049	+0.009	+0.018	+0.003		+0.011	+0.013	+0.012
85– 90	5.040	+0.010	+0.025	5.061	+0.021	+0.039	+0.014	+0.005	+0.022	+0.015	+0.018
90– 95	5.019	−0.011	+0.014	5.015	−0.025	+0.014	+0.000		+0.013	+0.011	+0.012
95–100	5.016	−0.014	+0.000	5.026	−0.014	+0.000	+0.000	+0.000	+0.000	+0.000	+0.000
Mean . .	5.030		Mean . .	5.040							

In Fig. 2 the equal-volume curve is laid off for the midway points of the 5° subdivisions.

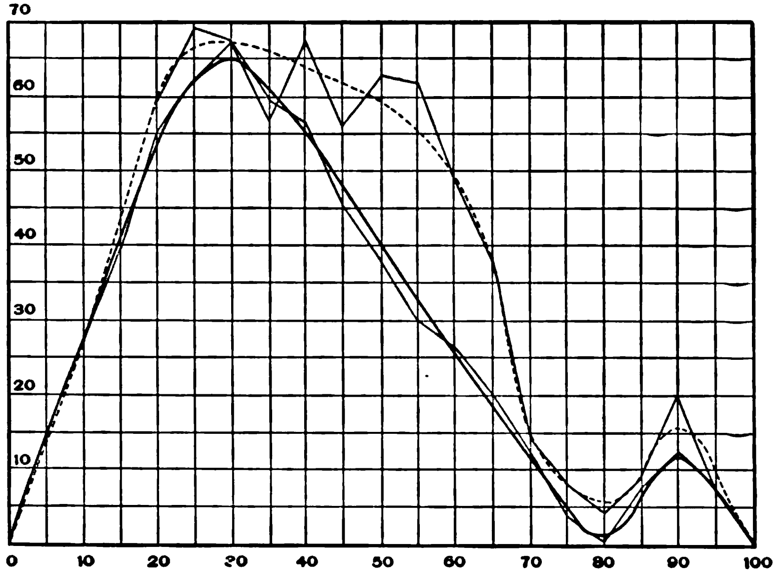


Fig. 1.

In Fig. 3 the zero of the equal space curve is taken at 5°. There are two methods of obtaining the data for this comparison. First, we can

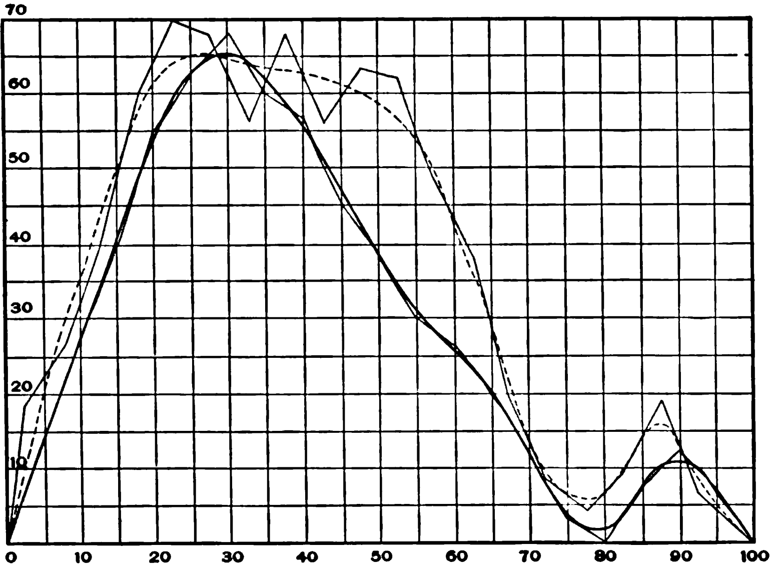


Fig. 2.

start the observed measures from this point, obtaining the results given in the last column but one of Table III. Secondly, we may employ the method given on page C 68, Vol. I., of the *Memoirs* for the conversion from one system to another.

Let $x =$ the correction for any point whatever in any given system. Two points a and b in this system have the corrections x_a and x_b. These corrections are transformed into another system y_m, giving the corrections y_a and y_b by the following formula: —

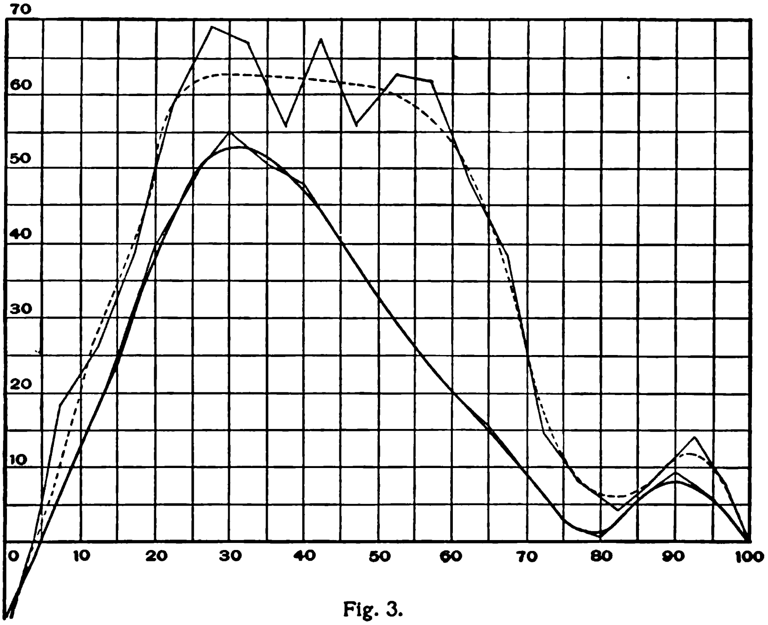


Fig. 3.

$$(4)\ y_m = \frac{x_n - x_a - A(m-a)}{B} + y_a = x_n - x_a - A(m-a) + y_n \text{ nearly,}$$

$$\text{in which } A = \frac{(x_b - x_a) - (y_b - y_a)}{(b + y_b) - (a + y_a)}, \quad B = \frac{(b + x_b) - (a + x_a)}{(b + y_b) - (a + y_b)} = 1 \text{ nearly.}$$

In order to transform the zero point of the scale division to the 5° point we have the following: —

$$\begin{array}{lllll} a = 5 & x_a = +28 & y_a = 0 & a + x_a = 5.028 & a + y_a = 5.00 \\ b = 100 & x_b = 0 & y_b = 0 & b + x_b = 100.000 & b + y_b = 100.00 \end{array}$$

$$A = \frac{-0.028}{100 - 5} = -0.030 \quad y_n = x_n - x_m + 0.030\,x_m.$$

For $n = 70^\circ$ $y_n = +110 - 0.028 + 15 \times 0.030 = +0.086.$
For $m = 0^\circ$ $y_n = 0 - 0.028 - 0.001 = 0.029.$
For $x_n = 60^\circ$ $y_n = +55 - 0.028 + 55 \times 0.030 = +0.041.$

In this manner the values in the second column from the last of Table III. were computed, and the means of the computed and the observed values are given in the last column. The mean values thus obtained were used in laying off the equal space-curve in Fig. 3. The measured differences between the two curves in this figure will give the corrections given in Table I.

As far as these observations indicate, the values in column I. are a little nearer the correct values than those given in columns II. and III., since the sum of the squares of the vertical column for I. is less than from either II. or III.

Internal and External Pressure.

We now take up the corrections due to external and internal pressure upon the mercury in the bulb of a thermometer.

(5) Let $d =$ the density of the water in which a thermometer is supposed to be immersed to a depth, below the surface, a distance h.
$u =$ the distance of the middle point of the bulb to the zero point of the scale.
$u_1 =$ the length of one degree expressed in millimeters.
$\delta_0 =$ the density of mercury at 0° C.
$B =$ the observed reading of the barometer.
$B' =$ the value of B reduced to 760 mm. at the level of the sea and at the latitude of 45°, by the formula

$$B' = B \frac{1 + 0.0000081\ T}{1 + 0.0001815\ T} \times \frac{g\phi}{g_{45^\circ}},$$

in which the values of $\frac{g\phi}{g_{45^\circ}}$, taken from the following table, are to be substituted.

ϕ	$\frac{g\phi}{g_{45^\circ}}$	ϕ	$\frac{g\phi}{g_{45^\circ}}$	ϕ	$\frac{g\phi}{g_{45^\circ}}$
35°	0.99912	41°	0.99964	47°	1.00018
36	0.99921	42	0.99972	48	1.00027
37	0.99929	43	0.99982	49	1.00030
38	0.99937	44	0.99981	50	1.00045
39	0.99946	45	1.00000	51	1.00054
40	0.99955	46	1.00009		

TABLE IV.

CORRECTION TO 2403 FROM DIFFERENT CURVES.

	I.	II.	III.	Mean = m_1.	B = Marek.	$\frac{B+m^1}{2}=m_2$.	$m_2 - \text{I} = v_1$.	$m_2 - \text{II} = v_2$.	$m_2 - \text{III} = v_3$.	v_1^2	v_2^2	v_3^2
0	+ 0	− 3	+00	− 1	+ 0	+ 0	+ 0	+ 0	+ 0	0	0	0
5	+ 0	+ 6	+10	+ 5								
10	+ 2	+12	+20	+11	−10	+ 0	− 2	−12	−20	4	144	400
15	+ 2	+16	+20	+14								
20	+ 6	+20	+10	+15	+ 2	+ 8	+ 2	−12	−12	4	144	144
25	+ 9	+24	+ 5	+13								
30	+ 4	+18	− 2	+ 7	− 7	+ 0	− 4	−18	+ 2	16	324	4
35	+ 9	+22	+ 3	+11								
40	+16	+29	+14	+20	+18	+19	+ 3	−10	+ 5	9	100	25
45	+25	+38	+26	+31								
50	+30	+48	+35	+40	+38	+39	+ 1	− 9	+ 4	1	81	16
55	+44	+54	+38	+45								
60	+43	+54	+34	+44	+21	+32	−11	−22	− 2	121	484	4
65	+37	+50	+19	+38								
70	+13	+34	+ 4	+17	+ 1	+10	− 3	−24	+ 6	9	576	36
75	+ 5	+14	+ 4	+ 8								
80	+ 7	+10	+11	+ 9	+ 9	+ 9	+ 2	− 1	− 2	4	1	4
85	+ 6	+ 3	+10	+ 6								
90	+12	+ 6	+ 1	+ 6	+ 5	+ 5	− 7	− 1	+ 4	49	1	16
95	+ 5	+10	+ 2	+ 6								
100	+00	+00	+00	+00	+00	+ 0	+ 0	+ 0	+ 0	0	0	0
Sums										217	1855	649

Let P_e = exterior pressure for a vertical position.

P_i = interior pressure for a vertical position.

β_e = the coefficient of exterior pressure = a quantity whose reading increases in proportion to an increase of 1 mm. in the exterior pressure.

β_i = the coefficient of interior pressure = a quantity that diminishes in proportion as the interior increases 1 mm.

Designating the exterior pressure by γ_1, the interior pressure by γ_2, and their sum by γ, we have

$$\gamma = \beta_i(u + nu_1) - \beta_e\left(B' + \frac{h}{13.6} - 760\right).$$

The plate upon which is represented the apparatus for comparing thermometers, as shown facing page 237, will indicate the method of obtaining the value of β_e with a very brief explanation. The bulb of a thermometer is immersed in a bottle of water with an air-tight connection. The air space at the top of the bottle is connected with an air pump by means of a small lead pipe, a branch of this pipe being connected with a glass tube which is immersed in a reservoir of mercury, which serves as a manometer. The operation of obtaining the value of β_e is as follows: The thermometer is first read under atmospheric pressure. The air is then pumped from the bottle till the column in the manometer remains at a stationary height, when the height of the column is read in millimeters, and a second reading of the thermometer is taken. We have thus an observed difference in the height of the mercurial column corresponding to an observed difference in pressure. Or: —

$$(P_1 - P_0)\beta_e = t_1 - t_2,$$

$$\beta_e = \frac{t_1 - t_2}{P_1 - P_0}.$$

Care must be taken in these observations to allow considerable time to elapse after a change of pressure takes place, in order for the glass to take its normal condition under strain, otherwise much too large values of β_e will be obtained. As much as ten minutes should be allowed for this purpose. The mean of a sufficient number of observations gave for this thermometer

$$647\,\beta_e = 0.033^\circ.$$

$$\therefore \quad \beta_e = 0.00005.$$

The value of the coefficient β_i is found by taking the difference between successive readings of a thermometer first in a vertical and then in a horizontal position.

All the observations which have been made at Breteuil agree in giving substantially the same value for β_i as for β_e for the same thermometer.

Since the coefficients for exterior and interior pressure are identical, the expression for the corrections can be simplified, giving, for a vertical point, —

$$(760 - B')\beta = \gamma_1,$$
$$(u + nu_1)\beta = \gamma_2.$$

The Fundamental Interval of a Thermometer.

If the reading of a thermometer in melting ice, when corrected for calibration error and for external and internal pressure, is zero, and if the reading in steam, at 760 mm. pressure, is exactly 100°, it is obvious that no correction for fundamental interval will be required. If under these conditions the thermometer, immersed in steam, at normal pressure, reads, *e.g.*, 100°.20, and the thermometer reads 0°.05 in melting ice, the difference between the two corrected values will give a quantity which is designated the fundamental interval of a thermometer. This difference must be distributed over the entire length of the distance between 0° and 100°. The chief difficulty in determining the true reading of a thermometer immersed in steam is the separate determination of the pressure of the steam. This is accomplished by means of a water barometer, as seen in the Regnault apparatus shown in Plate I. The method of procedure at Breteuil is as follows: —

Suppose that the reading of the barometer is 745.90 mm., that the reading of the water barometer is 12.7 mm. = 0.93 mm. for mercury, that the corrected reading of the zero point is — 0.363, and the corrected reading for steam 99°.369.

We first find the temperature corresponding to the pressure 745.90 + 0.93 = 746.83 mm. from the table following: —

Pressure.	T	Δ	**Pressure.**	T	Δ
mm.	°		mm.	°	
740	99.2577	0.0376	750	99.6310	0.0371
741	99.2953	0.0374	751	99.6681	0.0370
742	99.3321	0.0375	752	99.7051	0.0370
743	99.3702	0.0373	753	99.7421	0.0370
744	99.4075	0.0374	754	99.7791	0.0369
745	99.4449	0.0373	755	99.8160	0.0369
746	99.4822	0.0372	756	99.8529	0.0368
747	99.5194	0.0373	757	99.8897	0.0368
748	99.5567	0.0371	758	99.9265	0.0368
749	99.5938	0.0372	759	99.9633	0.0367
750	99.6310		760	1.0000	

Pressure.	T	Δ	Pressure.	T	Δ
mm.	°		mm.	°	
760	100.0000		770	100.3649	
761	100.0367	0.0367	771	100.4012	0.0363
762	100.0733	0.0366	772	100.4374	0.0362
763	100.1099	0.0366	773	100.4736	0.0362
764	100.1465	0.0366	774	100.5098	0.0362
765	100.1830	0.0365	775	100.5559	0.0361
766	100.2194	0.0364	776	100.5820	0.0361
767	100.2559	0.0365	777	100.6180	0.0360
768	100.2923	0.0364	778	100.6540	0.0360
769	100.3286	0.0363	779	100.6900	0.0360
770	100.3649	0.0363	780	100.7259	0.0359

From this table we find the temperature corresponding to 746.83 mm. $= 99^\circ.512$.

True value $$1^\circ = \frac{99.512}{99.369 - (-363)} - 1 = -0.00221.$$

Or, we may proceed as follows: —

Corrected 100° reading $= 99^\circ.3690$
Correction for pressure $= \dfrac{12.7 \times 0.0371}{13.6}$ $= -\ 0.0345$
Reading for pressure of 760 mm. $= 99^\circ.3345$
Reduction from 745 to 760 mm. $= 14.1 \times 0.0371 = +\ 0.5231$
True reading at 100° $= 99^\circ.8576$
True reading at 0° $= -\ 0.3630$
Fundamental interval $= 100^\circ.2206$
Corrected $1^\circ = \kappa$ $= -\ 0.00221$

There is one reason which seems to have considerable weight in favor of the latter method of reduction. If the reduced readings of the water barometer are subtracted from the observed temperature readings, the differences should be constant if the water barometer indicates the real steam pressure. It is just here that the observer is liable to serious error. It will be seen from the following example that the true reading of the barometer is obtained only after there has been a continuous constant steam pressure for at least twenty minutes. In the observations which follow it has been the practice to allow the thermometer to remain under a little greater than the normal pressure for about fifteen minutes, and after that under a constant pressure for about fifteen minutes longer.

The following is an example: —

OBSERVATION JANUARY 22, 1896.

T	**Water manometer.**	*t*	Δ*t*	**True *t*.**
h. m.	mm.	°		°
9 25	3	100.218	−0.008	100.210
9 28	2	100.235	−0.005	100.230
9 30	7	100.292	−0 019	100.273
9 32	8	100.302	−0.021	100.281
9 36	9	100.320	−0.024	100.296
9 38	9	100.334	−0.024	100.310
9 42	9	100 327	−0.024	100.303
9 44	8	100.337	−0.024	100.313
9 46	9	100.348	−0.024	100.324
9 48	9	100.348	−0.024	100.324
9 52	5	100.340	−0.014	100.326

Following the practice at Breteuil, the reading for the zero point has been obtained from exposure to melting ice immediately *after* the exposure to steam. For this particular thermometer the time required for recovery from the depression of the zero point is about five hours, hence in the present instance it would make little difference whether the reading before or after exposure was selected. The latter reading has been selected on the score of uniformity of practice. These observations apparently furnish an additional reason for this choice, although additional observations at low pressures are needed to confirm the apparent fact that for low pressures all the thermometers investigated by Dr. Rogers have their readings lowered at the zero point, while no such effect is seen when the reading for the zero point is taken immediately after exposure to steam. It will be seen by an examination of the following table that the fundamental interval of this thermometer remains nearly constant.

Date.	B'	100°.		Depression.	t.
		Before.	After.		
	inches	°	°	°	°
Jan. 17	769.7	+0.000	−0.027	+ 0.027	100.110
Jan. 20, morning .	770.0	+0.004	−0.026	+ 0.030	100.108
Jan. 20, afternoon .	770.0	+0.006	−0.027	+ 0.033	100.166
Jan. 21, morning .	764.9	+0.003	−0.027	+ 0.030	100.122
Jan. 21, afternoon .	763.4	+0.004	−0.026	+ 0.030	100.147
Jan. 22, morning .	766.1	+0.007	−0.023	+ 0.030	100.152
Jan. 22, afternoon .	766.9	+0.000	−0.015	+ 0.015	100.138
Jan. 23, morning .	771.6	+0.011	−0.008	+ 0.019	100.169
Jan. 23, afternoon .	770.6	+0.000	−0.018	+ 0.018	100.152
Feb. 10, morning .	746.2	−0.070	+0.004	− [0.074]	100.129
Feb. 11, morning .	741.9	−0.107	−0.018	− [0.089]	100.135
Feb. 12, morning .	751.7	−0.003	−0.025	+ 0.022	100.166
Feb. 13, morning .	762.7	+0.006	−0.018	+ 0.024	100.150
Feb. 14, morning .	746.4	+0.004	−0.003	+ 0.007	100.152
Feb. 15, morning .	759.1	+0.011	+0.000	+ 0.011	100.136
Mean				+ 0.019	100.142

Determination of the Fundamental Interval by means of a Thermometer whose Fundamental Interval is known.

Let us suppose that a thermometer whose constants are fully determined is immersed to a depth h in water which has a nearly constant temperature. The temperature t will be determined by the equation

$$t = n + x_n + \gamma_1 + \gamma_2 - \zeta + \eta\kappa,$$

in which $\zeta =$ the reading for the corrected zero point $= n + x_n + \gamma_1 + \gamma_2$ for 0°, and $\eta = n_1 + x_n + \gamma_1 + \gamma_2$.

Very soon afterwards place in the liquid another thermometer whose constants are all known except ζ and κ. We shall then have, since the temperature of the liquid is supposed to remain constant,

$$t = [n_1 + x_2 + \gamma_1 + \gamma_2] = \zeta_x - n_1\kappa_x.$$

Immediately afterwards put this thermometer in melting ice and obtain a corrected reading for the zero point by the equation

$$t_0 = n_0 + x_n + \gamma_1 + \gamma_2 - \zeta_x.$$

Since for this point $\kappa_x = 0$, for the second thermometer we shall have

$$\zeta = (n_0 + x_n + \gamma_1 + \gamma_2)$$

$$\kappa_x = \frac{t}{(n_1 + x_n + \gamma_1 + \gamma_2) - \zeta}.$$

We take as an example the data for thermometer A given on p. D 11, Vol. I. It is required to find ζ and κ for thermometer G. The constants for these two thermometers are given on pp. D III. and D IV. For thermometer A we have

$$\begin{array}{llllll} t = n & + x_n & + \gamma_1 & + \gamma_2 & - \zeta & + \eta\kappa \\ 13.025 & + 0.1666 & + 0.001 & + 0.014 & - 0.027 & - 0.029 = 13.150. \end{array}$$

For thermometer G we have

$$13.150 = 13.544 + 0.070 + 0.000 + 0.011 - \zeta = 13.626.$$

For t_0 we have

$$\begin{array}{lllll} \zeta = n & + x_n & + \gamma_1 & + \gamma_2 & \\ \zeta = + 0.365 & + 0.002 & + 0.000 & + 0.005 & = + 0.373, \end{array}$$

then

$$\kappa = \frac{13.150}{13.626 - (+0.372)} - 1 = - 0.00740,$$

$$\eta\kappa \text{ for } 13^\circ.6 = - 0.101.$$

For a check we have

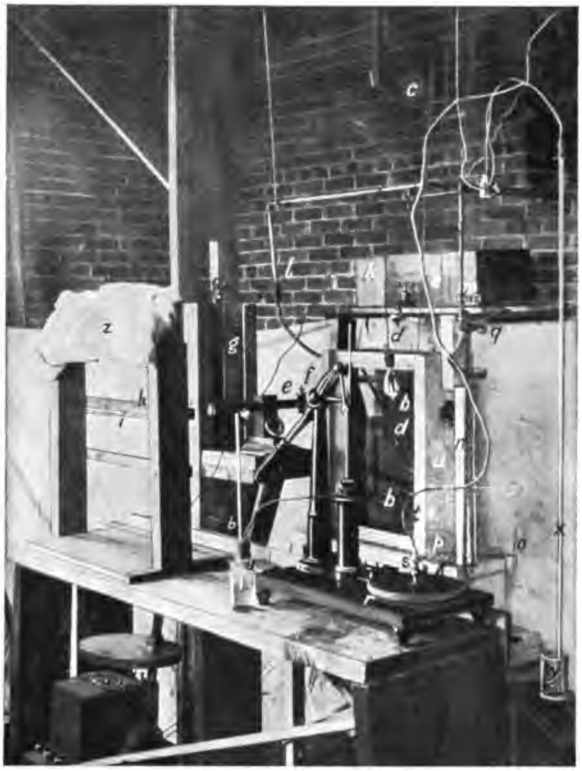
$$\begin{array}{lllllll} t = n_1 & + x_n & + \gamma_1 & + \gamma_2 & - \zeta & + \eta\kappa & \\ 13^\circ.150 = 13.544 & + 0.071 & + 0.000 & + 0.011 & - 0.372 & - 0.101 & = 13^\circ.153. \end{array}$$

We take as a second example the comparison of 2403, G. 2405, and Tounelot 62. The fundamental intervals for these thermometers are given below. Those for G. 2405 and T. 62 have been furnished by Dr. Rogers.

THE FUNDAMENTAL ELEMENTS.

	2403	2405	T. 62
	mm.	mm.	mm.
u = distance from center of bulb to zero of scale . . . . =	69	60	63
n' = the number of degrees of any reading . . . . =	...	...	...
u_1 = the value of one degree in millimeters . . . . =	4.47	3.29	3.69
$n'u_1$ = the distance from zero of scale to an observed reading . . . . . . =	$n' \times 4.47$	$n' \times 3.29$	$n' \times 3.69$
$u + n'u_1$ = distance of any reading from center of bulb . . . . =	...	...	...
β = coefficient of external pressure . . =	0.00005	0.00016	0.00014
β_i = coefficient of internal pressure . . =	0.00005	0.00016	0.00014
ζ = corrected reading of zero point . . =	− 0.0016	− 0.001	+0.400
κ = constant derived from the fundamental interval. . . . . . . =	−0.00142	−0.00142	−0.0038

PLATE I.



Apparatus for the Comparison of Thermometers, and for Determining the Coefficient of External Pressure.

(*a*)(*a'*) = water tank with windows of heavy plate glass in front and rear.
(*b*)(*b'*) = paddle wheels driven by the pulley (*c*).
(*k*) = ice box.
(*l*) = connection with water pipe.
(*m*) = connection with steam pipe.
(*q*) = connection with water tank.
(*g*) = upright groove, in which the sliding block *j* is moved by a weight passing over a pulley.
(*e*) = telescope of very short focus.
(*f*) = filar micrometer.
(*h*) = frame upon which the thermometer (*i*) is mounted in a horizontal position.
(*z*) = sheet cotton for shading (*i*).
(*d*) = wire from which the thermometers are suspended in the water tank.
(*n*) = Regnault's steam comparing apparatus.
(*o*) = water barometer for measuring the excess over normal steam pressure.
(*u*)(*s*)(*r*)(*x*)(*y*) = apparatus for measuring the coefficient of external pressure.

DATA DERIVED FROM OBSERVATION.

(All the thermometers were immersed in water at a depth $h = 510$ mm. Observed barometer reading, 745.6 mm. Att. ther. = 8°.22. Reduced barometer = $B' =$ 744.6 mm. 550 mm. for water = 38 mm. for mercury.)

		2403	2405	T.62
		°	°	°
n	=	+6.727	+6.730	+6.259
x_n	=	+0.008	−0.002	−0.044
$\beta_e \times 15.4 = \gamma_1$	=	+0.001	+0.002	+0.001
$-\beta_e \times 22\,6 = \gamma_1$	=	−0.001	−0.003	−0.002
$\beta_i \times 10.19 = \gamma_2$	=	+0.005	+0.013	+0.012
$-\zeta$	=	+0.016	+0.000	−0.461
n	=	+6.756	+6.740	+6.765
$\eta\kappa$	=	−0.009	−0.008	−0.025
T	=	+6.747	+6.732	+6.740

In conclusion we desire to express our thanks to Dr. Rogers for assistance rendered in this investigation and for the use of the apparatus belonging to the Shannon Physical Laboratory of Colby University.

REFERENCES TO MEMOIRS OF THE INTERNATIONAL BUREAU.

Reference Nos.	I.	II.	V.
(1)	—	C 37	85
(2)	—	C 45	8
(3)	—	C 45–C 47	18–21
(4)	—	C 68	11
(5)	D 5–D 7	C 34	26–37
(6)	—	C 62	—
(7)	A 10–A 13	—	—
(8)	D 5	—	—
(9)	D 7	—	—
(10)	D 5	—	—
(11)	A 46–A 48	C 52–C 53	—
(12)	D 9–D 11	—	—
(13)	D 5–D 11	C 54–C 63	49–62
(14)	D 11	—	—
(15)	D 10–D 11	—	—
(16)	D 5	—	—

A Study of the Apparent Capacity of Condensers for Short-charge Periods.

By Hubert V. Carpenter.

Condensers have frequently been studied to find the quantity of electricity q contained in them after charging or discharging for varying lengths of time through very high resistances. Curves obtained from the results have been published,[1] and show the validity of the formula[2]

$$q = Q(e^{-\frac{t}{RC}}),$$

where q represents the quantity contained after discharging for a time t through a resistance R, and Q is the quantity corresponding to the theoretical capacity C for the given potential. Curves are also given[1] for different periods of charge through high resistance, which agree with the corresponding formula[2]

$$q = Q(1 - e^{-\frac{t}{RC}}).$$

This latter formula, verified by experiment, indicates that, with any value of R greater than zero, the quantity held by any condenser depends upon the time of charge, and that it increases toward the limit Q as the time increases. For any low value of R, however, the exponential term becomes negligible with extreme rapidity. Take, for example, the case in hand of condenser No. 1, with C equal to 10^{-6} farads and R about 0.4 ohm. Assume $t = 0.0001$ second and substitute in the formula

$$q = Q(1 - e^{-\frac{t}{RC}}) = Q(1 - 2.718^{-\frac{0.0001}{0.4 \times 10^{-6}}}) = Q[1 - 2.718^{-250}].$$

This shows that the exponential term may be safely neglected in experimental work, and that q is practically equal to Q.

There is, however, a difference in the apparent capacity of condensers that seems to depend upon the time of charge in much the same way as shown by the exponential term. This difference is overcome far too slowly, however, to be explained by the formula, and we must therefore look for some other cause. The usual explanation is, that the charge slowly "soaks into," or "is absorbed by," the dielectric; but just how or why this absorption takes place is hard to explain.

In order to determine the rate at which this soaking in takes place, and its amount for different types of condensers, the writer, in connection with Mr. E. W. Poole, undertook an experimental study of it. Three condensers were studied. Condenser No. 1 was a subdivided mica condenser, marked 1 microfarad, made by Queen & Co. No. 2 was a condenser, insulated with paraffined paper, constructed in the laboratory. No. 3 was made by the Stanley Electric Manufacturing Co., and has a very thin dielectric, whose composition is not stated. The resistance R in the charging circuit was made equal to 0.4 ohm. This low resistance was chosen in order to eliminate the effect of the exponential term in the formula for quantity.

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[1] Nichol's Lab. Manual, Vol. II., p. 156.

[2] Bedell and Crehore's Alternating Currents, p. 72.

In order to get data for very short charge periods, it became necessary to devise some method of charging which would allow accurate determinations of the time to be made. A method of doing this was suggested to us by Mr. B. E. Moore, who gave us many valuable suggestions throughout the work. This method consisted in the use of a Carhart pendulum interrupter[1] arranged to charge the condenser for a short period, and then discharge it instantly through a ballistic galvanometer having a long period of vibration. Corrections were made as shown in Fig. 1. The pendulum, swinging from left to right, overturns first the detent of K_1 and thus connects the battery to the condenser. When key K_2 is released, the battery circuit is broken, and the condenser is at once discharged through the galvanometer. Key K_2 was placed at the lowest point of the arc described by the pendulum, and key K_1 was moved along the arc to the left of K_2. The horizontal distance between the two keys at the points where they were struck by the pendulum was measured with a micrometer screw, and the corresponding time of charge could then be computed with great accuracy. By this method we were able to measure the time of charge in ten-thousandths of a second for any length of time from 0.0006 to 0.3 of a second. Below 0.0006 second the torsional vibration of the pendulum affected the accuracy of the time measurement. The time necessary for the lever of key K_1 to pass from the lower contact to the upper was appreciable. This error was eliminated by adjusting the distance K_1–K_2 until no deflection resulted, and then applying this distance as a correction to the distances as measured.

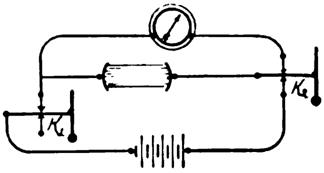

Fig. 1.

Observations were made with three different charging potentials to indicate any relation that might exist between the rate or amount of absorption and the potential difference.

A general idea of the data, Table I., may be obtained more easily from the curves, Fig. 2, where the abscissæ represent time of charge and the ordinates the corresponding deflections of the ballistic galvanometer.

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[1] Carhart, PHYSICAL REVIEW, Vol. II., p. 392; or Carhart and Patterson's Electrical Measurements, p. 106.

These deflections are, of course, proportional to the quantities Q, when corrected to $2 \sin \frac{\theta}{2}$, as they should be for absolute values. Since, however, the effect of such correction would only be to reduce each deflection about two millimeters, the ratio of the highest to the lowest value for any particular curve would not be appreciably affected.

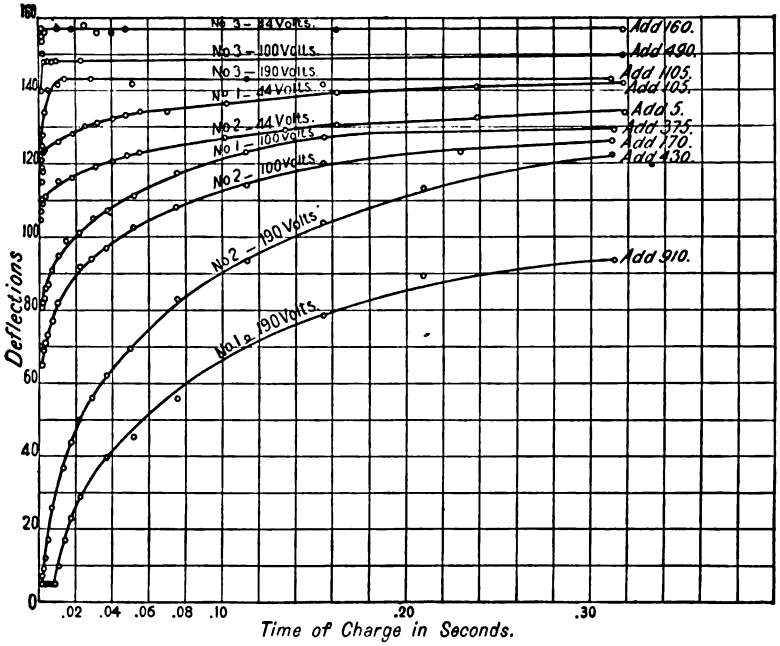

Fig. 2.

Time of charge.	Deflections at 44 volts.			Time of charge.	Deflections at 44 volts.		
	No. 1.	No. 2.	No. 3.		No. 1.	No. 2.	No. 3.
seconds				seconds			
0.0006	220.0	110.0	314	0.0409	237.0	126.0	316
0.0014	223.0	119.0	315	0.0486	238.0	127.0	317
0.0020	224.0	110.0	317	0.0560	239.0	128.0	—
0.0036	229.0	116.0	316	0.0714	239.5	129.0	—
0.0110	231.0	120.0	317	0.1023	241.0	132.0	—
0.0185	233.0	121.0	317	0.1636	244.5	135.5	317
0.0260	235.0	123.0	318	0.2390	246.0	137.5	—
0.0335	236.0	124.0	316	0.3196	247.0	139.0	317

Time of charge.	No. 1.		No. 2.		No. 3.	
	100 volts.	190 volts.	100 volts.	190 volts.	100 volts.	190 volts.
0.0015	456	825.0	235.0	—	630	—
0.0022	460	915.0	240.0	435.0	640	1230.0
0.0029	458	915.0	239.0	438.0	638	1233.0
0.0037	461	915.0	241.0	442.5	638	1239.0
0.0051	462	915.0	243.0	447.0	638	1245.0
0.0074	466	915.0	247.0	456.0	—	1245.0
0.0111	470	919.5	252.0	462.0	—	1246.5
0.0148	474	927.0	257.0	468.0	—	1248.0
0.0224	476	939.0	262.0	480.0	638	1246.5
0.0299	480	945.0	264.0	486.0	—	1245.0
0.0374	482	949.5	267.0	493.5	—	1246.5
0.0523	486	955.5	273.0	499.5	—	1245.0
0.0754	492	966.0	278.5	513.0	—	1245.0
0.1145	498	982.5	284.0	523.5	—	1248.0
0.1560	502	988.5	290.0	534.0	—	1248.0
0.2300	500	999.0	293.0	543.0	640	1248.0
0.3120	504	1003.5	296.0	552.0	640	1248.0
5.0000	524	1050.0	314.0	579.0	640	1257.0

Distance of scale = 110 cm.

Discussion of Results.

The data and curves show: (1) that the rapidity of absorption varies greatly for the different condensers; (2) that it varies with the potential difference; and (3) that the quantity absorbed after the first 0.005 second of charge is not strictly proportional to the potential difference. The total quantity absorbed cannot be determined, since absorption begins instantaneously upon closing the circuit. Steinmetz states the following law: Condensers having solid dielectrics should absorb under an alternating electromotive force an amount of energy proportional to the square of the potential difference. This has been shown experimentally to be approximately true within a wide range by J. Sahulka.[1] In our case, however, the time of discharge is so long that the above law cannot be applied.

The rapidity with which No. 3 absorbs its full charge indicates a very different dielectric than is found in the other two. The chief difference probably lies in the thickness of the insulation. The per cent of full charge that is absorbed after the first 0.005 second is, at 190 volts, by the Stanley condenser equal to 0.95 per cent; by the paraffin, 22.80 per cent; by the mica, 12.86 per cent.

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[1] Wiener Sitz. Ber., Vol. 102.

This would seem to indicate that the quantity absorbed varies much more rapidly than in exact inverse ratio to the thickness, because this would make the paraffin insulation about twenty-five times as thick as that of the Stanley. This ratio of thickness is, however, possible.

The data show also that for short periods the value of Q is not strictly proportional to the potential difference. For example, take the condenser No. 2 with $t = 0.01$ second. The ratio of Q for 190 volts to Q for 100 volts is not 1.9, but is 1.76. At 190 volts the ratio of the full-charge capacity of No. 3 to the full-charge capacity of No. 2 is equal to 2.16, while for a charge period of 0.01 second the ratio is 2.62. The corresponding ratios of No. 1 to No. 2 are 1.81 and 1.99, respectively. These ratios bring out again the difference in the rate of absorption between the three condensers. It is apparent that the capacity of a condenser may be very different in an alternating-current circuit from its value when determined by full-charge methods. A change in the frequency of the alternations might easily cause an appreciable change in the apparent capacity.

This method could well be used to study the rate at which the different dielectrics give up their absorbed charges by changing the connections. It would then be interesting to combine the two and limit both charge and discharge to an equal period.

UNIVERSITY OF ILLINOIS,
DEPARTMENT OF ELECTRICAL ENGINEERING,
July, 1896.

NOTE ON THE OSMOTIC THEORY OF THE VOLTAIC CELL.

BY H. M. GOODWIN.

IN a recent paper on "The Chemical Potential of the Metals"[1] Mr. Bancroft, from certain experiments of his own, and from the investigations of Paschen on dropping electrodes, and of others, concludes that, "*first*, the potential difference between a metal and an electrolyte is not a function of the concentration of the salt solution, nor of the positive ion, except in certain special cases; *second*, it is a function of the electrode, of the negative ion, and of the solvent," etc. Without quoting further, these conclusions will be seen to be so contradictory to the well-established theory of Nernst and Ostwald that, in view of the mass of experimental evidence published in confirmation of this theory during the last few years, a serious consideration of them must seem superfluous. I wish to point out and show, however, that certain predictions of the author, on which he bases a criticism of the results published by me on the electrometric determination of the solubility of difficultly soluble salts,[1] are in direct contradiction with facts, and that, therefore, the objections raised are ungrounded.

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[1] PHYSICAL REVIEW, Vol. III., 250, 1896.

In the article referred to I have calculated the solubility of silver chloride, bromide, and iodide from measurements of the electromotive force of elements of the form —

$$\mathrm{Ag} \left| \, x\,\mathrm{AgNO_3} \, \right| \begin{matrix} x\,\mathrm{KCl} \\ \mathrm{AgCl} \end{matrix} \Big| \, \mathrm{Ag}.$$

The fundamental formula for all reversible cells applies to this element, its electromotive force E being given by the formula

$$E = \frac{RT}{e_0} ln \frac{p_1}{p'},$$

where p_1 and p' are the partial osmotic pressures (or concentrations) of the *kations* at the two electrodes, R the gas constant, T the absolute temperature, and e_0 the electrochemical equivalent. By Nernst's solubility principle the product of the silver and chlorine ions must be constant at the second electrode (electrode of the second kind) ; *i.e.*

$$p'(p' + p_2) = s^2,$$

where s is the solubility of silver chloride in water, and p_2 the concentration of the chlorine ions coming from the potassium chloride. Since p_2 is very great compared with p', we have, approximately,

$$p' = \frac{s^2}{p_2},$$

or

$$E = \frac{RT}{e_0} ln \frac{p_1 p_2}{s^2},$$

from which s may be computed. In the deduction of this formula it is assumed that the electrolytic solution pressure of the electrodes is eliminated, and that the fundamental law of the osmotic theory of the cell holds true ; namely, that the potential difference between a metal and a liquid is determined by the *kation* concentration of the solution and is independent of the anion concentration.

Mr. Bancroft, however, questions the validity of this law — states, in fact, just the reverse of it. He bases his reasoning on the well-known fact that metals like mercury (which form insoluble halogen salts) show a different P.D. according to the soluble halogen salt with which they are in contact. Thus the absolute P.D. between mercury and a normal solution of potassium chloride, bromide, and iodide are, according to Paschen's[2] results with dropping electrodes, 0.539, 0.483, and 0.400 volts respectively. Mr. Bancroft is apparently unable to see that such electrodes are practically electrodes of the second kind.

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[1] Zeitsch. für Phys. Chem., 13, 641, 1894; Technology Quarterly, 8, 166, 1895.

[2] Paschen, Wied. Ann., 43, 590, 1891.

He says (p. 257): "If the potential difference between mercury and potassium chloride or potassium bromide solutions be due to the amount of mercury as ion which has gone into solution, we must say that the amount varies as we change from potassium chloride to potassium bromide [certainly it does], or, in other words, that the negative ion has an effect." In regard to the last part of this sentence I have shown very conclusively that the effect of the *anion* is in all cases of electrodes of the second kind an *indirect* but calculable one, inasmuch as their concentration determines by Nernst's solubility principle the concentration of the kations. Farther on he says: "I do not see that the relative solubilities of mercurous chloride and bromide can be used to help out matters, because we do not have a saturated solution at all." But this is practically what we do have, as is shown by the fact that the addition of a depolarizer (*e.g.* as in the normal electrode) does not appreciably alter the P.D. As a matter of fact, the above-cited results are in complete accord with Nernst's theory and might have been predicted from it. Since the iodides are less soluble than the bromides, and these again than the chlorides, by the solubility principle the kation concentration must diminish in the same order while the *anion* concentration remains practically constant, whence the decrease in the P.D.

Moreover, from a purely experimental standpoint, no more conclusive proof of the independence of the P.D. of the anion could be desired than that furnished by Neumann.[1] He measured the P.D. between thallium and over twenty different thallium salts, both organic and inorganic (including the chloride), and found in all cases the same value to within about 0.001 volt. Yet Mr. Bancroft will not be convinced by these figures of the fallacy of his law, since Neumann did not include in his measurements thallium bromide and iodide, the two salts whose slight solubility precluded their use for electrodes of the first kind.

As further evidence of the confounding of electrodes of the first and second kind, Mr. Bancroft applies conclusions deduced from the latter to the former. On page 260 he says: —

"If we consider the cell $Zn|ZnC_2|ZnBr_2|Zn$, the two solutions being assumed to be of the same concentration and dissociation and the wandering velocity of the bromine ion being further assumed to be identical with that of the chlorine ion, we should expect an electromotive force of 0.08 volt." With the bromide replaced by the iodide an electromotive force of 0.152 volt would be "expected." By reason of my failure to take this fact into account, my computation of the solubility of the silver halogens is, in his opinion, of no significance. Now if Mr. Bancroft had but made an experiment or two on cells of the above type, he would have found that instead of his predicted results the electromotive force both of the chloride-bromide and of the chloride-iodide cell is practically zero, — just as the osmotic theory requires (since the concentration of the zinc ions [the *kations*] at both electrodes is practically the same).

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[1] Zeitsch. f. Phys. Chem., 14, 225, 1894.

In the following table are given the results of some measurements I have made on cells of this type, which, aside from their bearing on Mr. Bancroft's statements, are of interest as furnishing additional evidence of the absolute confidence with which one may predict results from the osmotic theory. The halogen solutions were standardized by Vollard's method and then made up to the desired strength. The electrodes were liquid amalgams prepared by electrolyzing pure salt solutions of the metals over pure mercury kathodes. The cells used were of the H form, similar to those described in my previous paper.

Measurements were made by the usual Poggendorff method, exact compensation being obtained on a slide wire bridge by a capillary electrometer sensitive to 0.0001 volt. The apparatus was calibrated against a Clark cell from the Reichsanstalt. Measurements were made at the room temperature, which varied from 18° to 22° C. The values given are the mean of a series of measurements on three or four independent elements. The electromotive force was nearly constant from the start to within 0.001–0.002 volt.

TABLE I.

Cell.	Concentration 0.1 normal.	Concentration 0.05 normal.	Concentration 0.01 normal.
$ZnCl_2$–$ZnBr_2$	0.0030 volt	0.0021 volt	0.0023 volt
$ZnCl_2$–ZnI_2	0.0025	0.0019	0.0021

It is seen that the electromotive force of both elements is not only nearly the same but also practically zero, exactly as the osmotic theory requires. The 0.002 volt observed is probably due to the P.D. at the liquid junction. It is of a negligible amount compared with the 0.15 volt predicted by Mr. Bancroft. It need hardly be pointed out that the observed independence of the electromotive force of the absolute concentration is also a verification of the theory. These experiments, like those of Neumann, prove conclusively that in these reversible cells the *anion has absolutely no effect* on the electromotive force.

I give also some measurements on similar reversible cadmium cells, which are not without interest. The electromotive force of these cells should of course also be zero, provided the assumption regarding the equal dissociation of the salts holds true. This, however, is well known, both from freezing-point determinations and conductivity measurements, ***not*** to be the case. The halogen salts of cadmium are all dissociated to an abnormally small extent, and moreover to an unequal extent, the iodide being least, the bromide and chloride more dissociated. Hence, the chloride solution should contain more cadmium ions than either the bromide or iodide solutions, and hence be positive to them. The nitrate, on the other hand, is nearly normally dissociated, and contains therefore many more ions than the chloride, — hence in a chloride-nitrate cell the chloride solution should be negative. These predictions of the theory were found to be completely verified by experiment, as the following measurements show: —

TABLE II.

$CdCl_2$-$CdBr_2$	+0.005 volt
$CdCl_2$-CdI_2	+0.015
$CdCl_2$-$Cd(NO_3)_2$	−0.024

The solutions were all 0.1 molecular normal.

The electromotive force of the chloride-iodide cell is no longer small and equal to that of the bromide cell, but three times greater; the reversal of the electromotive force when the nitrate is used as one electrolyte is also evident.

In view of these results, Mr. Bancroft's objections to my interpretation of the results on silver halogen electrodes are evidently unfounded. I am utterly unable to understand why he should have passed over in silence the most striking and unquestionable proof I gave of the theory of difficultly soluble depolarizers, namely, the complete agreement of the electromotive force of the element,

$$\mathrm{Tl}\left|\begin{matrix}\mathrm{TlCl}\\ \text{in KCl}\end{matrix}\right|\begin{matrix}\mathrm{TlCl}\\ \text{in KBr}\end{matrix}\Bigg|\,\mathrm{Tl},$$

calculated from the solubility of the depolarizers ***analytically*** determined, with that actually observed, a result absolutely impossible on the assumptions of his theory.

In closing, it should be stated that the failure to discuss the other conclusions of Mr. Bancroft's paper does not imply an acceptance of them. In many cases their fallacy is as apparent as in the case here considered.

ROGERS LABORATORY OF PHYSICS,
MASSACHUSETTS INSTITUTE TECHNOLOGY,
June, 1896.

The Division of an Alternating Current in Parallel Circuits with Mutual Induction.[1]

By Frederick Bedell.

A DIVIDED circuit with mutual induction between the two branches is shown in Fig. 1, being the same as a transformer with the primary and secondary circuits connected in parallel. The electromotive force equations for the two circuits are similar, the internal electromotive forces in each being equal to the same impressed electromotive force. The electromotive force of mutual induction will be positive or negative according to the sense or direction in which the coils are connected. If the coils are connected as in Fig. 2, so that the ampere turns of the two coils assist each other, the electromotive forces of self and mutual induction will be of the same sign, and the coefficient of mutual induction will be positive. If the coils are connected so as to oppose each other, as in Fig. 3, the electromotive force of mutual induction will be opposite in sign to that of self-induction. The coefficient of mutual induction may accordingly be $+M$ or $-M$. Writing the electromotive force as a function of the time, we have the following equations: —

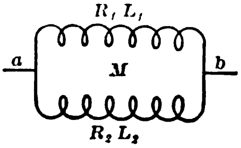

Fig. 1.
Divided circuit with mutual induction.

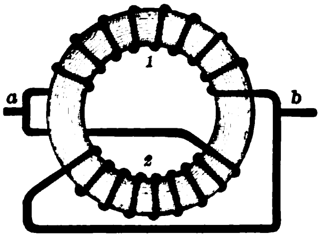

Fig. 2.
Coils additive.

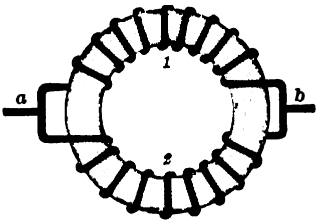

Fig. 3.
Coils opposed.

$$e = f(t) = R_1 i_1 + L_1 D i_1 \pm M D i_2; \qquad (1)$$
$$e = f(t) = R_2 i_2 + L_2 D i_2 \pm M D i_1. \qquad (2)$$

By differentiation, the above equations become

$$f'(t) = R_1 D i_1 + L_1 D^2 i_1 \pm M D^2 i_2;$$
$$f'(t) = R_2 D i_2 + L_2 D^2 i_2 \pm M D^2 i_1.$$

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[1] A paper read at the Liverpool meeting of the British Association for the Advancement of Science, 1896.

Eliminating i_2 by determinants, we have

$$\begin{array}{cccc} i_2 & Di_2 & D^2 i_2 & \text{Other Terms.} \\ \hline 0 & \pm M & 0 & R_1 i_1 + L_1 D i_1 - f(t) \\ 0 & 0 & \pm M & R_1 D i_1 + L_1 D^2 i_1 - f'(t) \\ R_2 & L_2 & 0 & \pm M D i_1 - f(t) \\ 0 & R_2 & L_2 & \pm M D^2 i_1 - f'(t) \end{array} = 0.$$

Canceling out the first column and third row, as indicated by fine type, we obtain

$$M^2[\pm M D^2 i_1 - f'(t)] \mp M R_2 [R_1 i_1 + L_1 D i_1 - f(t)] \\ \mp M L_2 [R_1 D i_1 + L_1 D^2 i_1 - f'(t)] = 0. \tag{3}$$

From this equation the value for the current i_1 is obtained: —

$$i_1 = \frac{R_2 f(t) + (L_2 \mp M) f'(t)}{(L_1 L_2 - M^2) D^2 + (R_1 L_2 + R_2 L_1) D + R_1 R_2}. \tag{4}$$

A similar expression is found for i_2.

For an harmonic electromotive force

$$e = f(t) = E \sin \omega t;$$
$$f'(t) = E\omega \cos \omega t.$$

Since $D^2 \sin \omega t = -\omega^2 \sin \omega t$, we may write $-\omega^2$ for D^2.

In equation (4) we may make these substitutions for $f(t), f'(t)$, and D^2. By combining the sine and cosine terms into one term, and multiplying numerator and denominator by D, we have

$$i_1 = \frac{DE\sqrt{R_2^2 + (L_2 \mp M)^2 \omega^2} \sin\left(\omega t + \text{arc tan} \frac{(L_2 \mp M)\omega}{R_2}\right)}{\alpha D - \omega\beta}; \tag{5}$$

where

$$\alpha = R_1 R_2 - \omega^2 (L_1 L_2 - M^2),$$
$$\beta = \omega (R_1 L_2 + R_2 L_1).$$

To free equation (5) from the operator D, perform the operation D as indicated in the numerator (by differentiating the numerator), and multiply numerator and denominator by $\alpha D + \omega\beta$. Substitute $-\omega^2$ for D^2 in the denominator and again perform the operation D in the numerator. We thus obtain an expression for i_1 which is free from D; this may be written in the following form: —

$$i_1 = I_1 \sin(\omega t + \phi_1); \tag{6}$$

where
$$I_1 = \frac{E}{\dfrac{\sqrt{\alpha^2+\beta^2}}{\sqrt{R_2^2+(L_2 \mp M)^2\omega^2}}}. \tag{7}$$

$$\phi_1 = -\text{arc tan}\frac{\beta}{\alpha} + \text{arc tan}\frac{(L_2 \mp M)\omega}{R_2}. \tag{8}$$

Similarly, for the current in the second branch,

$$i_2 = I_2 \sin(\omega t + \phi_2); \tag{9}$$

where
$$I_2 = \frac{E}{\dfrac{\sqrt{\alpha^2+\beta^2}}{\sqrt{R_1^2+(L_1 \mp M)^2\omega^2}}}. \tag{10}$$

$$\phi_2 = -\text{arc tan}\frac{\beta}{\alpha} + \text{arc tan}\frac{(L_1 \mp M)\omega}{R_1}. \tag{11}$$

The ratio of the two currents is

$$\frac{I_1}{I_2} = \frac{\sqrt{R_2^2+(L_2 \mp M)^2\omega^2}}{\sqrt{R_1^2+(L_1 \mp M)^2\omega^2}}. \tag{12}$$

Their phase difference is

$$\phi_1 - \phi_2 = \text{arc tan}\frac{(L_2 \mp M)\omega}{R_2} - \text{arc tan}\frac{(L_1 \mp M)\omega}{R_1}. \tag{13}$$

Where the coils are opposed and nearly similar the angle of phase difference between the currents depends largely upon the amount of magnetic leakage.

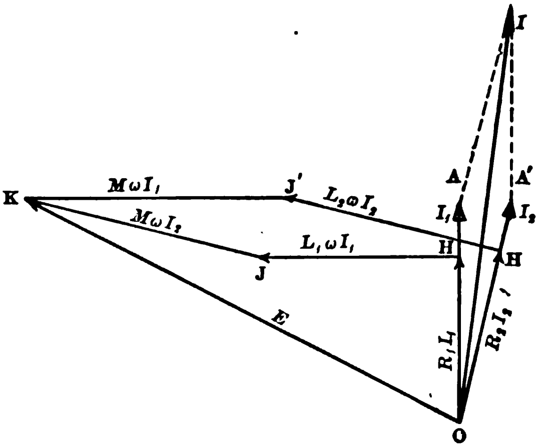

Fig. 4.

Coils additive; *M* positive.

The diagram for the electromotive forces in a divided circuit in which the effects of self and mutual induction are additive is shown in Fig. 4. The two currents are I_1 and I_2, respectively, the main current being their vector sum I.

In branch one, OH is the power electromotive force (in the absence of branch two); HJ and JK are the electromotive forces of self and mutual induction, being at right angles to the currents I_1 and I_2, respectively. The corresponding electromotive forces for the second circuit are shown in the same diagram. The sum of the several electromotive forces in either circuit add up to E, the impressed electromotive force.

Ordinarily the branch current and the main current would lag behind the impressed electromotive force by a large angle approaching 90°.

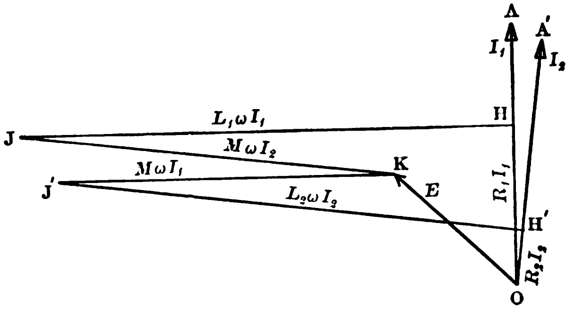

Fig. 5.

Coils opposed: M negative.

When the two circuits are identical, the branch currents I_1 and I_2 would have the same magnitude and direction; the points H′ and J′ would coincide with the points H and J, respectively. An inspection of the figure will show that for this case, in the absence of magnetic leakage, the effect of the mutual induction would be to double the tangent of the angle of lag.

Figure 5 shows the corresponding diagram for the case in which the electromotive forces of self and mutual induction are opposed. The current I_1 produces an electromotive force HJ in the first branch, and J′K in the second branch; these electromotive forces being at right angles to the current I_1, and opposite to each other in direction, on account of the sense in which the two coils are connected. Similarly, H′J′ is opposite in direction to JK. The effects of self and mutual induction consequently tend to neutralize each other. The electromotive force OK, required to cause the currents I_1 and I_2 to flow, is accordingly made less on account of the mutual induction, and would be reduced to the value of the ohmic electromotive force in the case of two similar coils wound oppositely. In this case the angle of lag would be reduced to zero, and the coil would be non-inductive.

The equivalent resistance and self-induction for the two coils together, whether they are additive or opposed, may be found by resolving E into two components, one in the direction of the main current, and the other at right angles to it. These components will be equal to $R'I$ and $L'\omega I$, respectively. Their magnitudes may be obtained graphically, and from them the values of the equivalent resistance R' and the equivalent self-induction L'. The equivalent resistance and self-induction of either branch may be obtained in the same manner.

The conclusions relating to the special case of two identical coils wound without magnetic leakage, stated above in connection with the graphical construction, may be reached by substituting in the formulæ the values $R_1 = R_2 = R$; and $L_1 = L_2 = \pm M = L$. Also, $\beta + \alpha = 2\,L\omega + R$. The two cases are as follows: —

When M is positive. — When M is positive, the coils being wound in the same direction,

$$\phi = -\text{arc tan}\,\frac{2\,L\omega}{R} + \text{arc tan}\;0 = -\text{arc tan}\,\frac{2\,L\omega}{R}. \tag{14}$$

The current in each branch is

$$I_1 = I_2 = \frac{E}{\sqrt{R^2 + (2\,L\omega)^2}}. \tag{15}$$

For this case, therefore, the effect produced is the same as though the mutual induction were zero and the self-induction of each circuit doubled, the tangent of the angle of lag being doubled and each current being diminished accordingly.

The main current is

$$I = I_1 + I_2 = \frac{2\,E}{\sqrt{R^2 + (2\,L\omega)^2}} = \frac{E}{\sqrt{\left(\frac{R}{2}\right)^2 + L^2\omega^2}}, \tag{16}$$

which is the same that would flow in a single coil, with the same self-induction and half the resistance as either branch.

When M is negative. — When the coils are identical and opposed,

$$\phi = -\text{arc tan}\,\frac{2\,L\omega}{R} + \text{arc tan}\,\frac{2\,L\omega}{R} = 0; \tag{17}$$

$$I_1 = I_2 = \frac{E}{R}. \tag{18}$$

Each coil acts in this case as though non-inductive, the magnitude of the current being in accordance with Ohm's law, and the angle of lag being zero.

Cases where the two coils are not identical may be treated by substituting particular values for the resistance and coefficients of induction in the general equations.

ITHACA, N.Y., June, 1896.

ON THE SPECIFIC GRAVITY AND ELECTRICAL CONDUCTIVITY OF THE NORMAL SOLUTIONS OF SODIUM AND POTASSIUM HYDROXIDES, AND HYDROCHLORIC, SULPHURIC, NITRIC, AND OXALIC ACIDS.

BY E. H. LOOMIS.

THE various tables of specific gravity of these common alkaline and acid solutions differ in some cases so considerably that the attempt to compute the specific gravity of a particular solution, of known strength, by use of the tables, results in much uncertainty.

This uncertainty has made it desirable to measure the specific gravity of certain normal solutions about which the most doubt exists. I have accordingly undertaken to prepare such solutions of NaOH, KOH, HCl, HNO_3, H_2SO_4, and $(COOH)_2$, for the purpose of measuring their specific gravity. I have also taken this opportunity to measure their electrical conductivity.

The various solutions were prepared by titration. The titration standards were normal solutions of sodium carbonate and oxalic acid whose preparation will be first described.

Normal Sodium Carbonate. — Sodium acid carbonate ($NaHCO_3$, Trommsdorff, c.p.) was glowed, and thus converted into Na_2CO_3. A saturated solution of this product was then treated with a current of pure CO_2 until the precipitation of $NaHCO_3$ was complete. This precipitated $NaHCO_3$ was then carefully heated at a temperature slightly under red glow until its weight became constant. The Na_2CO_3 thus obtained was deemed sufficiently pure for the given purpose.

The molecular weight of this salt ($O = 16$) is 106.12. Thus a normal solution requires 53.06 g. per liter of the solution.

The liter flask employed at 18° C. contained 1000 c.c.

A simple calculation shows that such a flask should hold apparently 997.667 g. of water whose temperature is 18° C. when the weighing is made in the air with brass weights. The mark on the flask was fixed in this way. The volume of such a flask is a true liter, *i.e.* that of 1000 g. of water at 4° C.

Normal Oxalic Acid Solution. — The c.p. acid was purified by recrystalizing twice from water, and, finally, by recrystalizing from dilute alcohol, according to Mohr. The crystals were then freed from the mother liquor, first, by a mechanical process, and, finally, by rolling and pressing between folds of filter paper.

The acid thus prepared still left a faint stain when volatilized on platinum foil. It was deemed, however, sufficiently pure for the purpose. The molecular weight of oxalic acid $(COOH)_2$, $2\,H_2O$ is 108. Accordingly, 54 g. were dissolved in water, and the solution diluted to a liter, at 18° C. These normal solutions of Na_2CO_3 and $(COOH)_2$ were then balanced against each other, and found to be equivalent. Thus the accuracy of each was confirmed.

The other normal solutions were prepared by titration with one or the other of these directly, or with some normal solution which had been directly standardized with them.

All of the titrations were performed by Mr. Hulett of the Princeton Chemical Laboratory, and I wish here to express my deep obligation to him for the great care with which this work was done, and for his additional assistance in the preparation and purification of many of the materials.

Sodium and Potassium Hydroxide. — The hydroxides were prepared by direct union of the metals with water in a silver vessel.

The metals were carefully freed from "scale" in the open air at about 0° C. The solutions were filtered through asbestos.

The NaOH solution was perfectly colorless, while the KOH solution, when concentrated, retained a slight straw color which indicated traces of organic matter. All reasonable precautions were taken to exclude CO_2 from the solutions during these operations.

Both the NaOH and KOH solutions were brought to normal strength by titration with a normal solution of H_2SO_4 which had already been standardized directly against the normal Na_2CO_3.

In these and the subsequent titrations, methyl orange was used as indicator, and, whenever possible, a second titration, with phenolphtalein as indicator, was made for the sake of checking the work.

It should be added that the final reduction of the given solution to normal strength was made by using a one-tenth normal solution of the particular standard with which the comparison was made.

After the required amount of water or compound had been added, a final titration was made to insure against any possible errors.

Hydrochloric Acid. — The c.p. acid (Baker and Adamson) was heated, and the expelled HCl gas was conducted through a platinum tube into water. The HCl solution was then diluted to normal strength by titration with the normal NaOH. It was then compared with the standard Na_2CO_3 directly, and found to be in exact balance.

Nitric Acid. — The c.p. acid (Baker and Adamson) was found to be free from chlorine and was then redistilled through platinum.

The acid was then reduced to normal strength by direct titration with the standard Na_2CO_3.

Sulphuric Acid. — The c.p. acid of Baker and Adamson was deemed of sufficient purity for the purpose. It was titrated directly with the standard Na_2CO_3.

Determination of Specific Gravity.

Water at 4° C. was taken as unity, and the temperature of the solutions was uniformly 18° C. The method employed was that of Sprengel, modified as I have already described in a former paper.[1]

The Electrical Conductivity.

The measurements were made in accordance with the Kohlrausch method. The resistance vessel was of the double type (Kohlrausch, *Leitfaden d. Physik*, Leipzig, 1892, 4th figure, p. 304).

Its "resistance capacity" was found (adopting the Kohlrausch value for normal NaCl as 696.10^{-8}) to be 0.00158 Hg units.

The vessel and contents were kept at 18° C. ± 0°.05 by immersion in a water bath kept carefully at this temperature for about thirty minutes before the measurement was made.

The Results.

In the following table the specific gravity of the various solutions is found in column 2, and the electrical conductivity K, multiplied by the factor 10^7 is found in column 3.

Conductivity of mercury at 4° C. is taken as standard.

1 Compound.	2 Sp. Gr. $\frac{18^\circ}{4^\circ}$.	3 $K \cdot 10^7$.	1 Compound.	2 Sp. Gr. $\frac{18^\circ}{4^\circ}$.	3 $K \cdot 10^7$.
NaOH	1.0418	145	HNO_3	1.0324	278
KOH	1.0481	170	H_2SO_4	1.0306	183
HCl	1.0165	279	$(COOH)_2$	1.0199	55

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[1] Loomis, PHYSICAL REVIEW, Vol. III., p. 270, 1896.

In addition to the NaOH solution, of which the measurements are given in the table, a second normal solution was prepared, by Mr. Hulett, by two successive purifications of Trommsdorff's c.p. NaOH by the well-known ethyl-alcohol process. The entire process was conducted in air freed from CO_2.

The product was perfectly colorless, and nearly transparent.

This hydroxide is believed to be as nearly pure as the alcohol method permits.

The specific gravity of this normal solution is 1.0417, and the electrical conductivity is 146.10^7. These results are not in agreement with those found for the solution made directly from the metallic sodium. The latter method undoubtedly yields the purer hydroxide, and the results for this are given in the table.

PHYSICAL LABORATORY, PRINCETON UNIVERSITY.

NOTES.

The Buffalo Meeting of the American Association for the Advancement of Science. — At the Buffalo meeting of the Association, which was held August 24 to 29, the attendance was small. The programme in Physics, as has been the case at many of the smaller recent meetings, did not, however, suffer from the smallness in general attendance. While there were no papers of supreme importance, the list was an interesting one throughout. The attendance upon the sittings of the section was excellent, and the number of physicists present at the meeting who are actively engaged in the research was noticeably large.

The title of Chairman Mees' address was "Electrolysis and Some Outstanding Problems in Molecular Dynamics."

The following is the list of papers presented in Section B: "Polarization and Internal Resistance of a Galvanic Cell," by B. E. Moore; "The Lead Storage Cell," by B. E. Moore; "A Theory of Galvanic Polarization," by W. S. Franklin and L. B. Spinney; "On the Counter Electromotive Force of the Electric Arc," by W. S. Franklin; "On the Element of Diffraction in Fresnel's Experiments with Two Mirrors and with the Bi-prism," by Ernest R. von Nardroff; "Segmental Vibrations in Aluminum Violins," by Alfred Springer; "Preliminary Note on a Proposed New Standard of Light," by Clayton H. Sharp; "A Photographic Study of the Roentgen Rays," by W. A. Rogers; "Note on the Duration of the X-Ray Discharge in Crookes' Tubes," by Benjamin F. Thomas; "Preliminary Communication concerning the Anomalous Dispersion of Quartz for Infra-red Rays of Great Wave-length," by Ernest F. Nichols; "An Experimental Study of the Charging and Discharging of Condensers," by F. E. Millis; "Notes on Certain Physical Difficulties in the Construction of Modern Large Guns," by W. LeConte Stevens; "On the Photographic Trace of the Curves described by the Gyroscopic Pendulum," by Ernest Merritt; "On the Distribution of High Frequency Alternating Currents throughout the Cross-Section of a Wire," by Ernest Merritt; "On the Compactness of a Beam of Light," by Ernest R. von Nardroff; "Some Points in the Mechanical Conception of the Electromagnetic Field," by W. S. Franklin; "Mechanical Models of the Electric Circuit," by Brown Ayres; "Graphical Treatment of Alternating Currents in Branching Circuits," by Henry T. Eddy; "Description and Exhibition of a Convenient Form of the 'Interferential Comparer,' and of an Interferential Caliper Attachment for Use in

Physical Laboratories," by W. A. Rogers; "Description and Exhibition of a Bench Comparator for General Use in Physical Laboratories," by W. A. Rogers; "On the Rule for the Dynamo and Motor," by Alexander Macfarlane; "Note on the Effect of Odd Harmonics upon the Virtual Values of Periodically Varying Quantities," by Frederick Bedell and James E. Boyd; "Experimental Determination of the Relative Amounts of Work done in changing the Lengths of Two Metal Bars under the Same Thermal Conditions, by an Envelope of Heated Air, and by Pure Radiations in a Vacuum," by W. A. Rogers; "A New Alternating-Current Curve Tracer," by Edward B. Rosa; "On a Piece of Apparatus constructed by Joseph Henry," by C. E. West; "Visible Electric Waves," by B. E. Moore; "Electrical Waves in Long Parallel Wires," by A. D. Cole; "The Influence of a Static Charge of Electricity on the Surface Tension of Water," by Edward L. Nichols and John Anson Clark; "Determination of the Specific Heats of Nitrogen by Adiabatic Expansion," by W. S. Franklin and L. B. Spinney; "The Analysis of Vowel Sounds by Means of the Sympathetic Vibrations of a Rigid Body," by L. B. Spinney; "Description and Exhibition of a Portable Apparatus for recording Curves of Alternating Currents and Electromotive Force," by H. J. Hotchkiss.

The officers for 1897 are: —

President: Wolcott Gibbs, of Newport, R.I.

Vice-Presidents: A. Mathematics and Astronomy, W. W. Beman, of Ann Arbor, Mich.; B. Physics, Carl Barus, of Providence, R.I.; C. Chemistry, W. P. Mason, of Troy, N.Y.; D. Mechanical Science and Engineering, John Galbraith, of Toronto, Canada; E. Geology and Geography, I. C. White, of Morgantown, W.Va.; F. Zoölogy, G. Brown Goode,[1] of Washington, D.C.; G. Botany, George F. Atkinson, of Ithaca, N.Y.; H. Anthropology, W. J. McGee, of Washington, D.C.; I. Social and Economic Science, Richard T. Colburn, of Elizabeth, N.J.

Permanent Secretary: F. W. Putnam, of Cambridge, Mass. (office, Salem, Mass.).

General Secretary: Asaph Hall, Jun., of Ann Arbor, Mich.

Secretary to the Council: D. S. Kellicott, of Columbus, Ohio.

Secretaries of the Sections: A. Mathematics and Astronomy, James McMahon, of Ithaca, N.Y.; B. Physics, Frederick Bedell, of Ithaca, N.Y.; C. Chemistry, P. C. Freer, of Ann Arbor, Mich.; D. Mechanical Science and Engineering, John J. Flather, of Lafayette, Ind.; E. Geology and Geography, C. H. Smyth, Jun., of Clinton, N.Y.; F. Zoölogy, C. C. Nutting, of Iowa City, Iowa; G. Botany, F. C. Newcombe, of Ann Arbor, Mich.; H. Anthropology, Harlan I. Smith, of New York, N.Y.; I. Social and Economic Science, Archibald Blue, of Toronto, Canada.

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[1] Deceased Sept. 6, 1896.

Treasurer: R. S. Woodward, of New York, N.Y.

The next annual meeting will be held in Detroit during the week preceding the British Association meeting in Toronto.

The British Association for the Advancement of Science.—Under the presidency of Sir Joseph Lister, the sixty-sixth annual meeting of the Association was held in Liverpool, September 16–23, with an attendance of over three thousand. The interest in the meetings of the Physics Section (A), presided over by Prof. J. J. Thomson, was particularly due to the vigorous discussions by a score of leading physicists, Lord Kelvin being chief, his comments, ingenuous and pertinent, never allowing an obscure point to pass, and ever stimulating the scientific pulse of the meeting.

The presidential address by J. J. Thomson, and the papers for two days, dealt almost solely with Roentgen and cathode rays. This prolonged discussion, in which Lenard and Bjerknes as well as English physicists took part, characterized the meeting, but lead to no generally accepted results.

On the recommendations of the committee appointed at Ipswich with Sir Douglas Galton as chairman, the Association voted to take active measures to secure from the government the necessary appropriation for the establishment of a National Physical Laboratory for the more accurate determination of physical constants and for other quantitative research, similar in many respects to the Physikalische-technische Reichsanstalt established and supported by the German government. Dr. Kohlrausch took part in the discussion of the requirements of such an institution. It is suggested that the laboratory be erected on the grounds of the Kew Observatory at Richmond Park.

After an animated discussion, the report of the Electrical Standards Committee was adopted, recommending the *erg* as the absolute unit of heat and 4.2×10^7 ergs as a secondary unit to be called the *calorie*. It is thus proposed to establish thermal units independent of thermometry.

The evening lecture to workingmen was admirably given by Dr. Fleming, his subject being "The Earth a Great Magnet."

The following papers were read in Section A: Isaac Roberts, "On the Evolution of Stellar Systems"; G. H. Darwin, "On Periodic Orbits"; P. Lenard, "On Cathode Rays"; J. J. Thomson and E. Rutherford, "On the Laws of Conduction of Electricity through Gases exposed to the Röntgen Rays"; A. W. Rucker and W. Watson, "On the Transparency of Glass and Porcelain to the Röntgen Rays"; S. P. Thompson, "On Hypophosphorescence"; S. P. Thompson, "On the Relation between Kathode Rays, X-Rays, and Becquerel's Rays"; H. H. F. Haysman, "Observations on the X-Rays"; Allan Cunningham, "Connection of Quadratic Forms"; H. M. Taylor, "On Great Circle Sailing"; S. H. Burbury, "On the

Stationary Motion of a System of Equal Elastic Spheres"; G. H. Bryan, "On Some Difficulties connected with the Kinetic Theory of Gases"; W. E. Plummer, "Comparison of the Results obtained from the Records of Robinson's, Osler's, and Dines's Anemometers"; A. Lawrence Rotch, "On the Exploration of the Upper Air by Means of Kites"; W. Hibbert, "On a One-volt Standard Cell with Small Temperature Coefficient"; W. H. Preece, "On Disturbance in Sub-Marine Cables"; Lord Kelvin, Magnus Maclean, and Alexander Galt, "On the Communication of Electricity from Electrified Steam to Air"; Lord Kelvin, J. T. Bottomley, and Magnus Maclean, "On Measurements of Electric Currents through Air at Different Densities down to One Five-Millionth of the Density of Ordinary Air"; Joseph Goold, "On Synchronisation of Sound Generators"; E. Rutherford, "On a Method of Detecting Electromagnetic Waves"; F. Bedell, "On the Division of an Alternating Current in Parallel Circuits with Mutual Induction"; E. H. Griffiths, "Note on the Measurement of Electrical Resistance"; S. A. Sworn, "Researches in Absolute Mercurial Thermometry"; Lord Kelvin, "On the Molecular Dynamics of Hydrogen Gas, Oxygen Gas, Ozone, Peroxide of Hydrogen, Vapor of Water, Water and Ice"; J. Chunder Bose, "On a Complete Apparatus for the Study of the Properties of Electric Waves"; W. M. Mordey, "Carbon Megohms for High Voltages"; W. M. Mordey, "On an Instrument for Measuring Magnetic Permeability"; G. H. Bryan, "On the Sailing Flight of Birds"; A. A. Rambaut, "On the Effect of Refraction on the Diurnal Movement of Stars, and a Method of allowing for it in Astronomical Photography"; R. Harley, "On the Stanhope Arithmetical Machine of 1780"; Elster and Geitel, "Photo-electric Sensitization of Salts by Kathode Rays"; A. P. Trotter, "A Direct-reading Form of Wheatstone Bridge"; W. N. Shaw, "The Total Heat of Water"; J. E. Keeler, "Measurements of the Velocity of Rotation of the Planets by the Spectroscopic Method"; L. A. Bauer, "On the Components Fields of the Earth's Permanent Magnetism"; F. T. Trouton, "The Duration of the X-radiation at Each Spark"; J. A. Harker and A. Davidson, "On Reostene, a New Resistance Metal"; R. Harley, "Results connected with the Theory of Differential Resolvents"; W. Barlow, "Homogeneous Structures and the Symmetrical Partitioning of them, with Application to Crystals"; J. Burke, "Change of Absorption accompanying Fluorescence."

Reports were submitted by committees on the following subjects: "On the Establishment of a National Physical Laboratory"; "On the $G(r, \nu)$ Integrals"; "On Bessel Functions and Other Mathematical Tables"; "On Meteorological Observations on Ben Nevis"; "On Solar Radiation"; "On Seismological Observations"; "On Meteorological Photographs"; "On the Comparison and Reduction of Magnetic Observations";

"On Electrical Standards (on Thermal Units)"; "On Magnetic Standards"; "On Electrolysis"; "On the Sizes of Pages of Periodicals."

The grants made amounted to £1355.

The Association meets next year at Toronto, August 18, under the presidency of Sir John Evans. American scientists will extend a hearty welcome to their British guests.

F. B.

NEW BOOKS.

Lehrbuch der Experimentalphysik. Von ADOLPH WÜLLNER. Fünfte vielfach umgearbeitete und verbesserte Auflage. Erster Band, "Allgemeine Physik und Akustik." Zweiter Band, "Die Lehre von der Wärme." Leipzig, B. G. Teubner, 1895 and 1896.

The publication of a carefully edited work which has for its aim the orderly presentation of the phenomena of physics, the judicious grouping of these phenomena according to their relationships, and the mathematical development of theory as far as it may be safely carried without allowing mathematics to dominate the physics, is not an event of everyday occurrence, and students of science should congratulate themselves upon the issue of a new edition of a work so well known and approved as the one under consideration.

There seems to be good ground for the belief that, of late years, the systematic study of physical phenomena has not received that attention its importance demands, and that mathematical physics and research work have claimed the student's time too early in his course. The existence of such a state of affairs can be accounted for by the very natural and commendable desire to produce as soon as possible a good piece of original work for publication or for a thesis, but it is responsible for much crude and immature work, and often fosters a spirit of conceit which the student's real attainments do not warrant.

Works on mathematical physics and laboratory manuals have been produced in abundance during the last ten years, but these, valuable as they are, do not supply the earliest want of the student. He needs an intimate acquaintance with the phenomena of nature and their relations. This, without doubt, can be gained most thoroughly and effectively at first by the study of text-books.

Without this fundamental knowledge the student's mathematical training loses half its value; for he wants that which gives him the ability to read the physical meaning of his equations, and his skill in the manipulation of apparatus lies idle or is misdirected through ignorance.

Wüllner's *Lehrbuch* is well adapted to the use of the young student who, having followed the general courses offered to undergraduates in our colleges, wishes to proceed with a thorough and systematic study of experimental physics.

The author takes little for granted, and treats elementary problems and describes the simplest instrumental appliances fully and in a manner which generally leaves little opportunity for criticism.

At the same time his subject is well written up to date, and includes a fair presentation of recent theory and experimental research.

As in previous editions, the text is supplemented by liberal and judicious reference to original papers.

These references in Vol. I. include publications to the end of 1892, and in Vol. II. to the end of 1893. This feature of the work gives it great value. English and American writers might well copy it more extensively.

It appears singular that the author finds it necessary to incorporate into the introductory chapter a little treatise thirteen and one-half pages long, explanatory of the methods and notation of the differential and integral calculus. It would seem that a person who was likely to use a work like the one under consideration could be assumed to possess a competent knowledge of elementary mathematics. The chapter is well done, however, as far as it goes. If its introduction is unnecessary, the error may be pardoned: it only makes a big book a little larger. It is not safe, however, to assume that the reading of this chapter will prepare the student for the mathematical work which follows in the body of the book. The reader will require a good working knowledge of the calculus, including some skill in handling differential equations, although in many cases the author gives in detail the steps by which he reaches his solution.

Among the commendable features of the work may be mentioned the liberal quotation of numerical tables giving actual experimental results which have been obtained in important investigations.

In many text-books these are omitted for want of room, and only single or average values are given. In such cases the student can form no idea of the accuracy of the work upon which conclusions are based. Every statement is absolute. There are no relative values, and incorrect notions are acquired as to what is proven and what is only highly probable. The illustrations also are excellent. They usually show actual apparatus instead of diagrammatic outlines, and in most cases a mechanic could build from them a working instrument which could be used without change for effective experimental work.

The two volumes of the fifth edition already published are not greatly changed in general appearance from those of the preceding issues. The paper is good, and the type and presswork leave nothing to be desired in a work of this kind. The corresponding volumes of the fourth edition appeared in 1882 and 1885, and it is interesting to compare them with the present issue and to note the thoroughness of the revision. We find as we should expect a liberal amount of new matter. The first volume is increased from 848 to 1000 pages, and the volume on Heat from 825 pages to 919. We find few chapters in which material changes have not been made. Many sections have been rewritten, some transferred to more appropriate positions, and others have been omitted entirely. In the mathematical part of the work these changes are numerous.

In the fourth and preceding editions Vol. I. was devoted to General Physics and Sound; Vol. II. to Light; Vol. III. to Heat; and Vol. IV. to Electricity and Magnetism.

In the fifth edition this order is changed and the treatment of Light is reserved for the fourth volume, following Electricity.

This change is obviously made to prepare the way for a proper presentation of the electromagnetic theory of light.

When we come to examine the work in detail, we find that the author has discarded the gravitation unit of force for the absolute unit in his treatment of the mechanical problems in the first volume, and has promoted his section on the absolute system of measurements from its position as an appendix to a more honorable place in the first chapter on Mechanics. He has also introduced into the second chapter a ten-page section on the general equations of motion, with examples showing their application.

It is in the second division of the first volume — that which treats of the internal constitution of bodies and the phenomena dependent upon it — that we should expect to find the author's work of revision most amply illustrated. These three chapters on Solids, Liquids, and Gases are largely expanded by the addition of new matter and by the more complete presentation of theory. Among the additions we find quoted observations by Thompson on the variation of the coefficient of elasticity with an increasing load; by Röntgen, Voigt, Cornu, and Cantone, on the diminution of the cross-section of rods under tension; by Amagat and Cantone on cubic compressibility; by Tomlinson, Pisati, and others, on the change in the coefficient of elasticity with the temperature; Hertz' theory of hardness, with Auerbach's measurements in absolute units; and Boltzman's theory of internal friction, with experimental evidence bearing on the subject. Capillarity and surface tension are treated more fully than in the previous edition, and a new section is added on the kinetic theory of fluids, with its application to the theories of solution, osmotic pressure, and diffusion as developed by Van t' Hoff, Nernst, and Riecke. That part of the first volume which treats of wave motion and sound — subjects whose experimental side has received comparatively little attention of late years — appears in substantially the same form as in the fourth edition. The few pages which have been added are largely devoted to the mathematical development of the subject.

Volume II. is divided into five chapters, which, as in the fourth edition, treat of the subdivisions of the subject in the following order: Expansion, Radiation and Conduction, Mechanical Theory of Heat, Specific Heat, Change of State, and Heat from Chemical Reactions.

The chapter on Thermometry has been enlarged by the addition of a section on the use of thermo-electric elements, especially in the case of extreme temperatures, and the author now includes the bolometer among the instruments employed in the study of radiant heat. Svanberg is credited with the invention of the method, and Langley is mentioned as using it thirty years later, and as giving the instrument its name. Contrary to his usual custom, the author does not describe the instrument as actually constructed, but contents himself with a diagram of the connections used by Lummer and Kurlbaum, and by giving an elementary demonstration of the electrical formulæ involved in its use.

It is a little singular that Callender's work with the platinum resistance thermometer should not be mentioned, and that an account of Boy's radiomicrometer should not be thought worthy of a place with the bolometer.

The author's treatment of this subject may be used as an illustration of a tendency often manifested in his work, which is a fair matter for criticism. From the student's point of view it is very desirable that every subject should in some place receive a fairly complete and consecutive treatment, to which reference may be made when desirable. The interdependence of the subdivisions of physics often makes this rule a difficult one to follow, but violations should be avoided when possible. The reader is frequently tempted to believe that Wüllner sins unnecessarily in this respect. In the case just referred to, we find important matter relating to the construction and calibration of thermo-electric elements for temperature measurements in at least four places in the second volume, with a reference to the one which is to follow for further information. The student has constant occasion to thank the author for a good index.

The chapter on the Mechanical Theory of Heat has not been materially changed from the form presented in the fourth edition. The author gives a good account of research on the subject, including the work of Rowland, and follows with a discussion of the first and second laws of thermodynamics, using to a great extent the notation and methods of Clausius, Zeuner, and Verdet.

The fifth chapter, which is the longest in the volume, presents an account of classical and recent experimental work upon the phenomena which accompany freezing, melting, and evaporation, and applies the results obtained to a discussion of the equations of condition proposed by Van der Waals, Clausius, and others.

Wüllner's manual as revised deserves to be ranked among the half-dozen text-books of physics which treat the subject in a fairly complete and connected manner. Each one of these treatises probably has some distinguishing characteristic, by virtue of which it may claim a place in the working library of the student of physics; but it is always the case that special emphasis laid on one feature will inevitably call attention to any insufficiency in the treatment of other subjects.

Mention has already been made of certain features of Wüllner's work which recommend it to the student's use. It must be confessed, however, that the very fullness and detail referred to sometimes becomes a fault, but we can pardon this error more easily than the opposite one.

If our author does not push his mathematical discussion of theory as far as some, if we find he has omitted much interesting matter which lies on the borderland between physics and chemistry, we must remember that it is the experimental side of the subject which he has undertaken to present, and that he expressly disclaims any intention of treating the work in chemical physics with any degree of thoroughness.

One feature of Wüllner's treatment of his subject should commend his work to teachers: he constantly emphasizes, by the methods he employs, the important fact that it is unsafe teaching in physics which does not make the study of phenomena and the conditions which determine their manifestation the principal work of the student.

A. S. KIMBALL.

WORCESTER POLYTECHNIC INSTITUTE.

Electrical Measurements: a Laboratory Manual. By HENRY S. CARHART and GEORGE W. PATTERSEN. 8vo. pp. xiii + 344. Boston, Allyn & Bacon, 1895.

This book provides a "graded series of experiments for the use of classes in electrical measurements," "selected with the object of illustrating the general methods of measurement rather than applications to specific departments of technical work"; and it will for some purposes form a valuable addition to the growing list of laboratory manuals.

After an introductory discussion of units, measurements of resistance, current, electromotive force, quantity and capacity, self and mutual induction, and magnetism are treated in separate chapters, the chapters themselves, and the experiments in each, being arranged in the order of the difficulty which they present to the student. In all, recent forms of the appropriate instruments of precision are figured, and in most cases, described and discussed in greater or less detail. "Antiquated apparatus" is excluded, with such rigor that the authors apologize for describing even the tangent galvanometer, doing so, however, "because of its historical importance, if for no other reason." Lord Kelvin's quadrant electrometer is dropped without apology.

In general, the discussion of the methods of measurement consists of a description of the method, a deduction of the formula for calculating the quantity to be determined from the data of observation, and a numerical example. In some cases, the description is prefaced by a statement of the general idea of the method; but in many, the reader does not see what he is doing until he has read the account of how it is to be done. Best values of direct measurements and best arrangements of apparatus for the attainment of accuracy are determined in some cases, but not generally. Thus, although the fundamental property of the Wheatstone bridge is proved, no investigation is given as to the best resistance of galvanometer, or the best arrangement of battery and galvanometer. The reader is told with regard to the latter that Maxwell says such and such an arrangement should be adopted; but if he wishes to know for what purpose, or how Maxwell knew, he must go to Maxwell himself. But little information is given as to the degree of accuracy attainable with the different instruments and methods, sources of errors and methods of eliminating them, or other practiced details. The authors have evidently not intended to make the work self contained; and they give references to other manuals and to original papers, which are probably intended to atone for the omissions referred to. These references, however, are not sufficiently numerous for this purpose.

The introductory matter suffers from the effort after condensation. For example: "One kind of quantity may always be expressed in terms of two or three other kinds" (p. 1). "From Kupffer's observations Miller deduces the absolute density of water as 1.000013" (p. 6). "In the C.G.S. system the constant in the expression for f [the force between two magnetic poles] becomes unity" (p. 9). "There is no obvious relation between the two" [the electrostatic and electromagnetic systems of units] (p. 11). There should be a reference here to Rücker's paper on suppressed dimensions, *Philosophical Magazine* (Vol. 27, p. 104). "The numerical value of the E.M.F. between two points of a circuit, when there is no source of E.M.F. on this part of the circuit, equals the difference of potential between the same points" (p. 14). (The student should be referred to Nichol's *Laboratory Manual* (Vol. I., p. 182), for an admirable statement of the proper usage of the term "electromotive force.") "Every conductor of electricity offers greater or less obstruction to its passage. The researches of Dewar and Fleming on the resistance of metals at the temperature of boiling oxygen go to show that the resistance of all pure metals is zero at — 274° C., or the 'absolute zero.' The resistance of pure metals is, therefore, very nearly proportional to the absolute temperature" (p. 14). The deduction (p. 20) of the laws of resistance (its variation with length and cross-section of conductor) from Ohm's law is not made from this law, as formally stated on p. 15, but from the law which properly bears the name of Ohm, and it involves other assumptions as well. Such slipshod expressions and reasonings, however, are confined to the introductory sections of the book, and are probably due to the desire to put much information into little space.

The book contains a very large selection of methods, and a selection which is not only large, but judicious, no important modes of measurement which are susceptible of accuracy having, so far as I have noticed, been omitted. With more references to original sources, it would thus form a valuable compendium for suggesting methods to the young investigator.

The chapters on the measurement of resistance and current occupy half the space, the former being especially full. Its weak point is the section on electrolytic resistance, in which the old form of Kohlrausch's method, with the electrodynamometer as indicated, is described, and no mention is made of the fact that for many years Kohlrausch has used the telephone as indicated. How great an improvement this is, in one respect, is shown by Wien in the last number of Wiedemann's *Annalen* (Vol. 58, p. 37). It is curious that with regard to this method the student is referred to a paper by Daniell, but not Kohlrausch himself. The chapter on electromotive force contains a detailed statement of the mode of constructing standard cells, which, being based on the wide experience of the senior author, is of great value. Methods involving the use of such cells are extensively introduced both in this and in other chapters. The discussion of quantity and capacity, induction and magnetism, occupy less space than the chapters which precede, but are equally full and are more exhaustive, so far as the number of methods referred to is concerned, than is usual in laboratory manuals.

Though the book assumes some knowledge of the calculus, the greater part of it is intelligible without such knowledge. In some cases in which the calculus is used it might, for the benefit of the non-mathematical reader, have been dispensed with without loss save of elegance. In one case in which it has been dispensed with — the discussion of the logarithmic decrement — the cogency of the reasoning suffers. The example appended to this discussion shows that such examples are not always worked out so as to give the most accurate result, the student being probably supposed not to aim at so high a degree of accuracy as to make this desirable.

The book is well gotten up, the type being good, the typography accurate, and the illustrations and diagrams excellent. It is provided, as all such books should be, with a full index, which very much facilitates reference.

J. G. MacGregor.

Lehrbuch der Electrochemie. By LE BLANC. 8vo. pp. 226. Leipzig, Oskar Leiner, 1896.

A careful and earnest study of the contents of this volume would certainly acquaint the student with the present standpoint of the subject of electrochemistry. The mathematics introduced are very simple. Only such simple differentials as $\frac{dE}{dT}$, expressing the rate of change of electromotive force with the absolute temperature, appear. The style of the author is very clear, and the subject-matter very systematically arranged. The book is primarily written for students beginning the study of theoretical electrochemistry. The reviewer has been able to make use of much of the contents to give technical students a thorough idea of the modern view of battery processes.

In the first chapter, the hypotheses of Grotthus, Clausius, and Arrhenius upon electrolytic conduction are discussed. The Grotthus hypothesis is shown to be contrary to the second law of thermodynamics. The kinetic theory of Clausius must be abandoned, also, since by this theory the "equivalent" conductivity should not increase with the dilution, as demanded by the Arrhenius dissociation theory. Chapter II. treats of the theory of Arrhenius; Chapter III. discusses the independent wandering of the ions; and Chapter IV. brings to our notice the conductivity of electrolytes. The phenomena noted in these chapters are linked together upon the theory of Arrhenius. The author points out that the van't Hoff law was instrumental in bringing Arrhenius to a comparison of the lowering of the freezing points of electrolytes and the electrical conductivity of the same. Upon these Arrhenius founded the theory of electrolytic dissociation.

The chapter upon electromotive force takes up nearly half of the book. Series are divided into the usual classes, reversible and non-reversible. A reversible process is carried through, and the analogy between osmotic and gas pressure pointed out. The expression used by Ostwald, "electrolytic solution pressure," is preferred to the name "solution tension" given it by Nernst. It would be almost an injustice to the book were one not to mention the neatness and simplicity with which the author points out that, just as the vapor pressure of water in contact with the liquid is the vapor pressure of saturation, so also the solution pressure of a substance or metal in contact with its solution is the osmotic pressure of saturation. Going out from this standpoint, series are treated after the Nernst method as machines working by osmotic pressure. Entropy is introduced in this connection as a "capacity factor" (not specific heat). This may afford an elementary, but scarcely a complete, idea of the phenomena of entropy.

In a reversible process applied to batteries the term entropy recurs in the later equation of von Helmholtz,

$$CE - Q = CT\frac{dE}{dT},$$

where C, E, Q, and T represent current, electromotive force, "heat-toning," and absolute temperature, respectively. The equation shows that the temperature coefficient of a series will be positive or negative, according as the chemical energy is less or greater than the electrical energy.

The galvanic series are treated in the following order. Concentration Series, embracing (1) various concentration of substances giving ions, (2) various ionic concentrations, and (3) concentration double series; Liquid Series, General Considerations upon Concentration — and Liquid Series, Thermo Series, and Chemical Series.

In the Concentration Double Series,

$$\mathrm{Zn} \mid \mathrm{ZnCl_2(conc)} \mid \mathrm{HgCl} \mid \mathrm{Hg} \mid \mathrm{HgCl} \mid \mathrm{ZnCl_2(dilute)} \mid \mathrm{Zn},$$

we have two cells of the same kind in opposition. It is maintained that this is not the same as

$$\mathrm{Zn} \mid \mathrm{ZnCl_2(conc)} \mid \mathrm{ZnCl_2(dilute)} \mid \mathrm{Zn}.$$

In the latter case when a current passes, besides zinc going into solution at one electrode and precipitating at the other, there is a wandering of the $\overset{+}{\mathrm{Zn}}$ and $\overset{-}{\mathrm{Cl}}$ ions from the concentrated to the dilute solution. In the calomel double series such a wandering is impossible. For this double series the electromotive force equals

$$\frac{3}{2}\frac{RT}{c}\log_e\frac{p}{p_1},^{1}$$

where p and p_1 are the osmotic pressures in the concentrated and dilute solutions respectively, and c is the number of coulombs, 96540, per gram equivalent. Now, in the second series, it can be easily shown that the electromotive force equals

$$\frac{u-v}{u+v}\frac{RT}{2c}\log_e\frac{p}{p_1},$$

where u and v are the wandering velocities of the kathion and anion respectively. The condition for equality is

$$\frac{u-v}{u+v} = 3.$$

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[1] This equation is the author's, the following one the reviewer's. The latter is also a particular form of an equation of the author in Liquid Series.

If n denote the transference number of the kathion, then

$$n = \frac{u}{u+v}; \text{ and } 1 - n = \frac{v}{u+v}.$$

The solution of these equations gives:

$$n = 2; \quad u = \tfrac{2}{3}; \text{ and } v = -\tfrac{1}{3}.$$

This means that two equivalents of the kathion, and one equivalent of the anion, pass into the dilute solution for every equivalent of current generated. One kathion is, however, given up at the electrode, which leaves a net gain, in the dilute solution, of one positive, and one negative, equivalent. Since one anion equivalent appears at the other electrode, the concentrated solution loses two equivalents of both ions,—one equivalent taking part in current conduction, and one equivalent passing to the dilute solution. The condition of affairs could not, indeed, be much simpler. Whether or not it is the actual condition could be found by testing the solutions after the passage of a definite current. The objection that the anion and kathion do not wander in such ratios can scarcely be raised, for one might also say, the two ions do not usually wander in the same direction. If diffusion and potential difference change the direction of an ion, why can they not modify its relative velocity?

The drop electrode method of measuring the electromotive force of each terminal is discussed, and special attention called to the Ostwald constant electrode

$$\text{Hg} \mid \text{HgCl in normal KCl} = -0.56 \text{ volts},$$

which may be combined with any other electrode and solution. From the electromotive force of the whole and that of the known mercury electrode, the unknown electromotive force of the other electrode is determined. The drop electrode method is thus obviated. The method should give good results if the known single potential be constant and reliably determined. Several observers obtain the same voltage for the electrode and certify to its constancy. Many persons will find the author's graphic treatment of this subject of interest.

The chapter on polarization is next in importance and interest. The subject matter and conclusions are largely drawn from the author's own investigations. The electromotive force of polarization of H and O between black platinum (platinirites platinum) electrodes is 1.07 volts; that between plain platinum electrodes is 1.70 volts. In the former case we have equilibrium between the gas absorbed by the electrode and the solution; in the latter case the electrolyte must contain much more gas at the electrode before it is in equilibrium with the outside pressure. The electromotive force of 1.70 volts could then be lowered by reducing the atmospheric pressure. This is found to be true. It is to be noted, too, that 1.70 volts is the electromotive force of acids and bases yielding hydrogen and oxygen, and this value is independent of the concentration. For acids whose polarization value is below 1.70 volts, the value tends toward 1.70 volts as a maximum on diluting the solution. On reaching this value, hydrogen and oxygen are given off.

Le Blanc's conclusion upon these phenomena is in sharp contrast to the widespread view of secondary action. For example, in a solution of sulphuric acid and water, he opines that all ions take part in the conduction in proportion to their respective concentration, but at the electrode only those ions are given up which are easiest given up,[1] *i.e.* those which have the lowest solution pressure, or lowest electromotive force of polarization. The ions are liberated which require the least expenditure of energy. How then can an electromotive force of polarization above 1.70 volts exist, *e.g.* that of zinc sulphate equal to 2.35 volts? The theory protects itself very cleverly. In this and similar cases, the evolution of hydrogen at the kathode would involve the production of hydroxyl ions, the maximum quantity of which are already present, and the electromotive force therefore rapidly rises to the evolution of another product.

Chapter VIII. is a short additional contribution of nine pages upon practical galvanic batteries and the lead accumulator. This is the first attempt that has been made to offer any explanation of the lead accumulator from the present standpoint of electrolytic processes. Devoting four and one-half pages to such a subject, one cannot give any complete theory of the lead cell, but the reviewer is inclined to think that this method of treatment can be so amplified as to explain much of the peculiar behavior of this cell. The author shows that the principal reaction of the cell upon discharge is

$$PbO_2 + 2\ H_2SO_4 + Pb = 2\ PbSO_4 + 2\ H_2O + 87{,}000\ \text{cal.}$$

The old theory also agrees to this reaction. Under the assumption of a zero temperature coefficient, this would give us an electromotive force of 1.885 volts. The electromotive force during discharge is for a considerable period of time not far from this value. Hence the small error in applying the Kelvin formula. However, the discharging electromotive force is often 2.1 volts. The author could have shown that under these circumstances the cell should have an appreciable temperature coefficient. An application of the von Helmholtz formula before mentioned shows that the cell would then cool during discharge, as noted by Dr. Duncan. The charging electromotive force goes still higher. The temperature coefficient would therefore be still higher, and the heating would be greater than the cooling in the previous case. Professor Ayrton has shown that this heating amounts to seventeen per cent of the entire energy of the cell, even after allowance is made for the C^2R loss within the cell. The fall of potential during discharge can be easily explained upon the principle of "solution tension" and osmotic pressure. The author treats this too briefly entirely. The theory points out the necessity for a concentration electromotive force, opposing both the charging and discharging currents. The author has made no allusion to this action, which alone would make it impossible to obtain a high efficiency when large currents are taken from the cell.

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[1] This view seems to have first been held by Hittorff.

It is to be regretted that there is no index to the book. The references to the original literature could have been increased to an advantage.

The author acknowledges his indebtedness to his personal contact with and writings of Professor Ostwald. The reviewer regrets very much that he has had the opportunity of making only a hasty inspection of the recent large and interesting volume of Professor Ostwald upon the same subject.

B. E. MOORE.

UNIVERSITY OF ILLINOIS,
June, 1896.

The Alternate Current Transformer. Vol. I. By J. A. FLEMING, F.R.S. 8vo, pp. 606. London, The Electrician Printing and Publishing Company, 1896.

In this new edition of a well-known work, the author has followed closely the lines of the earlier edition, but has amplified the treatment in places, and has eliminated some of the portions which have become antiquated. No book on the transformer is more replete with information than this work, and none are better qualified than Dr. Fleming to speak with authority on the subject. It is with disappointment, therefore, that the reader finds the same lack of continuity and disregard for sequence which characterized the earlier edition. The book contains too much patchwork for student use, and the facts contained are rather diffuse and scattered for the busy engineer.

F. B.

The Principles of the Transformer. By FREDERICK BEDELL, Ph.D. 8vo, pp. xii+416. Figs. 250. The Macmillan Company, 1896. (*Received.*)

Volume IV. *January–February, 1897.* *Number 4.*

THE

PHYSICAL REVIEW

THE FREEZING-POINTS OF DILUTE AQUEOUS SOLUTIONS. III.

BY E. H. LOOMIS.

THE following measurements were made during the winter months of 1895–96 in the Physical Laboratory of Princeton University.

The conditions under which the work was done were the same as during the preceding period of observation (winter, 1894–95), except that the lower and more nearly uniform temperature throughout the past winter made the maintenance of the room-temperature at about 3° C. much easier. In fact, the room-temperature rarely varied more than 0°.5 C. during an entire day.

This constancy of temperature has reduced the experimental error to one-half its former value, as appears from the fact that the average variation in a series of five entirely distinct and independent observations of the freezing-point of a given solution is now 0°.0005 C., while its former value was about 0°.001 C.

That the experimental errors are considerably lowered by this increased constancy of the room-temperature appears also in the fact that the successive observations of the freezing-point of water present less irregularity than in the previous work. (See Plate opposite p. 295, Fig. 3.)

This reduction of experimental error accounts for the greater regularity in the curves of molecular depressions in the region of extreme dilution, as compared with the similar curves for the previous results. There seems to be no doubt that this reduction of the experimental error is due wholly to the greater constancy of the temperature of the mercury thread in the projecting stem of the thermometer. I think it now more probable than ever before, that a final degree of accuracy is to be looked for only under the condition of absolute constancy of the room-temperature at 0° C. On such days as permitted a close approach to this condition, I found the various manipulations were much more easily made and the different measurements more nearly approached exact concordance. It has not been found necessary to make the slightest change in the apparatus or method, and the measurements here to be submitted may thus be regarded as a mere continuation of those of the preceding winter (1894–95).

The present series of observations comprises the compounds named in columns 1 and 2 in the following table. In column (3) *m* denotes the concentration of the solution in gram-molecules per liter.

To avoid confusion the number of grams of the "water-free" compound per liter of the solution is given in column (4). In column (7) is given, under $S\frac{18^\circ}{4^\circ}$, the specific gravity of the solution whose concentration is found in the column which stands immediately before it. In addition, the table gives the name of the maker of the chemicals used and the particular process by which the original solutions were prepared.

The Chlorides, — Preparation of Solutions.

Lithium Chloride was prepared by treating lithium carbonate, which was obtained in a high state of purity from Merck & Co., with dilute HCl. The solution was then carefully neutralized, care being taken to expel all CO_2. The specific gravity of the solution was taken with a Mohr balance, and the solution diluted to approximate normal strength by use of Kohlrausch's tables of specific gravity.[1]

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[1] Kohlrausch, Wiede. Ann. 6, p. 38, 1879.

TABLE I.

1	2	3	4	5	6	7		8	9
Compound.	Chemical formula.	m	Grams per liter of solution at 18° C.	Specific gravity,[1] Water at 4° C. = 1.	Electrical conductivity,[2] $K \cdot 10^8$. 18° C.	m	$s\frac{18^\circ}{4^\circ}$		Standard solution prepared.
Lithium chloride	LiCl	0.9917	42.13	1.0226	589	0.1983	1.0034	Prepared in laboratory	Kohlrausch tables
Calcium chloride	$CaCl_2$	0.5021	55.68	1.0438	635	0.2008	1.0170	" " "	" "
Strontium chloride	$SrCl_2$	0.4986	78.98	1.0672	633	0.1994	1.0264	Trommsdorff	" "
Stannic chloride	$SnCl_4$	0.250	64.98	1.0486(15°)	195_0	0.200	1.0384	Eimer & Amend	Gerlach's tables
Nitric acid	HNO_3	1.0000	63.04	1.0324	278_5	0.2000	1.0054	Prepared in laboratory	Hulett's titration
Potassium hydroxide	KOH	1.0000	56.14	1.0481	170_0	0.2000	1.0091	" " "	" "
Sodium hydroxide	NaOH	1.0000	40.06	1.0418	145_2	0.2000	1.0079	" " "	" "
Potassium di-hydrogen phosphate	KH_2PO_4	0.333	45.38	1.0299	198	0.2000	1.0175	Eimer & Amend	Direct weighing
Di-sodium acid phosphate	Na_2HPO_4	0.2000	28.42	1.0260	—	0.2000	1.0260	Merck & Co.	" "
$NaNH_4HPO_4$	$NaNH_4HPO_4$	0.333	45.70	1.0345	282	0.200	1.0207	" "	" "
Tri-sodium phosphate(?)	Na_3PO_4(?)	0.33(?)	54.73	1.06	—	—	—	Prepared in laboratory	" "
Sodium silicate	Na_2SiO_3	0.523	64.08	1.0636	663	0.21	1.026	Trommsdorff	Kohlrausch tables[3]
Acetic acid	$C_2H_4O_2$	1.0000	60.01	1.0073	12	0.2000	1.0004	Merck & Co.	Hulett's titration
Oxalic acid	$(COOH)_2$	0.5000	45.00	1.0199	551	0.2000	1.0073	" "	Direct weighing
Succinic acid	$C_4H_6O_4$	0.4975	58.71	1.0167	17	0.1990	1.0060	Eimer & Amend	" "
Tartaric acid	$C_4H_6O_6$	0.9970	149.57	1.0653	87	0.1994	1.0122	Marquart	" "
Citric acid	$C_6H_8O_7$	0.9983	209.67	1.0777(15°)	64	0.1997	1.0145	Merck & Co.	" " .

[1] Temperature of solutions is 18° except when otherwise indicated.

[2] Conductivity with reference to mercury at 0° C.

[3] Kohlrausch, Zeit. Phys. Chem. XII., p. 773.

The specific gravity of this approximate solution was then exactly determined as described in former papers. This value is given in Table I., 5. The gram-molecular strength of the solution was easily computed by comparing this value with that given by Kohlrausch for a normal LiCl solution,[1] by means of his specific gravity tables. This gram-molecular strength of the original solution is given under (m) in Table I., 3. The diluter solutions whose freezing-points were to be measured were uniformly made by the use of most carefully marked flasks and pipettes. It needs perhaps to be said that these flasks and pipettes were marked for absolute volume when "dry." Careful rinsing insures the complete discharge of their contents into the measuring flasks.

Calcium Chloride. — An excess of $CaCO_3$ in the form of natural crystals of Iceland-spar was treated with dilute HCl, until the effervescence ceased. The whole was then boiled to expel CO_2 and the filtered solution carefully neutralized by addition of a slight quantity of HCl. It was then diluted to approximate normal strength, and finally its exact gram-molecular concentration was calculated as in the case of LiCl.

Strontium Chloride. —There happened to be a quantity of finely crystallized strontium chloride already in the laboratory (Trommsdorff, c.p.), which was used without recrystallizing. The gram-molecular strength of the final solution was determined as in the foregoing cases. It will be observed in Table I., 6, that the conductivity of this solution differs not inconsiderably from that given by Kohlrausch.[2] This is undoubtedly due to the fact that I did not recrystallize the salt. This error is, however, too slight to affect the measurements of the freezing-points.

Tin Chloride (stannic). —The salt was obtained from Eimer & Amend, and was not recrystallized. The normal solution was prepared by use of Gerlach's tables of specific gravity.[3] The temperatures of these particular tables of Gerlach are $\frac{15^\circ}{15^\circ}$.

The chlorides already studied in my former work are NaCl, KCl, NH_4Cl, $BaCl_2$, $MgCl_2$, and HCl.

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[1] Kohlrausch, Leitfaden d. Physik, Leipzig, 1892, p. 404.

[2] Kohlrausch, Leitfaden, p. 404.

[3] Hoffmann's Tabellen, Berlin, 1877, p. 130.

Observations on the first four have been extended so as to include solutions of greater concentration as follows: —

NaCl, $m = 0.70$,	NH_4Cl, $m = 0.70$,
KCl, $m = 0.70$,	$BaCl_2$, $m = 0.50$.

The following tables present the experimental results for the chlorides (Table II.). Column (1) contains the value of m for the various dilutions. The observed depression of the freezing-point is found in (2), while the molecular depression is found in (3).

In order to compute from the present data the molecular depression on the basis of gram-molecular concentration per 1000 grams of water, as Raoult and others are accustomed to do, one needs to know the specific gravity of the solution. Accordingly, this value is given for the solution near the concentration $m = 0.20$ in column 7, Table I. The difference between the value of the molecular depression so calculated and that given in the tables, which is based on gram-molecular concentration per liter of the solution, is altogether inappreciable in the more dilute solutions, and reaches its maximum value of 1.5 per cent in the case of tin chloride. I have accordingly omitted these second values of $\frac{\Delta}{m}$

Graphic Representation of Results.

The values of $\frac{\Delta}{m}$ as functions of m are represented in Fig. 2. (See Plate opposite p. 295.)

Ordinates represent values of $\frac{\Delta}{m}$, abscissæ, values of m. The scale here is the same as that adopted in the former papers.

Discussion of the Chlorides.

1. It will be remembered that a very striking fact is presented in the molecular depressions of the freezing-point by HCl and $MgCl_2$, as was indicated in the author's account of observations on these compounds;[1] namely, the fact that the molecular depression for each of these compounds reaches a pronounced minimum at about $m = 0.10$. It now appears that *all* the chlorides thus

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[1] Physical Review, III., p. 281.

Table II.

LITHIUM CHLORIDE.

[Values in parentheses obtained by interpolation.]

1 Gram-molecules per liter of solution (m).	2 Depression of freezing-point (Δ).	3 Molecular depression of freezing-point $\frac{\Delta}{m}$.	4 Theoretical value of Δ.	5 Theoretical value of $\frac{\Delta}{m}$	6 Difference $\Delta_{th}-\Delta_{ex}$.	7 Difference $\Delta_{th}-\Delta_{ex}$ in per cent of Δ_{ex}.
	deg.		deg.		deg.	
0.0099	0.0363	3.67	0.0358	3.61	−0.0005	−1.4
0.0198	0.0710	3.59	(0.0706)	(3.56_5)	0.0004	0.6
0.0496	0.1770	3.57	0.1730	3.48_7	0.004	2.3
0.0992	0.3520	3.55	0.3388	3.41_5	0.013	3.8
0.1983	0.7018	3.54	(0.6626)	(3.34)	0.039	5.6
0.4959	1.809	3.65	1.581	3.19	0.228	12.5
CALCIUM CHLORIDE.						
0.0100	0.0513	5.13	(0.0510)	(5.10)	−0.0003	−0.5
0.0201	0.1013	5.04	(0.1000)	(5.00)	0.0013	1.3
0.0502	0.2437	4.85	0.238	4.74	0.006	2.5
0.1004	0.4823	4.80	(0.460)	(4.58)	0.022	4.5
0.2008	0.9718	4.84	(0.882)	(4.39)	0.090	9.3
0.5021	2.605	5.19	2.048	4.08	0.557	21.3
STRONTIUM CHLORIDE.						
0.0100	0.0508	5.08(?)	(0.0495)	(4.95)	−0.0013	−2.6
0.0199	0.1015	5.10	(0.0966)	(4.83)	0.0049	4.8
0.0499	0.2445	4.90	0.234	4.68	0.011	4.5
0.0997	0.4834	4.85	(0.452)	(4.53)	0.031	6.4
0.1994	0.9608	4.82	(0.862)	(4.31)	0.099	10.3
0.4986	2.532	5.08	1.989	3.99	0.543	21.5
TIN CHLORIDE ($SnCl_4$).						
0.01	0.1261	12.61				
0.02	0.2487	12.44	No data from which these values may be calculated.			
0.05	0.5973	11.95				
0.10	1.1457	11.46				
0.20	1.968	9.84				

TABLE II. (*continued*).

Compound.	m	Δ	$\frac{\Delta}{m}$	Compound.	m	Δ	$\frac{\Delta}{m}$
		deg.				deg.	
NaCl	0.70	2.399	3.43	NH_4Cl . . .	0.70	2.384	3.41
KCl	0.70	2.346	3.35	$BaCl_2$	0.50	2.412	4.83

far examined, tin chloride excepted, manifest this same minimum, though in the case of NH_4Cl, NaCl, and KCl it is much less pronounced. To obtain a general survey of this fact, the curves of molecular depression for the various chlorides have been collected in Fig. 1, where the scale of values for m (abscissæ) has been decreased to one-half its value elsewhere in the plate in order to include the solutions of greater concentration. It is to be remarked that Arrhenius' observations on these chlorides quite generally exhibit this minimum value. Since these early observations lay no claim to a degree of exactness attained in the later methods, it is the more surprising that they so plainly reveal the existence of these minimum values which characterize the chlorides. Although Arrhenius called no attention to them, and evidently believed they were due to experimental errors, it is still difficult to understand why subsequent observers with more precise methods should have failed to find them. The existence of this minimum value in the case of the binary chlorides may be easily demonstrated with no other apparatus than an ordinary $\frac{1}{10}°$ thermometer and beaker glass.

2. The former division of the compounds into two groups, one containing univalent radicals, as LiCl, the other containing bivalent radicals, as $CaCl_2$, is confirmed by the new results here presented.

The characteristics of these two groups, as found in the former work, remain unaffected by the addition of this new material.

3. There is no evidence in any of the chlorides, or any other of the compounds examined, of any "kinks" in the curves of molecular depressions, save in the region of extreme dilution,

where they are so small as to be best explained by experimental errors.

It may be safely said, now that I have examined some thirty-five characteristic compounds, that the existence of definite "Hydrates" at particular concentrations is not indicated by the changes in the freezing-point which attend changes in the dilution of the solution.

Bearing of these Results on the Dissociation Theory.

In the table of results for the chlorides, and in the subsequent tables for other compounds, are found the theoretical values of the depressions of the freezing-point, and the theoretical values of the molecular depressions, respectively. These values are based on Kohlrausch's value for $\frac{\mu_x}{\mu_\infty}$. In column (6) is found the difference between the theoretical and observed value of the depressions, while in (7) this difference is expressed in per cents of the total observed value. It thus appears that the observed values are higher than the theoretical. The differences, however, in the region of extreme dilution in the cases of LiCl and $CaCl_2$ are surprisingly small. In case of $SrCl_2$ the difference is very great, the observed depression being 0°.0013 C. higher than the theoretical value, and the observed value is certainly too low, judging from the curve of the molecular depressions as exhibited in Figs. 1 and 2.

It needs to be remarked that the molecular depressions calculated on the basis of the dissociation theory show no minimum value. This accounts for the enormous departure of the observed from the theoretical value in the region of greater concentration.

The case of tin chloride needs separate discussion.

First, it is to be observed that the depressions are much more than twice as large as those produced by any other chloride in the same concentration. While it is impossible to compute the theoretical value of the depression on the basis of the dissociation theory, since measurements on the conductivity of these solutions are wanting, it is, however, a striking fact that if it be *assumed* that the salt at the concentration $m = 0.01$ be *completely* dissociated, and that each molecule splits up into the maximum number of five parts, Sn–Cl–Cl–Cl–Cl, we would then have for the molecular depression only 5 × 1.89 or 9.45, while the observed value is 12.61. Obviously the depression is not to be explained in this way. It however suggests itself that the $SnCl_4$ does not exist at all in aqueous solutions, but that it assumes at once the properties of a mixture of $SnCl_2$ and 2 HCl, in solution. No value for the depression of the freezing-point by $SnCl_2$ has been found; but assuming that it is about that of $CdCl_2$ and $ZnCl_2$, as observed by Jones, *i.e.* 5.20, and taking the observed value for HCl, *i.e.* 3.61, we have for the depression due to $SnCl_4$, $5.20 + 2(3.61)$, or 12.42. This is very nearly the observed value. It should be said that many vain attempts were made to prepare solutions of $SnCl_2$, sufficiently free from $SnCl_4$, to enable the freezing-points to be measured.

The purpose was to measure the depressions in a solution of $SnCl_2$ at $m = 0.01$, and then add in succession a $\frac{1}{100}$ gram-molecule of HCl. Thus measurements would be made on

$$\begin{array}{ll} SnCl_2, & m = 0.01, \\ SnCl_2 + 1\,HCl, & m = 0.01, \\ SnCl_2 + 2\,HCl, & m = 0.01, \\ SnCl_2 + 3\,HCl, & m = 0.01, \end{array}$$

as was done in the cases of the phosphates, p. 288. This would have decided experimentally whether the suggestion just made in regard to the nature of $SnCl_4$ in solution is tenable. I hope to return to this difficulty in the near future. It may be well to call attention here to the great importance of exact measurements of the electrical conductivity of this salt, together with that of the various phosphates and the dilute solutions of $MgCl_2$ and NH_4NO_3 for the purpose of obtaining experimental values for the "degree of dissociation," $\frac{\mu_x}{\mu_\infty}$.

The Sodium and Potassium Hydroxides, Nitric Acid, and Sodium Silicate.

The materials used in the first three cases were those prepared in a high degree of purity for the purpose of determining their specific gravity and electrical conductivity.[1] The great care taken

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[1] Loomis, PHYSICAL REVIEW, IV., 1896, p. 252.

TABLE III.

POTASSIUM HYDROXIDE.

[Values in parentheses obtained by interpolation.]

1 m	2 Δ	3 $\frac{\Delta}{m}$	4 Δ theoretical.	5 $\frac{\Delta}{m}$ theoretical.	6 Δth-Δexp	7 Δth-Δexp in per cents of Δexp.
	deg.		deg.		deg.	
0.0100	0.0343	3.43	0.0371	3.71	+0.0028	+ 8.1
0.0200	0.0689	3.45	(0.0738)	(3.69)	0.0049	7.1
0.0500	0.1719	3.44	0.1825	3.65	0.0106	6.1
0.1000	0.3426	3.43	0.360	3.60	0.017	5.0
0.2000	0.6860	3.43	(0.714)	(3.57)	0.028	4.1
SODIUM HYDROXIDE.						
0.0100	0.0328	3.28(?)	0.0366	3.66	+0.0038	+11.6
0.0200	0.0691	3.46	(0.0724)	(3.62)	0.0033	4.7
0.0500	0.1727	3.45	0.1765	3.53	0.0038	2.2
0.1000	0.3414	3.41	0.350	3.50	0.0039	1.1
0.2000	0.6814	3.41	(0.696)	(3.48)	0.0015	0.2
NITRIC ACID.						
0.0100	0.0350	3.50	0.0373	3.73	+0.0023	+ 6.6
0.0200	0.0712	3.56	—	—	—	—
0.0300	0.1059	3.53	0.1107	3.69	0.0048	4.5
0.0500	0.1754	3.51	0.1835	3.67	0.0081	4.6
0.1000	0.3496	3.50	0.363	3.63	0.013	3.7
0.2000	0.6959	3.48	(0.720)	(3.60)	0.020	2.8
SODIUM SILICATE.						
0.0105	0.0676	6.46				
0.0209	0.1339	6.41				
0.0523	0.3068	5.87		No experimental data.		
0.1046	0.5533	5.29				
0.2092	0.9785	4.68				
0.5230	2.087	3.99				

to purify these materials was wholly unnecessary so far as the determination of their freezing-points was concerned, but as the material was at hand it was made use of for the present purpose.

The sodium silicate, Na_2SiO_3, was Trommsdorff's c.p. salt and was not recrystallized. It yielded a perfectly clear solution without filtering. It was reduced to normal solution by using Kohlrausch's values of specific gravity.[1]

The graphic representation of these results is found in Fig. 1. It is to be observed that the molecular depression for KOH, NaOH, and HNO_3, as exhibited in column (3), varies very little with the concentration, as has been found also to be the case with HCl. It is to be noted that the values for $m = 0.01$ in each instance are evidently too low and indicate experimental errors which reach, at least in the case of nitric acid, 0°.0006.

The value for NaOH, $m = 0.01$, I am unable to explain. The measurements were twice repeated, making in all thirteen entirely distinct observations on this solution. The average of all was 0°.0325. The value here given, 0°.0328, belongs to the same series to which the other measurements belong, and for this reason was taken. The error, I think, cannot be referred to possible errors of observation. It seems to be connected in some way with the solution. It is also difficult to believe that the conversion of the NaOH and KOH into the carbonates by the CO_2 of the air would be sufficient to account for the marked irregularity and for the seemingly low values throughout this region of greater dilution. The present method permits no experimental answer to this question.

Sodium Silicate. — The molecular depressions are very great, and the rate of decrease with increase in concentration, as in the case of $SnCl_4$, is enormous. Thus, while it is 6.46 at $m = 0.0105$, it has fallen to 3.99 at $m = 0.523$. It is not without interest to note that the *complete* dissociation of this salt in the solution $m = 0.0105$, provided each molecule splits into three parts after the analogy of Na_2SO_4, would still require the theoretical value to be only 3 × 1.89, or 5.67, which is much less than the observed value.

It seems probable that, as in the case of $SnCl_4$, the aqueous solution of Na_2SiO_3 has the properties of a mixture of NaOH and SiO_2 where the presence of the SiO_2 seems to lessen the degree of dissociation of the NaOH. The results of Kohlrausch's observations on the electrical conductivity seem to point to the same conclusion.

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[1] Kohlrausch, Zeit. Phys. Chem. XII., 1893, p. 774.

This point should receive a more careful study than I have thus far been able to give to it.

The Phosphates.

Table IV.

KH_2PO_4.			Na_2HPO_4.		
1 m	2 Δ	3 $\frac{\Delta}{m}$	1 m	2 Δ	3 $\frac{\Delta}{m}$
	deg.			deg.	
0.01	0.0358	3.58	0.01	0.0499	4.99
0.02	0.0720	3.60	0.02	0.0969	4.85
0.05	0.1740	3.48	0.05	0.2304	4.61
0.10	0.3365	3.37	0.10	0.4345	4.35
0.20	0.6434	3.22	0.20	Beyond limit at 0°	of solubility C.
$Na(NH_4)HPO_4$.			Na_3PO_4(?).		
0.01	0.0495	4.95	0.01	0.0715	7.15
0.02	0.0956	4.78	0.02	0.1369	6.85
0.05	0.2260	4.52	0.05	0.3048	6.10
0.10	0.4242	4.24	0.10	0.5661	5.66
0.20	0.7817	3.91	0.20	Not ob	served.

Preparation of Solutions.

The KH_2PO_4 (Eimer & Amend, c.p.) was carefully recrystallized, and after continued drying at 100° C. the necessary amount was weighed and made into a normal solution ($m = \frac{1}{3}$). Mr. Sill of the Princeton Chemical Laboratory found by the determination of the salt as $Mg_2P_2O_7$ that there were 45.06 g. of KH_2PO_4 per liter of the solution instead of 45.38 g.; thus the solution was apparently 0.7 per cent too weak. As this difference could be due to impurity of the salt as well as to the failure to completely dry the salt, I have not ventured to make any correction, and the results must be looked upon as affected with an error of 0.7 per cent.

The Na_2HPO_4 (Merck & Co., c.p.) was carefully recrystallized, and after heating at 120° C., until the weight became constant, the necessary amount was reduced to a normal solution by direct weighing. Mr. Sill's analysis showed that a liter of the solution, $m = 0.20$, contained 28.45 g. instead of 28.42 g. The solution was thus apparently 0.1 per cent too strong. This error is less than the probable error of the analysis, and the Na_2HPO_4 solutions may be looked upon as exact.

The $NaNH_4HPO_4$ (Merck & Co., c.p.) was recrystallized. It was dried with filter paper and finally in the air at a temperature a little above room-temperature.

Mr. Sill's analysis of the final solution, $m = \frac{1}{8}$, showed that it contained per liter 45.45 g. instead of 45.70 g., which is required for the solution, $m = \frac{1}{8}$. It was thus 0.55 per cent too weak. For the same reason as that given in the case of KH_2PO_4, no correction was applied to the results, though it is likely that the error here is wholly due to lack of complete drying of the salt.

The Na_3PO_4 was prepared in the laboratory by making a solution of molecular equivalents of Na_2HPO_4 and NaOH, taking care that an excess of NaOH was present. The solution was then evaporated to the point of crystallization. The abundant crop of crystals was then dissolved in one-half its weight of boiling water and left to recrystallize. The crystals thus prepared are small hexagonal prisms with jagged ends. These crystals were then dried at 120° C. to constant weight. The resultant compound should be Na_3PO_4, H_2O. The analysis of many solutions prepared on the basis of this formula for the dried salt and $Na_3PO_4 + 12H_2O$ for the crystals themselves gave me no confidence that the solutions could be looked upon as having more than a rough approximation to their supposed strength. Mr. Hulett made a very careful series of analyses of the crystals and found a very wide departure from the formula, $Na_3PO_4 + 12H_2O$. Time did not permit a final solution of the difficulty. The matter deserves a thorough study from the chemical side.

Discussion of Results for the Phosphates.

The depressions of the freezing-point, caused by phosphoric acid, H_3PO_4, are much less than those caused by the groups of compounds containing univalent and bivalent radicals, and this is the more surprising since phosphoric acid contains a trivalent radical.

For the purpose of examining into this matter more closely, I undertook to measure the freezing-points of the present series of phosphates.

1. The first fact which appears from the results is that the phosphates are entirely unlike the phosphoric acid and differ still more among themselves.

2. In proportion as the hydrogen of phosphoric acid is replaced by a univalent metallic radical, the depression of the freezing-point is increased. See curves for H_3PO_4, KH_2PO_4, Na_2HPO_4, and Na_3PO_4. (Fig. 2.)

3. If we assume that in the case of H_3PO_4 the molecule splits up into H· and HPO_4, we may calculate the freezing-points on the basis of the dissociation theory.

The theoretical values so obtained are given in column (4), while in (2) are found the observed values taken from former measurements. The agreement here exhibited makes our assumption fairly probable that in the process of dissociation only a single atom of hydrogen is "split off."

TABLE V.

H_3PO_4.

1 m	2 Δ	3 $\frac{\Delta}{m}$	4 Δ_{th}	5 $\Delta_{th}-\Delta_{exp}$
	deg.		deg.	
0.01	0.0282	2.82	0.0296	+0.0014
0.02	0.0536	2.68	0.0552	0.0016
0.05	0.1245	2.49	0.1275	0.0030
0.10	0.2358	2.36	0.2400	0.0040
0.20	0.4498	2.25	0.4580	0.0082

It is to be remarked further that the degree of dissociation is much smaller than in the case of any inorganic compound so far examined, being only 56 per cent in the solution $m = 0.01$. Kohlrausch value for μ_∞ is assumed (110).

4. Passing now to the KH_2PO_4, where we have one H atom replaced by K, we find that the depressions produced belong to the group, KCl, NaCl, etc., and it at once suggests itself that with this salt as with all members of this group the molecule splits into two parts, in this case K and H_2PO_4, and that the degree of dissociation is about the same as for other potassium salts of the group.

Unfortunately, no measurements on the conductivity of KH_2PO_4 in extreme dilution have been made, so far as is known to the writer.

5. The replacement of a second atom of hydrogen by a metallic atom, as for example in the case of the two phosphates, Na_2HPO_4 and $NaNH_5HPO_4$, at once brings us into the second group, *i.e.* into the group of electrolytes which contain bivalent radicals, as K_2SO_4, $BaCl_2$, etc.

In this group the dissociation of the molecule is said to be into three parts, and the degree of this dissociation is on the average about 80 per cent in the solutions $m = 0.01$. Assuming the dissociation to be into three parts, namely, Na–Na–HPO_4, we are able to compute from the present data the degree of dissociation by the formula $\frac{\Delta}{m} = 1.89\ (1 + 2\delta)$, in which δ is the degree of dissociation. In this particular case, Na_2HPO_4, the value of δ is accordingly

$$\frac{0.0499}{0.01} = 1.89\ (1 + 2\delta), \text{ or } \delta = 0.82.$$

For Na_2HPO_4 the degree of dissociation in the solution $m = 0.01$ is thus 82 per cent. Values for the electrical conductivity of this salt in solution are not yet known, and hence it is not possible to verify this result at present.

Replacing now the *third* H atom with Na, we have Na_3PO_4, and the observed depression for the solution $m = 0.01$ becomes 0°.0715. To account for this very great depression, the dissociation theory requires that the salt be dissociated 93 per cent, each dissociated molecule yielding four parts. Here, however, as in the other cases, no electrical conductivity measurements have as yet been made.

For the sake of inquiring as closely as possible into the interesting matter presented by these phosphates, observations were made on the freezing-points of various mixtures of Na_2HPO_4 and NaOH solutions. The results are given at the end of the following table. The results for H_3PO_4 and the other phosphates under discussion are also found in the same table. The coefficients indicate the number of $\frac{1}{100}$ gram-molecules of the respective constituents in the solutions.

1	2 Depressions.	3 Successive differences.	1	2 Depressions.	3 Successive differences.
	deg.			deg.	
1 H_3PO_4	0.0282		1 Na_2HPO_4 + 2 NaOH	0.0995	0.0289
1 KH_2PO_4	0.0358	0.0076			
1 Na_2HPO_4	0.0499	0.0141	1 Na_2HPO_4 + 3 NaOH	0.1279	0.0284
1 Na_3PO_4	0.0715	0.0216			
1 Na_2HPO_4 + 1 NaOH	0.0706	—			

The successive differences which the series presents is not without interest. Assuming that in place of the salt KH_2PO_4 we had NaH_2PO_4, and that the depression due to the sodium salt is sensibly the same as that of the potassium salt,[1] then we observe (see table) that the addition of the first atom of Na increases the depression by 0°.0076, the second by 0°.0141, the third by 0°.0216. We have now reached the compound Na_3PO_4, to which the 1 NaH_2PO_4 + 1 NaOH is seen to be equivalent, at least, so far as the limits of accuracy of the latter solution require.

The further addition of 1 NaOH, as in the solution, 1 Na_2HPO_4 + 2 NaOH, which may be looked upon as equivalent to 1 Na_3PO_4 + 1 NaOH, increases the depression by 0°.0289. Thus far the successive differences appear, on the average, to be successive multiples of 0°.0073. The further addition of 1 NaOH, as appears in 1 $Na_2HPO_4 + 3$ NaOH, which is equivalent to $Na_3PO_4 + 2$ NaOH, is attended by about the same increase as that caused by the addition of the preceding molecule of NaOH, *i.e.* 0°.0284. Thus it would seem probable that the relation of the sodium atom to the rest of the molecule in the series of compounds examined is less and less close as the number of sodium atoms increases, and this fact may be sufficient ground on which to account for the progressive alkaline properties of this series of salts. I regret that these relations did not present themselves until the work was in preparation for publication (May–June), and thus no farther observations could be made. I wish again to emphasize that the results for Na_3PO_4 are certainly affected by a very large error, perhaps amounting to 7 per cent. I have no means, at present, of knowing in which direction this error lies. My only apology for presenting the results is the fact that even so large an error as this, either + or –, would not affect the essential nature of the interesting relations which the present study of the phosphates has brought to light. The examination of this question from the standpoint of electrical conductivity is desirable, since it appears that the dissociation theory would here encounter some very decisive tests.

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[1] My attempts to prepare solutions of NaH_2PO_4, which could be found by analysis to be sufficiently accurate, have thus far failed. All analogy, however, seems to allow the assumption which is made above.

The Organic Acids. (Acetic, Oxalic, Succinic, Tartaric, and Citric.)

The Preparation of the Solutions. — The acetic acid was reduced to normal strength by titration with the standards used in the preparation of the solutions previously described.[1] The preparation of the oxalic acid is described in the same connection. The succinic, tartaric, and citric acids were obtained from the makers in as high a degree of purity as they furnish them, and after drying as thoroughly in each case as their nature would permit, they were reduced to solutions of desired strength by direct weighing. They were then each titrated and corrected in accordance with the result of the titration, on the assumption that their having less than the supposed strength was due to the difficulty of completely drying them. The corrections are relatively very small, even in the case of succinic acid, where they amount to 0.5 per cent.

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[1] Loomis, PHYSICAL REVIEW, IV., 1896, p. 252.

The results are given below in tabular form, and graphically, near the bottom of Fig. 2.

Table VI.

ACETIC ACID, $C_2H_4O_2$.

$\mu_\infty = 364$.

1 m	2 Δ	3 $\frac{\Delta}{m}$	4 Δ theoretical.	5 $\frac{\Delta}{m}$ theoretical.	6 $\Delta_{th}-\Delta_{exp}$	7 $\Delta_{th}-\Delta_{exp}$ in per cents of Δ_{exp}.
	deg.		deg.		deg.	
0.0100	0.0196	1.96	0.0196	1.96	+0.0000	+0.0
0.0200	0.0375	1.88	(0.0388)	(1.94)	0.0013	3.5
0.0300	0.0559	1.86	0.0579	1.93	0.0020	3.6
0.0500	0.0928	1.86	0.0960	1.92	0.0032	3.4
0.1000	0.1855	1.86	0.191	1.91	0.0055	3.0
0.2000	0.3732	1.87	0.382	1.91	0.0090	2.4
0.3000	0.5629	1.88	—	—	—	—
0.5000	0.9378	1.88	—	—	—	—
1.0000	1.886	1.89	1.90	1.90	0.01	0.5

OXALIC ACID, $(COOH)_2$.

$\mu_\infty = 363$(?).

0.0100	0.0328	3.28	(0.0351)	(3.51)	+0.0023	+7.0
0.0200	0.0640	3.20	(0.0676)	(3.38)	0.0036	5.6
0.0500	0.1519	3.04	—	—	—	—
0.1000	0.2848	2.85	—	—	—	—
0.2000	0.5329	2.66	—	—	—	—

SUCCINIC ACID, $C_4H_6O_4$.

$\mu_\infty = 356$.

0.0100	0.0202	2.02	0.0204	2.04	+0.0002	+1.0
0.0199	0.0391	1.96	0.0398	2.00	0.0007	1.8
0.0498	0.0965	1.94	0.0976	1.96	0.0011	1.1
0.0995	0.1876	1.89	—	—	—	—
0.1990	0.3751	1.89	—	—	—	—

TABLE VI. (*continued*).

TARTARIC ACID, $C_4H_6O_6$.

$\mu_\infty = 356$

1 m	2 Δ	3 $\frac{\Delta}{m}$	4 Δ theoretical.	5 $\frac{\Delta}{m}$ theoretical.	6 Δth-Δexp	7 Δth-Δexp in per cents of Δexp.
	deg.		deg.		deg.	
0.0100	0.0234	2.34	(0.0239)	(2.39)	+0.0005	+2.1
0.0200	0.0435	2.18	(0.0454)	(2.27)	0.0019	4.4
0.0499	0.1042	2.09	(0.1070)	(2.14)	0.0028	2.7
0.0997	0.2018	2.02	—	—	—	—
0.1994	0.3993	2.00	—	—	—	—

CITRIC ACID, $C_6H_8O_7$.

1 m	2 Δ	3 $\frac{\Delta}{m}$	4 Δ theoretical.	5 $\frac{\Delta}{m}$ theoretical.	6 Δth-Δexp	7 Δth-Δexp in per cents of Δexp.
0.0100	0.0226	2.26				
0.0200	0.0424	2.12				
0.0499	0.1029	2.06		No	data.	
0.0998	0.1999	2.00				
0.1997	0.3978	1.99				

In computing the theoretical values for the depressions, the Kohlrausch values for electrical conductivity are used in the case of acetic acid. In other cases the Ostwald[1] measurements are used. In all cases Ostwald's value for μ_∞ was assumed as the most probable. This value is given at the top of the separate tables. No electrical measurements, so far as is known to me, have been made on citric acid. It should be said, perhaps, that the particular acids examined were chosen for the sole reason that they may be had in a high state of purity and are easily made into solutions of exact molecular strength.

This series of organic acids is chiefly interesting on account of the relation which is presented by them between the observed depressions and those required by the dissociation theory.

It appears that in the case of acetic, succinic, and tartaric acids, the agreement between the observed and the theoretical values for extreme dilution is so nearly exact that, barring experimental errors, it may be regarded as complete. Further, these theoretical values of the depressions indicate that the Ostwald values for μ_∞ are very probable, although in the case of acetic acid the experimental values for the electrical conductivity as found by Kohlrausch would not perhaps warrant the assumption of so high a value.

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[1] Ostwald, Zeit. Phys. Chem., III., 1889, pp. 174, 281, 371.

The exception presented by oxalic acid is not to be overlooked. This acid presents difficulties in other directions, as well as in the present instance, and it may happen that fuller knowledge of its nature may be able to reconcile the "theoretical" with the observed depressions of the freezing-point.

The Constancy of the Thermometer.

In conclusion I wish to submit the entire series of observations on the zero point of the $\frac{1}{100}^\circ$ thermometer, which were made during the period of the present measurements.

This thermometer has been in use four years. It has been kept at room-temperature except during the period of the measurements, when it is kept constantly within the limits, −3° and +1° C.

In the following table the observed zero points are given in (4), and the values when corrected to 760 mm., and a room-temperature 0° C. are given in (5).

1 1896.	2 Barometer.	3 Room-temperature.	4 Observed zero point.	5 Zero point corrected.
		deg.	deg.	deg.
Jan. 6	769.9 mm.	0 C.	0.0438	0.0422
7	766.6	0	0.0439	0.0428
20	763.3	5	0.0468	0.0439
22	762.0	6	0.0469	0.0438
29	765.6	6	0.0475	0.0438
Feb. 3	754.6	5	0.0457	0.0442
10	753.1	5	0.0458	0.0447
13	754.4	5	0.0460	0.0446
17	770.1	0	0.0451	0.0433
19	749.3	5	0.0454	0.0449
22	766.1	2	0.0456	0.0437
26	750.3	2	0.0440	0.0447
Mar. 3	751.6	3	0.0447	0.0447
4	753.1	3	0.0447	0.0445
5	753.9	5	0.0465	0.0452
11	749.1	4	0.0450	0.0449
12	747.8	4	0.0449	0.0452
16	756.2	4	0.0458	0.0445
17	756.4	5	0.0466	0.0449

The results are graphically represented in Fig. 3. Ordinates represent thermometer readings, abscissæ time of observations.

The scale is so chosen that the observed changes in the thermometer readings are magnified 125-fold. In order to facilitate comparison between the observed course of the zero point and what it would have been in case it had made a strictly uniform rise of 0°.0022 during the period, a straight line has been drawn from 0°.0428 to 0°.0450. We observe, first, that the initial reading is 0°.0422. The last reading of the preceding winter was 0°.0417.

It thus appears that during the nine months of rest the zero of the thermometer remained constant. The total rise during the present period is about 0°.002 C.; during the preceding winter it was a little more than 0°.002 C. Further, it is noticeable that the course of the observed zero point is much less of a "zig-zag" than in former periods of use. This, I think, is due to a reduction of the experimental error of the method and to a greater constancy of the thermometer itself, due to its age and continued use.

The sharp changes between Feb. 13 and Feb. 22 must, I think, be referred to the thermometer itself and not to experimental errors, since the series of depressions which were measured on these particular days indicate no corresponding errors. The HNO_3 series was measured on Feb. 19, when the zero point seems to be too high. While this particular series of observations show unusually large experimental errors, these errors seem to suggest that the zero point was too low rather than too high. The other zero points in this period of marked variation of the thermometer were made in connection with solutions which are not yet ready for publication, which, however, show no cause for assuming that the zero points were erroneously determined. This matter will be noticed fully at the proper time.

Summary of Results.

1. The method developed in 1893 has been used, with no change whatever (except an enlargement of the "freezing-tube"), to de-

termine the depression of the freezing-point for the following solutions: —

$LiCl$, $CaCl_2$, $SrCl_2$, $SnCl_4$,
HNO_3, KOH, $NaOH$,
KH_2PO_4, Na_2HPO_4, $Na(NH_4)HPO_4$, Na_3PO_4,
Na_2SiO_3,
$C_2H_4O_2$, $(COOH)_2$, $C_4H_6O_4$, $C_4H_6O_6$, $C_6H_8O_7$.

2. It has been found that the average variation in a series of five entirely independent determinations of the freezing-point of a solution is now 0°.0005, or about half its former value. This indicates a reduction of the experimental errors, and this indication is confirmed by the greater regularity of the results in the region of extreme dilution. It is thought that this decrease in the experimental error is due wholly to a greater constancy of the room-temperature.

3. The observations on the four chlorides of the present series, together with additional observations on the five chlorides previously studied, establish the fact that the *molecular depression of the freezing-point for all chlorides reaches a minimum value.* This depression is very pronounced for the binary chlorides. See Fig. 1.

$SnCl_4$ yields such enormous depressions that the measurements could not be carried far enough to establish any conclusion in this regard. There is no reason, however, for believing that this chloride would not also reveal a minimum in the molecular depression if the measurements had been extended into the region of greater concentration.

4. Tin chloride in extreme dilution produces a molecular depression of 12.61. This is more than five times the van't Hoff constant (1.89). The hypothesis of Arrhenius fails thus to explain the depression, since the greatest possible number of "active molecules" which a molecule of $SnCl_4$ could yield would be *five*, and so the maximum molecular depression which this salt could produce would be 5×1.89, or 9.45. The observed molecular depression, however, is 12.61, at $m = 0.01$. It may be suggested that $SnCl_4$ in solution possesses the properties of a mixture

of $SnCl_2$ and $2\,HCl$. If this be so, the molecular depression would be about 12.42. This is not far from the observed result. The strong acid reaction of solutions of $SnCl_4$ favors the suggestion.

5. The study of the phosphates makes it probable that H_3PO_4 in aqueous solution is dissociated simply into two parts, — H and H_2PO_4.

This appears from the fact that its salt, KH_2PO_4, gives a depression which belongs to the first group, namely, to the class of electrolytes which yield two parts in the process of dissociation (NaCl, KCl, etc.). Thus the *salt* probably splits into K and H_2PO_4, and this points to the conclusion that the acid behaves similarly.

Further, it appears that the successive introduction of a univalent metallic radical into a salt of phosphoric acid increases the depression by a constant amount for each radical so added. This would be explained on the basis of the dissociation theory by saying that the introduction of each metallic radical increases by one the number of parts into which the molecule is dissociated, and causes no essential change in the *degree* of dissociation.

Thus in KH_2PO_4 there are two parts, K and H_2PO_4,
in Na_2HPO_4 there are three parts, Na–Na–HPO_4,
in Na_3PO_4 there are four parts, Na–Na–Na–PO_4.

6. Sodium silicate, Na_2SiO_3, presents the same difficulty as $SnCl_4$, *i.e.* the depression is much greater than the simple hypothesis of Arrhenius can account for.

7. The organic acids — acetic, oxalic, succinic, tartaric, and citric — exhibit the characteristics of the electrolytes, and not those of the non-electrolytes. The only exception is acetic acid, which in the more concentrated solutions behaves exactly like the non-electrolytes; namely, beyond $m = 0.10$, the molecular depression *increases uniformly* with the concentration.

8. A comparison of the experimental results with those required by the hypothesis of van't Hoff and Arrhenius shows that in the case of LiCl and $CaCl_2$ the agreement is practically complete. A most striking agreement is also presented by acetic, tartaric, and succinic acids.

The agreement is fairly good in the case of $SrCl_2$.

On the contrary HNO_3, KOH, and NaOH deviate very widely from the "theory." The differences between the observed values and those required by the hypothesis of Arrhenius are 8–11 per cent for the most dilute solutions. The fact that these differences become very small in the region of greatest concentration suggests the existence of some unknown source of error peculiar to the solutions.

In the remaining cases, $SnCl_4$, citric acid, and the salts of phosphoric acid, no measurement of the electrical resistances have been made, and thus no data are at hand for computing the theoretical values of the depressions.

These measurements are much needed, together with those for the extremely dilute solutions of $MgCl_2$ and NH_4Cl.

Physical Laboratory, Princeton University.

A METHOD FOR ENERGY MEASUREMENTS IN THE INFRA-RED SPECTRUM AND THE PROPERTIES OF THE ORDINARY RAY IN QUARTZ FOR WAVES OF GREAT WAVE LENGTH.[1]

By Ernest Fox Nichols.

IN previous determinations of the distribution of energy in the infra-red spectrum, either the principle of the thermoelement, or of the more modern bolometer, has been almost without exception the basis of measurement.[2]

In the present study, however, a modified form of the Crookes radiometer has proved so efficient that I take the liberty of describing briefly the apparatus and method by which the observations on quartz, given in the second half of the paper, were made.

I. Apparatus and Method.

1. *The Spectrometer.*

A diagram showing the relation of parts of the reflecting spectrometer used is given in Fig. 1. Rays enter the instrument through the bilateral slit s_1, are brought to parallelism by the concave mirror S_1, and, after reflection on the plane mirror S_2, traverse the prism P, are again collected by the concave mirror S_3, and brought to focus at s_2 in the tube F. The only unusual feature of the arrangement is the introduction of the plane mirror S_2 rigidly attached to the prism, a modification due to Wadsworth,[3] in which, contrary to the usual process, the telescope remains fixed, and the spectrum is made to travel across the field of view by rotating the combined prism and mirror. This adjustment is necessary to the present method, because, unlike the bolometer, the radiometer is not conveniently portable.

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[1] Read before the Berlin Academy, Nov. 5, 1896.

[2] Pringsheim [Wied. Ann., 18, p. 32 (1883)], in the study of certain wave lengths in the infra-red solar spectrum, used a Crookes torsion radiometer combined with a grating, but the form of apparatus differed, in many respects, from the arrangement here to be described.

[3] Phil. Mag., 38, p. 337, 1894.

The eye-piece in the tube F was removed, and a second bilateral slit s_2 was brought into the focal plane of the concave mirror S_3. Rays emerging from this slit were collected by the rock-salt lens

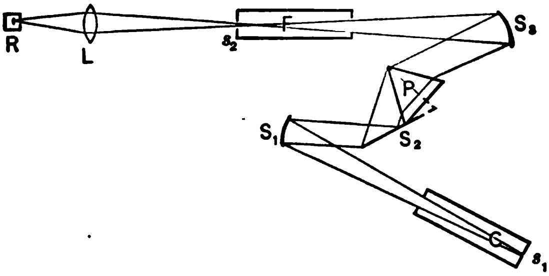

Fig. 1.

L, and brought to focus on one vane of the radiometer at R. The large fluorite prism P ($\phi = 60° 0'$) was calibrated in accordance with Paschen's[1] indices of refraction.

The calibration curve was tested in the visible spectrum, and found correct to 2 units in the fourth decimal place. In the infrared the curve was again tested for the middle of the absorption band in sylvite, given at $\lambda = 7.08\,\mu$ by Rubens,[2] and here observed at $\lambda = 7.088\,\mu$ according to the curve. At this point a sylvite plate 4 mm. thick transmitted only 6 per cent of the incident energy. This may be accepted as evidence of the purity of the spectrum obtained, and of the absence in this region of more than a small percentage of stray rays of shorter wave lengths.

2. *The Radiometer — Construction.*

The details of construction of the radiometer will be doubtless most easily understood from Fig. 2, which shows a section of the instrument facing the spectrometer.

The figure is drawn to scale, and represents the instrument at half its natural size. The outer case A, a block of bronze with an axial boring from the top to within 5 mm. of the base, is supported upon three leveling screws, and is closed at the top by a bell of glass *B* ground on the upper surface of the case. *H* is a stopcock to cut off the connection to a Raps' mercury pump used to produce the vacuum. The case *A* contains two lateral borings: *C*, closed by a plate of mirror glass through which the deflections of the suspension are read, and *F*, closed by a fluorite plate through which the rays to be measured enter the radiometer. The arrangement of parts in the window *F* is seen in Fig. 2 *a*, which shows a sectional plane perpendicular to the former one.

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[1] Wied. Ann., 53, p. 301, 1894.

[2] Wied. Ann., 54, p. 481 (footnote), 1895.

The circular fluorite plate *P*, 5 mm. thick, is held in a screw fastening with rubber packing at *d*, and the whole mounted on a brass plate cemented to the case. In the boring *F* a short brass tube *r* is soldered, which extends to within 2 mm. of the axis of *A*, and carries a thin plate of mica *k* on its inner end.

Directly behind the mica plate hang the two equal mica vanes *aa*, both blackened on the same side, which form the important part of the torsion suspension. The vanes are held together by thin whips of drawn glass, and on the line midway between them is fastened the glass fiber *ce*, which forms the axis of rotation and carries near its lower end the mirror *s*. A very fine quartz fiber attached at *c* is suspended from the bridge *g*, and gives the directive force to the system.

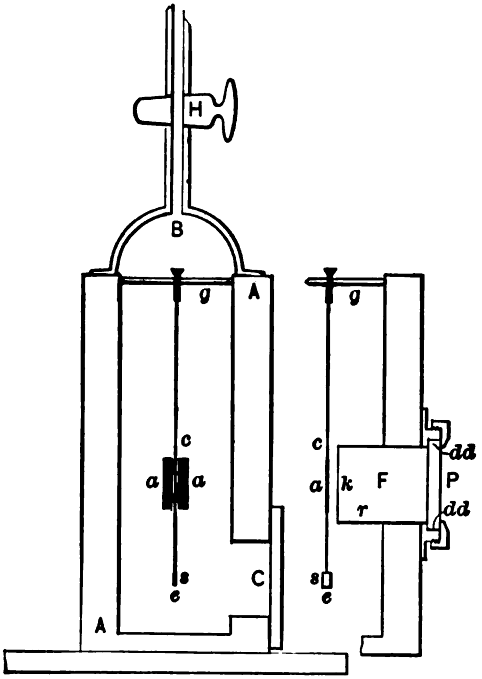

Fig. 2. Fig. 2*a*

The total weight of the suspension was 7 *mg*.

3. *The Radiometer as a Measuring Instrument.*

The sensitiveness of the radiometer was shown by Crookes[1] to be closely connected with the pressure of the gas in the inclosing vessel. A study at comparatively high pressures begun by Stoney and Moss[2] showed that the sensitiveness for the same pressure changed with the distance of the blackened vane from the opposite wall, increasing as the distance decreased.

Crookes[1] also observed that in the region of pressures where the radiometric sensitiveness was high, the logarithmic decrement of a torsion pendulum began to decrease rapidly with the pressure. These are the principal considerations that affect the present instrument, which, when the vanes were 2½ mm. from the mica plate and the pressure[3] in the vessel 0.05 mm., had a full period of vibration of twelve seconds, and was at its maximum sensitiveness. With further pumping the period of vibration could be brought down to eight seconds, but the shortening was accompanied by a decrease in sensitiveness. With greater pressure the period increased until it became aperiodic, and at atmospheric pressure if the upper end of the fiber was given 90° twist fully a minute elapsed before the suspension followed to its new position.

Near the maximum point the sensitiveness changes very slowly, so that the period of vibration is here the better test for the pressure sought.

If the vanes be brought within a millimeter of the mica window, for pressures greater than 0.1 mm., the sensitiveness is much increased, but the disadvantage is twofold, as the change involves an increased period, and the throws for small quantities of energy are greater in proportion than for large quantities.

For pressures near, or below, the maximum sensitiveness, small changes in the distance have no noticeable effect either on the sensitiveness or on the period.

Under the conditions which prevailed throughout the measurements the throws of the radiometer were proved proportional to the energy in several different ways; one of which was the measurement of the reflection percentage as dependent upon the wave length in the case of a fluorite surface under an incidence angle of 25°.[1]

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[1] Proc. Roy. Soc., Nov. 16, 1876.

[2] Proc. Roy. Soc., Feb. 22, 1877.

[3] Measured with the MacLeod gauge.

The results obtained are compared in Table I. with reflection computed by the Fresnel formula, using Paschen's[2] observed indices of refraction. Wave lengths appear in the first column, observed reflections in the second, computed reflections in the third, and the differences in the fourth.

Table I.

REFLECTION OF FLUORITE.

$r = 25°. \quad R = \frac{R_{\perp} + R_{\parallel}}{2}.$

λ	R obs.	R comp.	Δ	λ	R obs.	R comp.	Δ
0.59	0.031	0.032	−1	4.0	0.030	0.030	0
1.00	0.031	0.032	−1	4.5	0.029	0.029	0
1.50	0.031	0.031	0	5.0	0.029	0.028	+1
2.00	0.031	0.031	0	5.5	0.029	0.028	+1
2.36	0.030	0.031	−1	6.0	0.028	0.027	+1
2.50	0.031	0.031	0	6.5	0.028	0.026	+2
3.00	0.031	0.031	0	7.0	0.027	0.025	+2
3.50	0.031	0.030	+1				

The sensitiveness at which the radiometer was held during the measurements which follow, was such that the rays from a candle at a distance of 6 m., when allowed to fall on one of the vanes, gave a deflection of 60 divisions on a millimeter scale at a distance of 1 m. from the instrument. This corresponds to a throw of more than 2100 scale divisions for a candle 1 m. away.

The compensating action of the two vanes was so nearly perfect, that rays from a source directly in front of the radiometer, which, when allowed to fall upon either vane alone gave a throw of 60 scale divisions, gave less than a millimeter when both were exposed. Thus it was possible to have an extremely sensitive instrument which at the same time held steadily to a fixed zero point, and gave throws trustworthy to the tenth of a millimeter.

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[1] These observations were made by comparing the reflection of fluorite with that of silver as a standard, as in the case of quartz to be described later. As the reflection of fluorite is only about 3 per cent of the corresponding reflection of silver, throws — the larger of which was from thirty to forty times the smaller — were directly compared, and the test of proportionality was consequently a very severe one.

[2] Wied. Ann., 53, p. 301, 1894.

As compared with the linear bolometer or the thermopile, the present form of radiometer has the following advantages: —

1. It is uninfluenced by all magnetic and thermo-electric disturbances, which render work with a very sensitive galvanometer tedious and unsatisfactory.

2. It can be more accurately compensated against rays which do not come from the source to be measured, and is consequently more free from "drift."

3. The radiometer is free from any disturbance corresponding to the air currents that arise about the warmed bolometer-wire.

It has, however, the following disadvantages: —

1. It is not as easily portable as either the bolometer or the thermopile.

2. All the rays to be measured must traverse the window of the radiometer, and be subject to its selective reflection and absorption.

3. In its present form the radiometer cannot be separated from a mercury air pump for more than a week at a time and remain sensitive.

II. OBSERVATIONS ON QUARTZ.

Our knowledge of the optical properties of quartz in the visible spectrum has been extended in the one direction by observations of Sarasin[1] in the ultra-violet, and in the other, by observations of Mouton,[2] and later of Rubens,[3] in the infra-red. Rubens' observations extend to a wave length $\lambda = 4.2\mu$. Beyond this point direct measurements of refractive indices are impossible because of the heavy absorption which in quartz begins in this region.

The present study begins here, and consists of observations of the reflection and transmission of quartz between the wave lengths

[1] Archiv. des Sc. Phys., 3, 10, p. 303, 1883.

[2] Comptes Rend., 88, p. 1190, 1879.

[3] Wied. Ann., 53, p. 277, 1894, and 54, p. 480, 1895.

$\lambda = 4\mu$ and $\lambda = 9\mu$, where for another reason — the absorption of the fluorite prism used for dispersion — these observations come to an end.

1. *The Observation of Reflection.*

The reflection of quartz was not compared directly with the intensity of the incident beam, but for convenience with the reflection of silver, which had been previously determined.

The study of silver was made by comparing the beam reflected under an angle of 15° with the direct, by means of the "swinging arm" apparatus used earlier by Rubens.[1] The mirror studied was a silver deposit on glass.

The results are given in Table II., which contains also the values for the reflection percentages of silver observed by Rubens[1] and by Langley.[2] The agreement is seen to be very close. That the values here given are however slightly higher, in the visible spectrum, than those of either Rubens or Langley, can doubtless be

TABLE II.

λ	Nichols.	Rubens.	Langley.	λ	Nichols.	Rubens.
0.35	—	—	61	0.90	96.0	95.8
0.38	—	—	73	1.00	—	96.5
0.40	—	—	79	1.15	—	97.0
0.43	82.7	—	—	1.40	—	97.4
0.44	86.4	—	—	1.65	—	97.7
0.45	—	87.0	85	2.00	97.2	97.3
0.49	90.1	—	—	2.50	96.5	97.0
0.50	—	88.3	89	3.00	97.3	98.3
0.54	91.5	—	—	3.50	98.3	—
0.55	—	90.3	91	4.00	100.0	—
0.59	91.6	—	—	6.00	99.8	—
0.60	—	92.7	92	7.00	99.6	—
0.64	93.6	—	—	8.00	99.0	—
0.65	—	93.3	93	8.40	99.5	—
0.70	—	94.5	93	8.65	99.2	—
0.75	95.0	—	94	9.00	100.0	—
0.80	—	95.2	—	—	—	—

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[1] Wied. Ann., 37, p. 249, 1889.

[2] Phil. Mag., 27, p. 10, 1889.

explained by differences in polish, upon which the reflection in this region in great measure depends.

In the infra-red, on the contrary, the reflection is but little affected by ordinary differences in polish, and a silver mirror grown perceptibly yellow with age gave the same percentage of reflection at wave lengths 6μ and 8μ as a perfectly fresh one. For waves longer than 4μ observations show the reflection to be so nearly total, that in the comparison with quartz the intensity of the beam from the silver mirror was taken as unity.

A prism surface cut perpendicular to the optic axis of a negatively rotating quartz crystal was chosen for the study of the reflection of quartz.

To gain the very small incidence angle of 5°, the arrangement of apparatus shown in Fig. 3 was adopted, in which p is the zircon plate of the Linnemann burner which supplies the energy. L_1 is a rock-salt condensing lens, M_1 the device for holding the mirrors to be compared, and M_2 a silver plane mirror to direct the rays into the spectrometer slit s_1. A shutter was introduced at S and a diaphragm at D. The two plates forming the sides of the spectrometer slit were covered with squared paper, and the position of the real image formed by the lens L_1 of a platinum point at c served as a check to control the adjustment in the change from silver to quartz and the reverse.

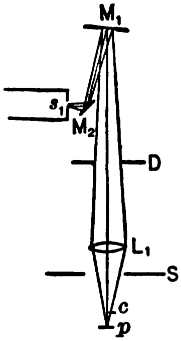

Fig. 3.

The surfaces to be compared were carefully mounted in two exactly equal carriers, each of which fitted into the holder M_1, and were held firmly in place by two springs.

To show the treatment of single observations and at the same time the behavior of the radiometer, the observations at wave lengths 8.72μ and 8.52μ are given entire. Of the fifteen series of observations, made at as many different wave lengths between 8μ and 9μ, the former furnishes the best agreement of single values among themselves, and the latter the worst.

TABLE III.

TWO SETS OF OBSERVATIONS OF THE REFLECTION OF QUARTZ *Q* COMPARED WITH THE REFLECTION OF SILVER *S*.

κ^2, the percentage of reflection, is obtained alternately from $\frac{2Q_n}{S_n + S_{n+1}}$ and $\frac{Q_n + Q_{n+1}}{2S_{n+1}}$.

$\lambda = 8.72\,\mu$. Slits 1.5 mm. wide.			$\lambda = 8.52\,\mu$. Slits 1 mm. wide.		
Scale readings.	**Throws.**	κ^2	**Scale readings.**	**Throws.**	κ^2
S_1 532.2 497.1	35.1	—	S_1 532.9 506.1	26.8	—
Q_1 531.8 509.9	21.9	62.7	Q_1 533.0 514.2	18.8	71.9
S_2 531.9 497.0	34.9	62.9	S_2 532.7 507.0	25.7	72.0
Q_2 531.9 510.0	21.9	62.9	Q_2 532.9 514.6	18.3	70.9
S_3 531.8 496.9	34.9	62.5	S_3 533.1 507.0	26.1	70.9
Q_3 531.7 510.0	21.7	62.5	Q_3 532.6 513.9	18.7	72.9
S_4 531.8 497.0	34.8	62.2	S_4 532.3 506.9	25.4	74.0
Q_4 531.4 509.9	21.5	62.3	Q_4 532.7 513.7	19.0	74.0
S_5 531.3 497.1	34.2	—	S_5 532.1 506.3	25.8	—
Average, 62.6 per cent.			Average, 72.4 per cent.		

All the observed values of the reflection percentages of quartz are collected together in the second column of Table IV., the respective wave lengths being given in column 1. Column 3 gives the wave-length interval, $\lambda_1 - \lambda_2$ in μ, corresponding to the angular width of the slits used, both of which were always opened equally wide. These values thus show in a sense the purity of the spectrum at each of the observed points.

TABLE IV.

λ	n^2 $i=5°$	$\lambda_1-\lambda_2$	T obs.	$\lambda_1-\lambda_2$	T_1 corr.	κ_0^2 10^{-4}
4.20	—	—	90.0	0.07	—	—
4.40	—	—	89.7	0.07	—	—
4.50	3.40	0.07	92.0 93.6	0.07 0.10	99.0	0
4.70	—	—	90.1	0.06	—	—
4.80	—	—	89.3	0.07	—	—
4.90	—	—	85.9	0.07	—	—
5.00	3.00	0.07	69.7 71.3	0.07 0.10	75.0	41
5.05	—	—	71.0	0.05	—	—
5.10	—	—	74.6	0.05	—	—
5.15	—	—	74.5	0.05	—	—
5.20	—	—	71.1	0.07	—	—
5.25	—	—	62.3	0.05	—	—
5.30	2.73	0.07	54.1	0.05	57.0	174
5.40	—	—	67.8	0.06	—	—
5.50	—	—	72.2	0.10	—	—
5.70	—	—	80.0	0.11	—	—
5.80	2.50	0.09	81.1	0.05	85.4	16
5.90	—	—	75.5	0.11	—	—
6.00	—	—	72.8	0.11	—	—
6.10	—	—	70.6	0.11	—	—
6.20	—	—	60.2	0.11	—	—
6.25	1.80	0.11	59.2 59.4	0.11 0.05	61.9	174
6.30	—	—	63.3	0.10	—	—
6.35	—	—	68.1	0.11	—	—
6.40	—	—	77.2 75.9	0.11 0.11	—	—
6.45	1.46	0.10	79.8 82.3	0.11 0.05	83.5	42
6.50	—	—	77.2	0.10	—	—
6.55	—	—	72.0	0.10	—	—
6.60	—	—	68.1	0.10	—	—
6.65	—	—	68.1	0.05	—	—
6.70	1.19	0.10	67.6	0.10	69.0	121
6.80	—	—	74.5	0.10	—	—
6.90	—	—	80.5	0.10	—	—
7.00	0.65	0.12	81.3	0.09	82.2	23
7.10	0.46	0.12	[1] 79.3	0.09	80.1	62
7.20	0.44	0.12	75.3 [2]	0.09	76.2	76
7.30	0.31	0.12	70.4	0.09	71.2	121
7.40	0.29	0.12	66.7	0.09	67.4	166

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[1] Graphically interpolated.

[2] Double dispersion applied to transmission spectrum in this region.

TABLE IV. (*continued*).

λ	R^2 $i=5°$	$\lambda_1-\lambda_2$	T obs.	$\lambda_1-\lambda_2$	T_1 corr.	κ_0^2 10^{-4}
7.50	0.42	0.12	59.5	0.09	60.1	286
7.60	0.86	0.09	51.5	0.09	52.5	466
7.70	1.34	0.09	35.8	0.09	36.9	1,149
7.80	3.25	0.09	18.0	0.09	19.3	3,215
7.90	[1] 6.00	0.09	12.2 [2]	0.09	13.9	4,747
7.92	6.69	0.09	—	—	—	—
8.00	14.0	0.08	5.6	0.09	7.8	8,136
8.02	17.2	0.08	—	—	—	—
8.05	[1] 23.0	0.08	3.1	0.09	5.8	10,240
8.10	—	—	> 0.5	0.09	—	—
8.12	43.3	0.08	—	—	—	—
8.22	63.3	0.08	—	—	—	—
8.32	70.4	0.08	—	—	—	—
8.42	75.0	0.08	—	—	—	—
8.52	72.4	0.08	—	—	—	—
8.62	51.1	0.08	—	—	—	—
8.65	56.5	0.08	—	—	—	—
8.67	58.4	0.08	—	—	—	—
8.72	62.6	0.12	—	—	—	—
8.80	67.1	0.12	—	—	—	—
8.82	64.0	0.12	—	—	—	—
8.90	60.5	0.08	—	—	—	—
8.92	58.0	0.11	—	—	—	—
9.00	51.5	0.08	—	—	—	—
9.02	49.2	0.11	—	—	—	—

Wave lengths and reflection percentages are shown again in a curve in the plate opposite page 312. The curve falls very gradually from $\lambda = 4.5$ μ to a minimum at 7.4 μ, where the observed reflection was only 0.29 per cent.

From $\lambda = 7.4\,\mu$ to $\lambda = 8.45\,\mu$ the curve rises with surprising rapidity, reaching a maximum value of 75 per cent near the latter wave length, after which it falls to a second minimum of 51 per cent near $\lambda = 8.6\,\mu$ and rises to a second maximum of 66 per cent at $\lambda = 8.8\,\mu$. It then falls regularly to $\lambda = 9\,\mu$, where the observations of reflection end.

The error in the reflection percentages here given is due wholly to the presence of a small percentage of rays of shorter wave length, never entirely absent from measurements in this region, and which here has the effect to bring the two maximum points too low, and the second one lower in proportion than the first. The first minimum at $\lambda = 7.4\,\mu$ is probably too high, as will be seen later.

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[1] Graphically interpolated.

[2] Double dispersion applied to transmission spectrum in this region.

2. *The Observation of Transmission.*

For the study of transmission, the lens L_1 (see Fig. 3) and the lamp were brought into line with the collimator of the spectrometer, and the intensity of the direct ray compared with that of the ray transmitted through a thin plate of quartz placed directly before the slit.

The plate used was a thin scale cut perpendicular to the optic axis of the crystal and mounted in a glass frame with a free aperture 10 × 5 mm. The spectrum of light reflected from the plate under an angle of 15° gave 10 + interference bands between the Fraunhofer lines *C* and *D*, from which the thickness 18 μ was computed.

Column 4 of Table IV. gives the observed percentages of transmission T for the wave lengths given in column 1. Column 5 contains the wave-length interval $\lambda_1 - \lambda_2$ corresponding to the angular width of the slits; and column 6 gives the values of T_1, *i.e.* the values of T corrected for the reflection at both surfaces, computed by the formula

$$T_1 = \frac{T}{(1 - 2\,u^2)^2}.$$

The observed transmissions extend from $\lambda = 4.2\,\mu$ to $\lambda = 8.05\,\mu$ and the observations are plotted as a curve in the plate with wave lengths as abscissæ and percentages of transmission as ordinates.

Beyond $\lambda = 7\,\mu$, to make sure of the purity of the spectrum, which later became important, a second fluorite prism and slit were introduced into the path of the rays before they entered the spectrometer, thus giving double dispersion. In the path of the curve between $4.2\,\mu$ and $7\,\mu$ are five maxima and four minima, of which three in each case are sharply defined. From $7\,\mu$ on, the curve sinks so rapidly that it was impossible to follow it beyond 8.1 μ, where, through a thickness only two and one-fourth times the wave length of the incident ray, only a doubtful 0.5 per cent struggled through.

Although the whole region between 8.1 μ and 9 μ was carefully searched, no trace of transmitted energy large enough to measure was discovered.

TABLE V.

λ	ν − Root.	ν + Root.	u^2 corrected.	ν accepted.	ν Rubens.	Δ
4.50	0.692	1.450	—	1.450	1.442	+ 8
5.00	0.706	1.417	—	1.417	1.411	+ 6
5.30	0.719	1.393	—	1.393	1.386	+ 7
5.80	0.730	1.368	—	1.368	1.343	+ 25
6.25	0.765	1.309	—	1.309	1.287	+ 22
6.45	0.785	1.274	—	1.274	1.257	+ 17
6.70	0.803	1.242	—	1.242	1.212	+ 30
7.00	0.853	1.172	—	1.167[1]	1.145	+ 22
7.10	0.873	1.144	—	1.125[1]	1.117	+ 8
7.20	0.877	1.141	0.150	1.080	1.089	− 9
7.30	0.899	1.113	0.025	1.032	1.056	− 24
7.40	0.899	1.112	0.000	1.000	1.020	− 20
7.50	0.880	1.137	0.135	0.930	0.979	− 49
7.60	0.835	1.199	0.568	0.861	0.933	− 72
7.70	0.798	1.256	—	0.798	0.881	− 83
7.80	0.702	1.432	—	0.702	0.819	− 117
7.90	0.611	1.645	—	0.611	0.746	− 135
8.00	0.478	2.172	—	0.478	0.657	− 179
8.05	0.366	—	—	0.366	0.603	− 237

3. *The Dispersion.*

In general, it is possible to compute ν, the index of refraction, for every wave length λ for which u^2 and D_1 are known, by substituting, in the Cauchy formula, the observed values of u^2 and the extinction coefficient κ_1. In reflection at normal incidence, the intensity of the ray $u_{\parallel}^2$ polarized in the plane of incidence and the ray $u_{\perp}^2$ polarized in the plane at right angles are equal, and for 5°, the observed incidence, the equality may, within the limits of error, still be assumed.

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[1] Interpolated graphically.

The simpler expression, that for $u_{\parallel}^2$, was consequently used in the computation, and the observed reflection, u^2, put into the formula

$$u^2 = \frac{(\cos i - \sqrt{\nu^2 - \sin^2 i})^2 + \kappa_1{}^2}{(\cos i + \sqrt{\nu^2 - \sin^2 i})^2 + \kappa_1{}^2},^{1} \tag{1}$$

which may be expressed as an explicit function of ν of the form

$$\nu^2 = \sqrt{\sin^2 i + \left\{\cos i\left(\frac{1+u^2}{1-u^2}\right) \pm \sqrt{\cos^2 i\left(\frac{1+u^2}{1-u^2}\right)^2 - (\kappa_1{}^2 + \cos^2 i)}\right\}^2}, \tag{2}$$

in which the extinction coefficient κ_1 for the incident angle i may, in this case, be put equal to κ_0, the extinction coefficient for normal incidence. These values, computed from the formula[2]

$$T_1 = e^{\frac{-4\pi d\kappa_1}{\lambda}}, \tag{3}$$

are given in column 6, Table IV. In the formula, $d = 18\,\mu$, the thickness of the plate, and λ is the given wave length.

As the value of $\kappa_0{}^2$ at $8.05\,\mu$, the highest observed, is only 102×10^{-4}, $\kappa_0{}^2 m$ (2) was put equal to zero, so that the refraction, in this case, depends solely upon the reflection, and the modified Fresnel formula is applicable within the limits of error.

Equation (2) gives two values of ν for every value of u^2, depending on which root of the inner radical is chosen. The index given by the positive root corresponds to a reflection u^2 for a ray striking at an angle of 5° from the rarer into the denser medium, while the negative root gives an index which would give the same reflection for the same incident angle for a ray crossing the boundary in the opposite direction. Both values are given in Table V., and again as curves in the plate. From the beginning out to $\lambda = 7.4\,\mu$, where u^2 reaches its lowest value, there is no question which index is the correct one, but beyond this point the choice is a doubtful one, as will be seen later. The equation as a whole possesses one important peculiarity: whenever u^2 is very small, minute changes in u^2 produce comparatively large changes in ν. At $7.4\,\mu$, for

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[1] Wüllner Lehrbuch d. Physik, Bd. II., p. 536, Leipzig, 1883.

[2] Wüllner Lehrbuch d. Physik, Bd. II., pp. 123–126.

example, $u^2 = 0.29$ per cent, and $\nu = 1.11$ or 0.9; while for $u^2 = 0$, $\nu = 1$.

Reflection percentages in the neighborhood of zero must therefore be very carefully treated, and the possibility of false values due to stray light, never entirely absent in such work, must be taken into account.

The wide slits which it was necessary to use, and the nearness in the reflection spectrum to a maximum point of at least two hundred and fifty times the intensity here involved, make the conditions for stray energy most favorable. The importance of using double dispersion, which furnishes the best precaution against such an error, was unfortunately first realized after the measurements had all been taken. The observed reflection percentages in this region are in consequence doubtless too large.

If the reflection of 0.29 per cent observed at $7.4\,\mu$ was due wholly to stray energy from near $8\,\mu$, which is not unlikely under the circumstances, and the reflection percentages from $7.2\,\mu$ to $7.6\,\mu$ be corrected by this amount, and ν again computed for the corrected values, points are obtained which are connected by broken lines on the plate.

The position of these broken lines is suggested by the whole course of the curves outside this doubtful region, and the writer regards these new values of ν as nearer correct than the old ones, in which case the most natural conclusion is that the curves intersect rather than touch at $7.4\,\mu$. Quartz and air are thus believed to change places here, and later, air to play the part of the denser medium; but it is impossible, on the basis of these results alone, to say with certainty which of the two possible dispersion curves is the correct one.

The course of either curve between $7\,\mu$ and $8\,\mu$, together with the heavy absorption and the change in reflection, which at $\lambda = 7.4\,\mu$ is probably less than that of any body known in the visible spectrum, and which at $\lambda = 8.4\,\mu$ rivals that of burnished silver for violet light, show conclusively that quartz in this region passes over completely from a non-metallic to a metallic body.

4. *The Ketteler-Helmholtz Dispersion Formula.*

Rubens,[1] supplementing his own careful observations with the other experimental material in hand at that time, computed for quartz the five constants in a form of the Ketteler-Helmholtz dispersion formula, which is rigorously applicable only where the absorption bands are two in number, and linear. The values of ν given by these constants appear in column 6 of Table V. and again in the plate.

Only the general character of the curve thus obtained and that of the preferred curve computed from the observations can be considered the same. Before $\lambda = 7.2\,\mu$ the latter curve lies higher, afterward lower than the former one.

While the indirect method here employed for measuring the refractive index is at best a very rough one, still the divergence of the two curves is systematic, and is so great that the discrepancy cannot be entirely accounted for by the inaccuracies of the method. For the same wave length the difference between the refractive indices given by the two curves finally reaches 39 per cent. On the other hand the wave-length difference given by the two curves for the same index of refraction nowhere exceeds $0.16\,\mu$ and the agreement is closer than might have been expected when it is considered that the furthest point observed by Rubens was at $\lambda = 4.2\,\mu$. On the basis of constants computed for the region lying before this wave length, which were introduced into a special form of the general equation, the curve has been continued a full octave farther to $\lambda = 8.05\,\mu$.

It is also possible that there are still other absorption bands beyond $\lambda = 9\,\mu$, not taken into account in the formula, which influence the dispersion in the region studied.

Before closing I wish to thank Professor E. Warburg for the friendly interest he has taken in these experiments, and for the means to further them put at my disposal. I wish to express, also, my deep appreciation of the kindly interest in this study shown by Professor Heinrich Rubens, under whose direction it was made, and to acknowledge my indebtedness to him for many helpful

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[1] Wied. Ann., 53, p. 277, 1894, and 54, p. 480, 1895.

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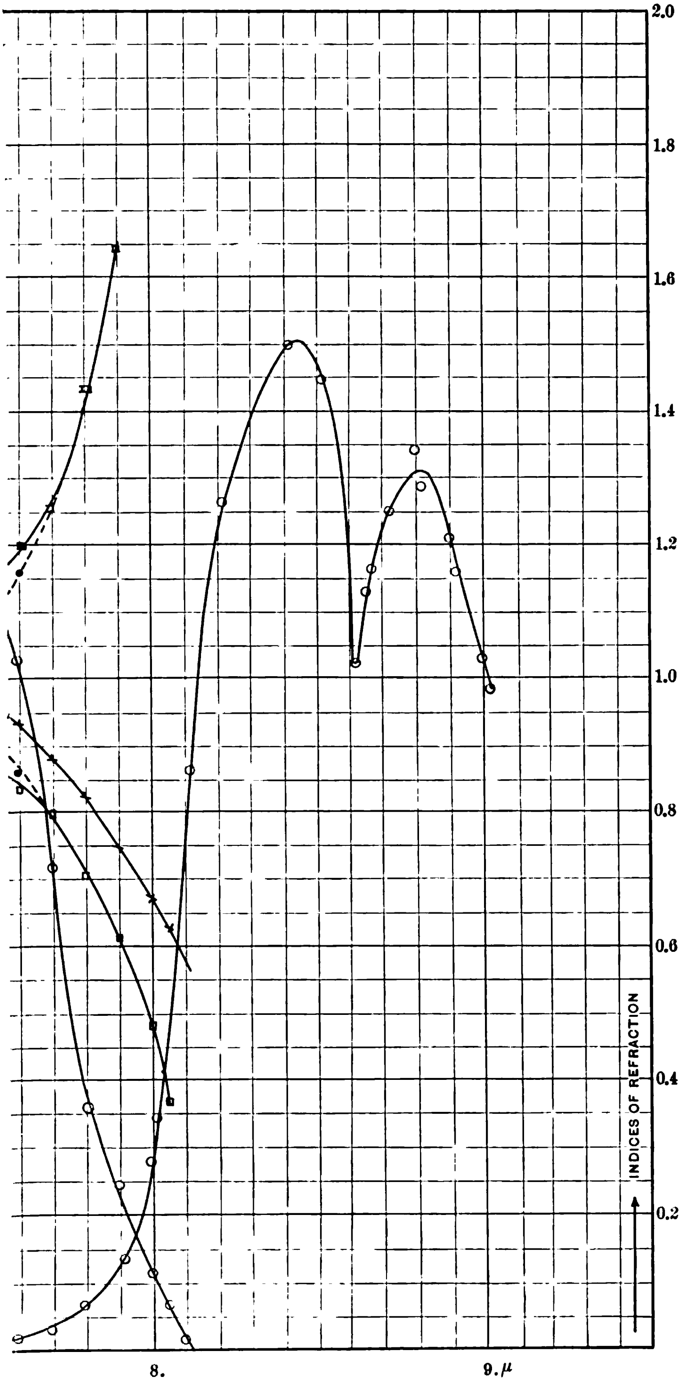
[1] Wied. Ann., 53, p. 277, 1894, and 54, p. 480, 1895.



RSION OF QUARTZ.

suggestions. I am further indebted to the kindness of Professor E. Pringsheim, at whose suggestion the work of building the radiometer was undertaken, for friendly advice concerning many details in the construction.

PHYSICAL INSTITUTE OF THE UNIVERSITY OF BERLIN,
June 1895–June 1896.

HEAT RAYS OF GREAT WAVE LENGTH.

By Heinrich Rubens and E. F. Nichols.

SINCE we have become accustomed to think of waves of electrical energy and light waves as forming component parts of a common spectrum, the attempt has often been made to extend our knowledge over the wide region which has separated the two phenomena, and to bring them closer together, either by cutting down the wave length of electrical oscillations — which, through the remarkable experiments of Righi[1] and later of Lebedew,[2] have been reduced to $\frac{1}{100}$ of the shortest waves measured by Hertz — or by the discovery and measurement of longer heat waves. In this direction no noticeable progress has been made since Langley's well-known experiments in 1886. On the contrary, since the determination of the dispersion of rock salt by Paschen[3] and one of the present writers,[4] it has been necessary to modify Langley's[5] estimate very considerably, so that the longest heat waves previously observed can scarcely have exceeded 0.015 mm., or about twenty-five times the wave length of sodium light. The farthest limit of exact measurement reached by Paschen,[6] in his determination of the dispersion of fluorite, was at 0.00943 mm., or approximately sixteen times the wave length of sodium light. The great difficulty which accompanies the study of infra-red waves of great wave length is that these rays form only the minutest part of the total energy emitted by the flame or incandescent body used as a source. Consequently, if their properties are to be studied, these waves must be separated from others which completely overlap and conceal them.

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[1] A. Righi, Rendiconti Acc. d. Lincei (5), 2, 505 (1893).

[2] P. N. Lebedew, Wied. Ann., 56, 1 (1895).

[3] F. Paschen, Wied. Ann., 53, 337 (1894).

[4] H. Rubens, Wied. Ann., 54, 476 (1895).

[5] S. P. Langley, Ann. de Chim. et de Phys. (6), 9, 433 (1886).

[6] F. Paschen, Wied. Ann., 53, 301 (1894).

To accomplish this end a prism of either rock salt or fluorite has commonly served.

The heat spectrum formed by a fluorite prism, because of absorption, contains no waves which exceed 0.01 mm. in length, while that from a rock-salt prism contains these waves and still longer ones in measurable quantity. That the diathermacy of this substance is by no means without limit, was conclusively shown by Gustav Magnus[1] twenty-six years ago, when he discovered that a rock-salt plate 5 mm. thick transmitted only about one-third of the energy emitted by a rock-salt crystal heated to a temperature of 150°.

The loss of energy due to the selective absorption of a prism is indeed avoided in the grating, but this escape is cumbered with disadvantages even more formidable; for not only are the diffraction spectra very weak in energy, but they overlap in such a way that it is almost impossible to separate them.

The new dispersion theories, however, suggest a method, by the use of which, unassisted by either prism or grating, homogeneous rays of great wave length may be obtained, and in sufficient quantity to make the determination of their properties and wave length possible. Helmholtz' dispersion theory, based on the laws of electromagnetic resonance, and that of Ketteler, based on the theory of elasticity, both lead to the same final equation which was shown in an earlier study by one of the present writers[2] to be in close agreement with the observed facts. In a region of the spectrum where there is little or no absorption, the formula common to both hypotheses is

$$n^2 = b^2 + \frac{M_1}{\lambda^2 - \lambda_1^2} - \frac{M_2}{\lambda_2^2 - \lambda^2},$$

in which n is the index of refraction corresponding to the wave length λ, and λ_1, λ_2, are wave lengths approximately in the middle of the two neighboring absorption bands, one in the ultra-violet and the other in the infra-red. b^2, M_1, and M_2 are other constants characteristic of the substance.

Theoretically, the absorption in the region of these two bands near λ_1 and λ_2 is very heavy, approaching the absorption of a metal for light rays; for a weaker absorption could not influence the dispersion. As absorption in this degree is necessarily accompanied by a metallic reflection, the wave lengths corresponding to the absorption band in any substance would appear magnified out of all due proportion in the spectrum of rays reflected from it. Suppose, for example, that within the region covered by the absorption band the reflection be twenty to thirty times as great as in other parts of the spectrum; as was shown by one of the present writers[1] to be the case in the reflection of quartz near wave length 0.0085 mm.; then in the spectrum of rays after three reflections from the substance, these waves would lose comparatively little in intensity, while the intensity of waves lying on either side would be brought down in the ratio of $(20)^3$ or $(30)^3$ to 1; *i.e.* as 8000 or 27000 to 1. If, further, a source of energy be chosen which emits these waves advantageously[2] — as will necessarily be the case if the body heated is of the same substance as the reflectors — the spectrum after three reflections will contain, in measurable quantity, only the wave lengths sought.

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[1] G. Magnus, Pogg. Ann., 139, 445 (1870).

[2] H. Rubens, Wied. Ann., 53, 267 (1894), and Wied. Ann., 54, 476 (1895).

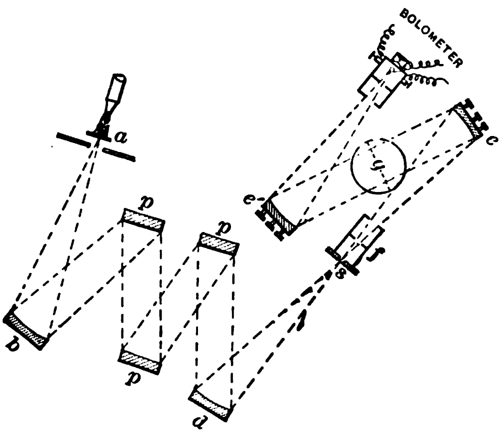

Fig. 1.

From the foregoing considerations the arrangement of apparatus shown in Fig. 1 will be easily understood. In the figure *a*, the source of energy, was a bit of platinum foil 15 × 15 mm., coated with the substance to be studied, and heated from behind by a blast lamp. A concave silver mirror at *b* reflected a bundle of parallel rays, which afterward underwent three successive reflections from the surface *p* of the substance in question. By means of the second concave silver mirror at *d*, the rays were brought to focus on the slit of a reflecting spectrometer. A transmission grating made of wires $\frac{1}{8}$ mm. thick could be placed at will on the spectrometer table at *g*, in the path of the rays, for the purpose of wave-length determination. To obtain a sufficiently sensitive bolometer, it was necessary to work with a galvanometer of the highest efficiency. As such the "Panzer Galvanometer," recently designed by Dr. du Bois and one of the present writers, gave most satisfactory results, and kept a very constant zero point, so that in spite of the magnetic disturbances occasioned by the Berlin street traffic, it was possible to recognize with certainty temperature differences of 0°.00001 of the exposed bolometer. The slit width chosen for experiments — equal to that of the bolometer — was 3.6 mm., which corresponded to an angular width of 0° 20′ on the spectrometer circle. The bolometer first tried consisted of thirteen strips of iron 0.25 mm. wide by 0.005 mm. thick, which, because of its large resistance and high temperature coefficient, had proved very sensitive in previous work. This instrument was replaced later by a platinum bolometer; for it was discovered that the layer of lampblack with which it was coated did not absorb more than about 5 per cent of the rays obtained from fluorite. The new instrument — supplied us through the kindness of President Kohlrausch — consisted of five strips of platinum $\frac{1}{2}$ mm. wide by 0.001 mm. thick, and was constructed in accordance with the design of Lummer and Kurlbaum.[1] The absorbing layer was in this case a coating of platinum black, deposited electrolytically. This instrument, although less sensitive for waves of shorter wave length, gave deflections twice as large as the earlier one for the long rays resulting from three reflections on fluorite. It thus appears that platinum black possesses an absorption covering a much wider region than lampblack, which fact has been shown in a different way by the experiments of Lummer and Kurlbaum.

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[1] E. F. Nichols, PHYSICAL REVIEW, Vol. IV. p. 297; Berichte d. Berl. Acad., Nov. 5, 1896.

[2] From "advantageously" it is not to be inferred that the greater part of the rays emitted by the source chosen shall correspond to the wave length of the absorption band, but rather that the emission of these rays in proportion to others shall be greater than from an ideal black body heated to the same temperature. In the case of glowing fluorite powder, for example, the wave lengths emitted which correspond to that region of absorption showing metallic reflection form only the smallest portion of the total energy. Still the ratio is larger than would be gained by using another powder in its place.

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[1] O. Lummer and F. Kurlbaum, Wied. Ann., 46, 204 (1892).

By the foregoing method, the two substances quartz and fluorite have already been studied. For both of these substances the constants in the Ketteler-Helmholz equation had been computed from direct observations of refractive indices, pushed as far into the infra-red as the absorption in each case permitted. From Paschen's[1] computation for fluorite, the middle of the infra-red absorption band was predicted at wave length 0.03 mm. Similar computation, by one of the present writers,[2] placed the band in quartz at wave length 0.01 mm. The peculiar relation of these constants to the remaining quantities which enter into the equation renders the correct placing of such a band somewhat uncertain; for, although its existence influences the dispersion in the observed region markedly, its exact position plays a comparatively minor rôle. From the theory, therefore, after three reflections from quartz, or from fluorite, only rays in the neighborhood of either 0.01 mm. or 0.03 mm. wave length should remain, and the

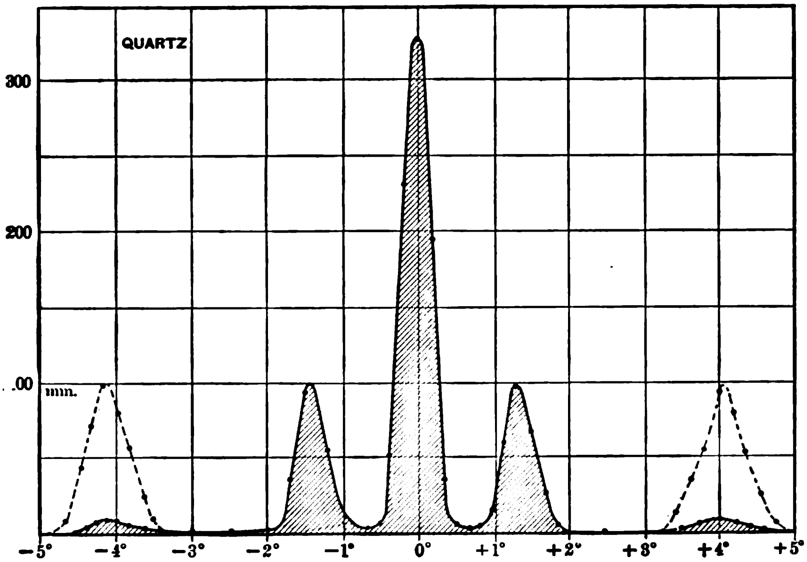

Fig. 2.

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[1] F. Paschen, Wied. Ann., 53, 812 (1894).

[2] H. Rubens, *l.c.*

experiments have proved, in the main, the correctness of the reasoning.

In Figs. 2 and 3, two series of observations, one on fluorite and the other on quartz, are shown graphically. The ordinates give the galvanometer deflections, and the abscissæ the corresponding readings of the circle of the spectrometer, which characterize the position of the bolometer in the spectrum.[1] In both figures, the central image stands out plainly, and symmetrically on either hand the diffraction images of the first order. In the case of quartz, the images of the third order are also easily recognized.[2] The grating constant, which was unusually large, was optically determined by the position of the diffraction images of the sodium line out to the image of the twenty-third order, which gave a constant 0.37165 mm. Measured on the dividing engine, the constant was 0.37167 mm.; 1° deviation, therefore, indicated a wave length of 0.0065 mm. approximately.

Experiment with Quartz.

The three reflecting surfaces in this case were from two prisms and a plate, all cut perpendicular to the optic axis of the crystal. The deviation of the maximum point of the diffraction image of the first order from the middle of the central image was 1° 22′, and that of the third order 4° 6′. The mean wave length of the observed rays thus computed gave in the one case 0.00887 mm., in the other 0.00882 mm. The agreement between the two values lies wholly within the probable error, and furnishes, at the same time, a valuable check upon the accuracy of the grating to a deviation of 4°.

In consequence of the wide slit and the equally wide bolometer used, it is impossible to draw conclusions concerning the distribution of energy in the diffraction image itself; but the fact that the image of the third order appears little or no wider than that of the first, shows that the rays must be fairly homogeneous, and can scarcely comprise a wave-length difference greater than 10 per cent. Using a bit of platinum foil coated with iron oxide at a temperature of about 1000° as a source, it was easy with the arrangement of apparatus described to effect a heating of the bolometer of 0°.02, which corresponded to a deflection of about 2 m. The properties of rays of equal wave length have long been known, as it is possible to obtain them in measurable intensity in the spectrum of a rock-salt, or even fluorite, prism.

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[1] In Fig. 3, which gives graphically the series of observations made with fluorite, the scale of ordinates chosen is ten times as great as that in Fig. 2, which shows a similar series with quartz.

[2] In order to show more plainly the position of the maxima in the spectra of the third order, the ordinates corresponding to the galvanometer deflections were multiplied by ten, which gave rise to the dotted-line curves. Because of the peculiar construction of the grating, the spectra of the 2d, 4th, 6th, ... orders are wanting.

Experiment with Fluorite.

The rays emitted by an incandescent platinum foil coated with fluorite powder after three reflections from fluorite retain scarcely more than 0.001 of their initial energy.

This remnant, when undispersed, was capable of raising the bolometer temperature about $\frac{1}{800}$° (120 mm. deflection), and the greatest patience and care were necessary, with this small quantity of energy, to establish the exact position of the maxima in the images of the first order lying on either side of the center.

In Fig. 3, which represents one of the several series of observations for this purpose, it is seen that the maximum energy in the diffraction spectrum of the first order lies 3° 45′ from the middle of the central image, which corresponds to a wave length of 0.0244 mm. The mean wave length resulting from other series varied from 0.024 mm. to 0.025 mm.

Heat rays of $\frac{1}{40}$ mm. wave length show in many respects quite different properties from those previously studied, which lie much nearer the visible spectrum. They are, for example, completely absorbed by a fluorite plate 2 mm. thick. A rock-salt plate 5 mm. thick lets through only about 3 per cent and an equally thick silvite plate about 5 per cent. In fact, after considerable searching we have not yet succeeded in finding a substance as diathermous for these rays as fluorite is for infra-red rays to a wave length of 0.007 mm.

The least absorbent substance so far discovered is vitreous chloride of silver in the form of thin plates, which Schultze-Sellack,[1] in the year 1870, showed to be more and more diathermous the greater the wave length of the incident rays. A plate of chloride of silver 0.5 mm. transmitted about 70 per cent of the incident rays, while a plate 2½ mm. thick absorbed 75 per cent of the remaining energy.

To study the optical behavior of lampblack in this region, a plate of chloride of silver, the absorption of which had been earlier determined, was blackened in the smoke of a candle until it was completely opaque to the sun's rays, and the absorption again measured, with the surprising result that the increase did not exceed 5 per cent, which shows that lampblack cannot be considered a black body for these rays.

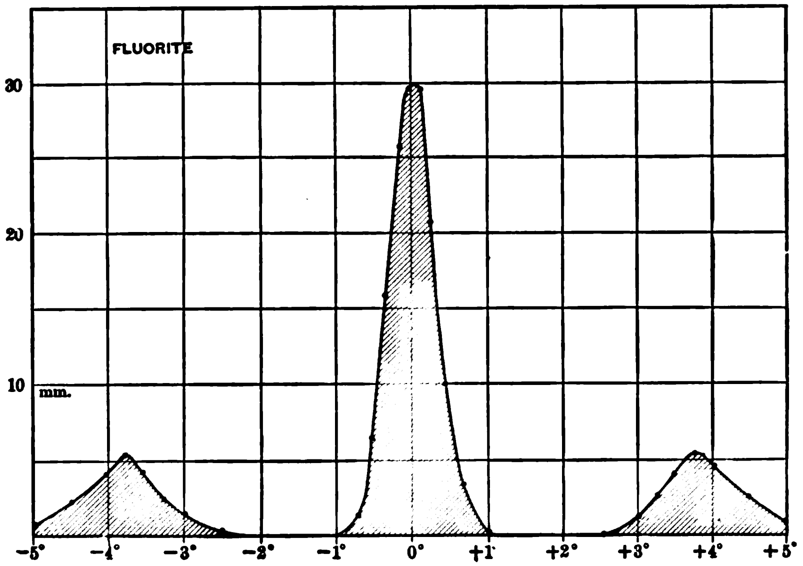

Fig. 3.

It appeared, further, of interest to find out whether these waves were destructively absorbed by water vapor and carbon dioxide gas, as is the case with shorter heat waves. To this purpose a blast of carbon dioxide gas and dry air were alternately blown into the collimator (Fig. 1, *f*) of the spectrometer, but no corresponding changes were perceived in the intensity of the rays. The same experiment was tried with water vapor with like result. If either absorb in this region the action must be very weak.

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[1] C. Schultze-Sellack, Pogg. Ann., 139, 182 (1870).

If the numbers obtained from the foregoing experiments for the mean wave length of the rays resulting from three reflections on quartz, and on fluorite, be compared with the values computed from the Ketteler-Helmholtz dispersion formula for the middle of the absorption bands in each case, the observed value for quartz is 10 per cent less than the computed, and for fluorite 20 per cent. This may perhaps be accounted for by the inaccuracies unavoidable in the computed values, but there may be errors affecting the experimental values as well. For example, nothing is so far known of the relation between the absorption of the platinum black, with which the bolometer was coated, and the wave length. In case the absorption decreases as the wave length increases, which is by no means unlikely, the same quantities of energy would give rise to smaller and smaller deflections of the galvanometer as the wave length increased. This would obviously cause a displacement of the true maximum toward shorter wave lengths. Further, the curve which represents the distribution of energy of a black body when heated first rises as the wave length increases, reaches a maximum, and then sinks asymptotic to zero. The maximum of the energy curve of the source used lies certainly at a wave length much shorter than those here considered; consequently within the absorption band the longer waves are again at a disadvantage compared with the shorter ones.

In any case, one is justified in regarding the agreement between the observed and computed wave lengths as close enough to confirm the utility of the theories involved.

In addition to the experiments with quartz and fluorite, similar trials were made with rock salt and sylvite, which, according to the course of their dispersions in the region where it has been studied, should, after multiple reflection, give rays very considerably longer than those here recorded; but we have not yet succeeded in getting sufficiently faultless surfaces of these substances to make the experiments successful. We still hope, however, to overcome these difficulties, and to refine the present method until we are in position to study waves of much greater wave length; and, further, to obtain a much higher degree of sensitiveness by substituting the improved radiometer[1] for the present bolometer.

Before closing, it will be of interest to call attention to the fact that, in respect to wave lengths, the rays corresponding to the infra-red absorption band in fluorite lie almost exactly midway between the shortest ultra-violet rays observed by Schumann[2] ($\lambda = 0.0001$ mm.) and the 6 mm. electrical waves of Lebedew, when the interval is reckoned according to octaves, as is customary in acoustics. On the other hand, if the spectrum be laid out according to uniformly increasing wave lengths, the space between the longest heat waves and the shortest electrical waves is 240 times greater than the length of the whole ultra-violet, visible, and infra-red spectrum taken together, and if reckoned according to the uniformly increasing number of oscillations is 250 times smaller than the combined ultra-violet, visible, and infra-red spectrum.

BERLIN: PHYSICAL INSTITUTE OF THE UNIVERSITY.

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[1] E. F. Nichols, *l.c.*

[2] V. Schumann, Wiener Acad. Ber., 102, 415, and 625 (1893).

MINOR CONTRIBUTIONS.

ON THE FORMATION OF LEAD SULPHATE IN ALTERNATING CURRENT ELECTROLYSIS WITH LEAD ELECTRODES.

BY SAMUEL SHELDON AND MARCUS B. WATERMAN.

IN a paper on the Capacity of Electrolytic Condensers[1] it was shown that condensers formed by submerging the ends of platinum wires in dilute sulphuric acid were of very large capacity. These were, however, not adapted to commercial ends, because the capacity was not constant with varying voltages, and especially because the efficiency was very low. Efforts to determine whether electrolytic condensers, formed with lead electrodes, might not have a much higher efficiency, acting much in the same manner as the lead plates of a storage battery, yielded the results imparted in this paper.

Condensers were made with electrodes of lead wire, 0.5 mm. in diameter, passing through glass tubes and imbedded, at the ends, in a cement formed of venice turpentine and shellac. Only the polished ends of the wires were in contact with the electrolyte (dilute H_2SO_4). Endeavors to determine the capacity of these condensers, by the method of reversals outlined in the above-mentioned paper, yielded nothing but non-concordant results. The cause of these discrepancies was suggested upon introducing the condensers into an alternating current circuit in a vain endeavor to determine whether the phase was shifted by their introduction. The immediate result, upon passing the current, was the dissolution of the electrodes and the formation of insoluble lead sulphate. The latter, after a long time, settling to the bottom of the electrolytic cell.

The results of an investigation concerning the causes which influence the dissolution of lead electrodes in H_2SO_4, when traversed by alternating currents, are given in the tables which follow.

There are three main causes which affect the quantity of lead dissolved per coulomb of electricity passing through the circuit. They are the current density at the electrodes, the frequency of the alternations of the current, and the temperature of the electrolyte.

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[1] PHYSICAL REVIEW, Vol. II., No. 12.

The Influence of Current Density.

Three pairs of electrodes, made of hollow cylinders of polished lead of about 0.8 cm. diameter, were submerged in a 5 per cent solution of H_2SO_4 to such a depth that the individual electrodes of the pairs exposed areas of 4, 8, and 12 square centimeters respectively to the action of the electrolyte. The interiors of the cylinders were filled with a cement not affected by the acid. Current from a small Westinghouse alternator was sent through these cells connected in series, and its magnitude was measured by an electrodynamometer. The time during which the circuit was closed was determined by means of a stop watch. Each pair of electrodes was weighed before and after each run. It was desirable that the run should be moderately long in order that the loss in weight of the electrodes might be determined with sufficient accuracy. On the other hand, it was difficult to maintain the temperature of the electrodes constant during a long run, owing to the heat generated by the current. The durations of the runs in each case were so chosen as to yield the best results. In calculating the number of coulombs which had passed through the cell in a given time, the assumption has been made that the shape of the current curve given by the alternator was sinusoidal. This assumption does not involve a very large error, as was proved by comparison of the curve, yielded by the alternator and obtained by the wiping contact method, with a sine curve of equal maximum value. The machine gave four complete alternations per revolution. The maximum positive and negative values of the current were, in arbitrary units, $+93$, -86.5, $+86$, -91, $+91.5$, -93, $+93$, -93. Plotting a curve of current i as a function of the time t, the area $\int idt$, inclosed between this curve and the time axis, represents the quantity of electricity passed through the circuit. During one revolution these areas, as determined by a planimeter and expressed in arbitrary units, were $+211$, -205, $+202$, -214, $+214$, -217, $+220$, -210. The area contained between a sine curve and the same portion of the time axis intercepted by the current curve, the maximum ordinate of the sine curve being 90.9, *i.e.* an average of the maximum current values, was 213. This is practically the same as the average of the current areas. It thus appears that no very great error is introduced, if the coulombs passing through the circuit in t seconds be taken as the product of the $\sqrt{\text{mean}^2}$ current, as obtained from the electrodynamometer, and $\frac{2t}{0.707\pi}$. The results given in Table I. are graphically represented by Fig. 1.

TABLE I.

Temperature.	Weight of lead dissolved. Milligrams.	Duration of run. Seconds.	Surface of electrode. Sq. centimeters.	$\sqrt{\text{mean}^2}$ current. Amperes.	$\int idt$. Coulombs.	At 18° milligrams of lead dissolved per coulomb.	Current density. Amp. per cm.^{2}
16.98	33.3	600	12.0	0.452	271.0	0.124	0.0380
18.00	15.6	180	12.0	0.543	97.6	0.160	0.0452
21.30	26.8	180	12.0	0.723	130.0	0.214	0.0600
18.00	27.2	180	6.5	0.543	97.6	0.279	0.0835
21.37	37.7	180	8.0	0.723	130.0	0.302	0.0900
17.00	47.6	120	12.0	1.127	135.2	0.348	0.0940
17.17	104.6	600	4.0	0.452	271.0	0.382	0.1130
22.37	96.3	118	12.0	1.710	202.0	0.503	0.1420
17.30	75.5	120	6.5	1.127	135.2	0.553	0.1730
20.84	94.9	120	8.0	1.422	171.0	0.575	0.1780
20.97	126.1	300	4.0	0.755	226.0	0.579	0.1890
22.50	123.7	118	8.0	1.710	202.0	0.646	0.2140
20.65	122.2	180	4.0	1.020	183.7	0.690	0.2550
20.17	133.0	120	4.0	1.422	171.0	0.799	0.3560
23.49	157.8	118	4.0	1.710	202.0	0.834	0.4280
17.24	206.4	60	4.0	3.810	228.6	0.895	0.9520
16.77	220.0	60	4.0	4.037	242.2	0.895	1.0100

The Influence of Frequency.

In investigating the influence of the frequency of alternations upon the quantity of lead dissolved per coulomb, two methods of altering the frequency were employed. By the first, the speed of the motor which drove the alternator was varied so as to give from 13 to 145 complete alternations per second. At the lower speeds the electromotive force of the generator was small. Accordingly, to obtain a uniform current density throughout the range of frequencies, a small current was employed. For alternations between 0 and 20 per second a direct current was employed, and its direction was commutated by a slow speed revolving commutator of two sections, which was driven by a small motor. The current was measured by a Weston ammeter. At these low frequencies the rate of formation of the lead sulphate is very sensitive to small changes in the frequency, and there seems to be an unstable chemical condition. For instance, during one run with three alternations per second, after about thirty seconds the electrodes took on the dark brown color of a storage battery positive, and after that no sulphate appeared to be formed. With two alternations per second the attack on the electrodes ceased after twenty seconds. At these low frequencies, therefore, the runs were of necessity of very short duration, and the loss in weight of the electrodes was very small. No great amount of accuracy can be claimed for the results. They are, however, interesting as indicating, in a general way, the relation which exists between the rate of loss and the frequency. As it was impossible to obtain exactly the same current density in the various observations, the differences must be allowed for, if the different observations are to be connected. No great error will be introduced by assuming that the milligrams of lead dissolved per coulomb are directly proportioned to the current densities.

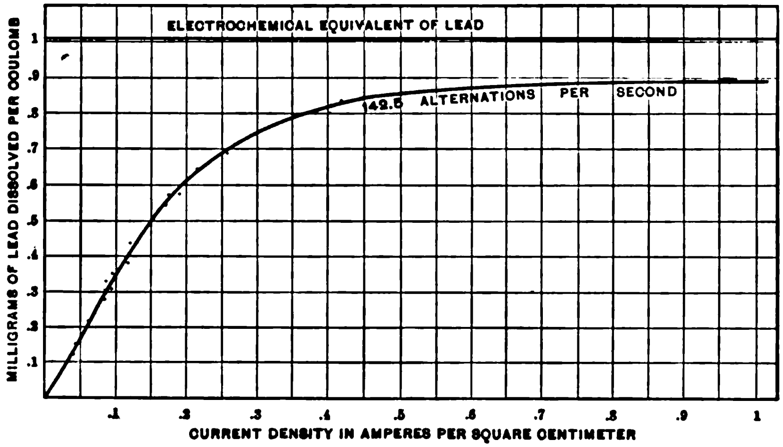

Fig. 1.

The results obtained are given in Table II., and are graphically represented in Fig. 2.

TABLE II.

Milligrams of lead dissolved per coulomb at 18°.	Number of alternations per second.	Current density in amperes per $cm.^2$	Milligrams of lead dissolved per coulomb at current density = 0.22.
0.202	2.0	0.209	0.212
0.334	3.0	0.221	0.333
0.523	4.0	0.222	0.520
0.618	5.0	0.220	0.618
0.714	5.5	0.223	0.705
0.756	10.1	0.226	0.735
0.800	20.1	0.226	0.779
0.785	84.2	0.246	0.703
0.646	142.5	0.224	0.635

The Influence of Temperature.

In determining the effect of temperature upon the amount of lead dissolved per coulomb of electricity, the commutated direct current was employed. The speed of the commutator was regulated to give such a number of alternations per second that small variations in frequency would have but little influence on the weight of lead dissolved per coulomb. A pair of electrodes, each exposing four square centimeters of surface to the action of the electrolyte, was placed in a beaker of 5 per cent H_2SO_4 at any desired temperature. Current at an electrode density of 0.215 amperes per square centimeter was passed through the cell for thirty seconds. The temperature during this interval remained practically constant. Maintaining, in this manner, the current density constant, the milligrams of lead dissolved per coulomb of electricity passed through the circuit was found to be 0.794, 0.744, and 0.683 for temperatures 17.134, 22.330, and 28.125° C., respectively. These results yield a temperature coefficient of —0.0125, *i.e.* the current density remaining constant, the milligrams of lead dissolved per coulomb of electricity passed decreases 1.25 per cent of the amount dissolved at 18° C. for each degree rise of temperature above 18°.

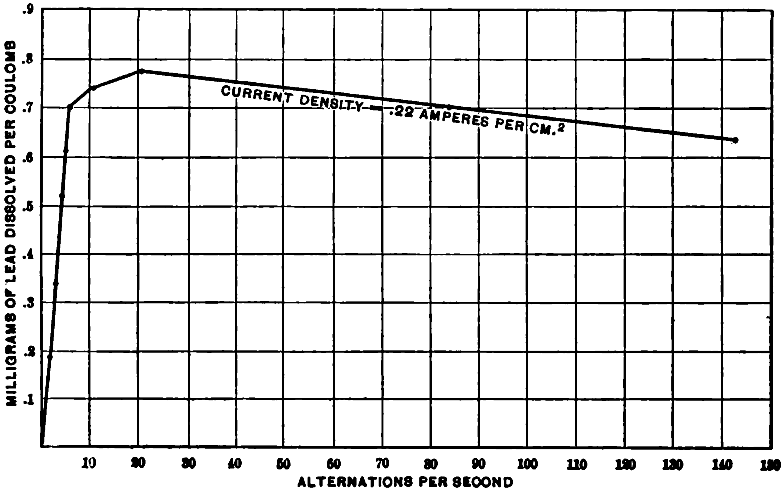

Fig. 2.

Conclusion.

The authors have no theory to offer as explanatory of their results. The facts brought out are unquestionably associated, in some manner, with the velocities or rates at which the involved chemical processes take place. The experiments on very low frequencies have developed the fact that PbO_2 is not soluble in H_2SO_4, or acted upon by the products of the electrolytic decomposition of H_2SO_4. The fact that $PbSO_4$ is formed with the alternating currents would seem to indicate that a lower oxide than the peroxide was first formed in quantity, and that this was afterwards acted upon by the electrolyte to form $PbSO_4$. If this be so, the velocity of formation of the lower oxide must be greater than the velocity of oxidation to PbO_2.

POLYTECHNIC INSTITUTE OF BROOKLYN, June, 1896.

POLARIZATION AND INTERNAL RESISTANCE OF A GALVANIC CELL.

By B. E. MOORE AND H. V. CARPENTER.

THE electromotive force of many cells decreases very rapidly upon closed circuit, and after a period of rest it nearly recovers its original value. In some cells this is so marked that they are called "open circuit" cells. This is rather a conventional use. It means only that the cells cannot be used on closed circuit any considerable length of time. Carhart, in his *Primary Batteries*, gives curves for change in electromotive force, current, and internal resistance of a great many such cells. The cells are run for a certain period, and then allowed to recover for the same length of time. The total electromotive force, E, and fall of potential, E', over a known external resistance, R, are measured. The current can be calculated from the equation $C = \frac{E'}{R}$, and the internal resistance, r, is determined from the equation $r = \frac{E - E'}{E'} \cdot R$.

While this method reveals the fact and magnitude of polarization, it by no means localizes it. That is, it does not tell us how much polarization occurs at each electrode. In cells of the Leclancé type it has generally been assumed that the polarization occurs at the carbon electrode, and that the potential difference between zinc and solution is constant. The fall in electromotive force on closed circuit, at first rapid, has generally been attributed to the formation of gas liberated from the positive ion at the carbon electrode. The later decrease would therefore be slower, owing to the slow absorption of gas at the carbon. Depolarizers are therefore frequently added to minimize the formation of gaseous products. On the other hand, when the cell is thrown on open circuit, it was reasoned that the gas actually in contact with the electrode would be rapidly given off, and thus occasion a rapid rise in electromotive force. However, the gas somewhat removed from the electrode surface would hinder the electromotive force from regaining its original value for some time. The following short experiment was carried out with the view of substantiating or of refuting this explanation.

A diamond carbon cell was taken. This is essentially a Leclancé cell with no depolarizer. Reliance is had upon the soft carbons to absorb the gas and communicate it ultimately to the air. The carbon electrode consisted of seven pencils, each 9 mm. in diameter, arranged in a circle 75 mm. in external diameter. The zinc electrode, 6 mm. in diameter, is located symmetrically as to the carbons. For the purpose of this experiment the zinc electrode was displaced 4.5 mm. from the center, and diametrically opposite it at the same distance from the center was placed a special carbon electrode of exactly the same electrode surface. The end of this carbon protruding from the solution was copper plated, and a wire was soldered to it for making connections. A fresh solution of chemically pure ammonium chloride, containing five ounces to the quart, constituted the electrolyte.

The object of the experiment was to measure the electromotive force and its changes upon closed and open circuit between the zinc and main carbons, to get the electromotive force between the zinc and special carbon at intervals, and also to obtain the electromotive force of the carbon series during the progress of the experiment. Such a procedure would evidently enable one to determine the amount of polarization at the electrodes of the main circuit, provided the tension of the special carbon remained constant. Taking off appreciable currents through the special carbon as a circuit is manifestly excluded. It became necessary, therefore, to use a condenser. Even the charging of the microfarad condenser at first disturbed the potential differences, and no carbon could be found which would show a zero electromotive force between the main carbons and special carbon. At last a strip of glass 2 mm. thick and 18 mm. wide was inserted between the zinc and special carbon, whose surface was only 3 mm. from the former at the nearest point. Repeated charging and discharging of the condenser now showed no change in the potential differences in different parts of the circuit. The special carbon ultimately used was taken from another similar cell and turned down to proper size. Its electromotive force in respect to the main carbons was -0.261 volts.

Figure 1 shows the arrangement of the connections. The zinc and special carbons are designated by a and b, respectively. The keys K_1, K_2, K_3, and K_4 are turned over by a pendulum swinging from left to right.[1] The time required for the pendulum to pass from K_1 to K_4 was about

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[1] Carhart, PHYSICAL REVIEW, Vol. II., p. 392.

0.1 second. Keys *c*, *d*, and *e* make contact at the upper points unless depressed. By setting keys K_2 and K_3 only, and releasing the pendulum from its detent, we obtain the electromotive force of the main circuit, if *c*, *d*, and *e* are not disturbed. Had *c* been depressed, it would have given us the potential between the carbon electrodes, or had *e* instead of *c* been depressed, we should have obtained the total electromotive force of the zinc and special carbon circuit. It is evident that when keys K_1, K_2, K_3, K_4 are set, and keys *c* and *d* are depressed, we can measure the fall of potential over R_1. Under these conditions the special carbons and main carbons compose the battery. We can also measure the fall of potential over R, when either of the other two circuits is closed, by setting K_1, K_2, K_3, and K_4. R and R_1 were resistances of 1 ohm and 10 ohms respectively.

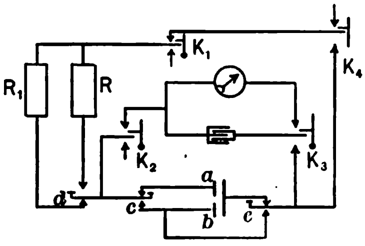

Fig. 1.

At the outset, repeated observations were taken upon the various circuits closed through the resistance R and R_1 for the above short-time period, 0.1 of a second. The ballistic galvanometer gave constant deflections in every case. Had there been a change of 0.5 mm. in the deflection, it could have been detected. To make a run upon the battery, the key K_1 was left in contact with the upper point, and key K_4 was set. In getting the total electromotive force of any circuit, it now becomes necessary to drop the detent to K_4 just before the liberated pendulum reaches key K_2. Key K_4 must promptly be set again. With a little practice the time required for this operation need not exceed two seconds. In measuring the fall of potential over R and R_1, open circuit was entirely avoided by stopping the pendulum with the hand before reaching key K_4. Depressing the keys *c* and *d* or *e* also opens the circuit, but the time is here also small, as they need only be depressed while the pendulum is making one swing from left to right. This only required one-half second. The time the battery was upon open circuit, during the run of thirty minutes, was therefore very small. Upon recovery key K_4 was set only when terminal potential was to be determined. Key K_1 was then also set, and the battery would therefore only be closed for 0.1 of a second.

In the table of results E, E_s, and E_c represent total electromotive forces upon the main circuit, the zinc and special carbon circuit, and the main carbon and special carbon circuit, respectively. E', E_s', and E_c' represent the corresponding terminal potentials. The latter is measured over R_1, which it became necessary to use in order to obtain appreciable readings for E_c'. We will designate the internal resistances of circuits giving E, E_s, and E_c by r, r_s, and r_c, respectively. The resistance, r, began at 0.12 ohms and ended at 0.09. The resistance, r_s, began at 1.27 ohms, reached the value of 1.7 ohms in 24 minutes, and ultimately fell to 0.78 ohms. The variation in r_c was more marked. At first it was 7.3 ohms. The next reading gave 88 ohms. During the recovery readings this resistance was of the same magnitude as at first. We attributed this great variation to some experi-

Table I.

POLARIZATION.

Standard Clarke gave a deflection on the galvanometer = 80 = 1.434 volts.

Time.		E	E'	E_c	E_c'	E_s	E_s'
min.	sec.						
0	00	1.368	1.020	−0.261	−0.144	1.614	0.738
0	15	—	1.017	—	—	—	—
1	00	1.260	—	—	—	—	—
1	45	—	1.013	—	—	—	—
2	45	—	1.011	—	—	—	—
3	30	—	—	0.395	—	—	—
4	15	1.206	—	—	—	—	—
5	30	—	1.010	—	—	—	—
6	30	—	—	—	0.045	—	—
7	30	1.161	—	—	—	—	—
8	00	—	1.004	—	—	—	—
8	30	—	—	—	0.054	—	—
9	30	—	—	0.450	—	—	—
10	00	—	1.004	—	—	—	—
11	00	—	—	—	—	1.647	—
11	45	—	—	—	—	—	0.558
14	30	—	0.990	—	—	—	—
15	30	1.062	—	—	—	—	—
17	00	—	—	—	0.101	—	—
19	00	—	0.962	—	—	—	—
19	30	1.071	—	—	—	—	—
21	00	—	—	—	—	1.623	0.603
23	45	—	—	—	—	—	—
27	00	—	—	—	0.102	—	—
28	00	—	—	0.531	—	—	—
29	00	—	—	—	—	1.602	—
30	00	1.008	—	—	—	—	—
31	00	—	0.927	—	—	—	—

mental error which had escaped our notice. It is apparent that the internal resistances are of the relative magnitudes 1, 10, and 80. The one to ten ratio could be attributed to the decreased carbon electrode surface and the insertion of the glass plate. This does not help us in the one to eighty ratio. Now Carhart has shown[1] that the internal resistance varies with the

TABLE II.

RECOVERY.

Time.		E	E'	E_c	E_c'	E_s	E_s'
min.	sec.						
0	30	—	0.972	—	—	—	—
1	00	1.062	—	—	—	—	—
1	15	—	—	—	—	1.656	—
1	30	—	—	−0.522	—	—	—
2	00	—	—	—	−0.279	—	—
2	30	1.098	—	—	—	—	—
3	30	—	1.008	—	—	—	—
4	30	—	—	—	—	1.674	—
5	30	—	—	—	—	—	0.864
7	00	—	—	0.495	—	—	—
7	30	—	—	—	0.279	—	—
9	30	1.152	—	—	—	—	—
11	00	—	1.053	—	—	—	—
13	00	—	—	0.468	—	—	—
15	00	—	—	—	0.270	—	—
16	00	1.170	—	—	—	—	—
16	45	—	—	—	—	1.680	—
17	30	—	—	—	—	—	0.900
26	30	—	1.107	—	—	—	—
27	00	—	—	—	—	1.680	—
27	30	—	—	—	—	—	0.918
28	00	—	—	0.414	—	—	—
28	30	—	—	—	0.243	—	—
29	00	1.224	—	—	—	—	—
29	30	—	—	0.396	—	—	—
30	00	—	1.109	—	—	—	—
77	00	1.242	—	—	—	—	—
77	30	—	1.152	—	—	—	—
78	00	—	—	0.360	—	—	—
78	30	—	—	—	0.243	—	—
79	00	—	—	—	—	1.680	—
79	30	—	—	—	—	—	0.954

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[1] See previous reference.

current. An introduction of 100 ohms in the main circuit would reduce the current to the same magnitude as the current in the E_c' circuit. To obtain appreciable differences in the readings for E and E' with this resistance in circuit, the connections were changed. The galvanometer was placed in series with 30,000 ohms and the battery. This gave a deflection of 366 scale divisions. When the battery was shunted with 100 ohms, the deflection was 362. This gives an internal resistance of 1.1 ohms, and the ratio is now about $\frac{1}{7}$ to $\frac{1}{7.5}$. With the galvanometer connected in this manner, a readable deflection could be obtained when the E_c series was shunted with one ohm. This gave a deflection of ten scale divisions, but it took the series four hours to regain the value first obtained when the battery was upon open circuit, viz., 73 scale divisions. This gives an internal resistance of 6.3 ohms, — a ratio of 1 to 5.5. The current is now, however, three times that in the main circuit with 100 ohms. This difference must be an electrode or contact of electrode and solution phenomena, as the two circuits giving E and E_c are in every respect symmetrical. Streintz[1] has succeeded in measuring the resistance of a cell by means of an alternating current, and finds that the kathode resistance alone shows a dependence upon the current strength, while the anode resistance scarcely varies with the same. As the special carbon is twice kathode in this experiment, the kathode resistance does not help us out. When the main

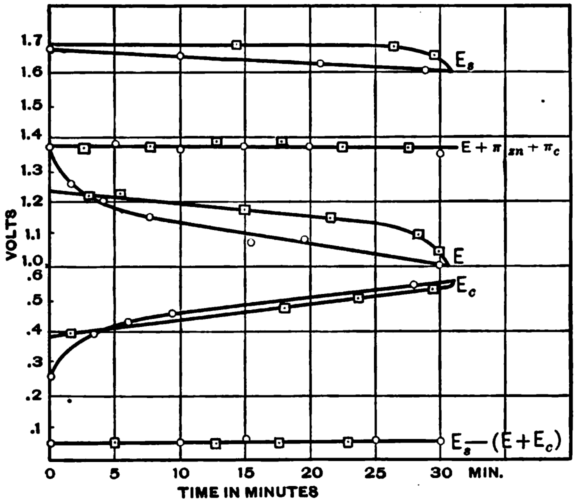

Fig. 2.

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[1] Wied. Annal., Vol. 57, p. 711, 1896.

carbons are kathode, the internal resistance is smallest, and one would scarcely think that on becoming anode the resistance would be so large. We cannot attribute the difference to polarization, as the difference is marked at the outset when no polarization has occurred.

The chief merit of this method is the means it gives us of determining single potential changes. The curves for E, $E_{,,}$ and E_c (see Fig. 2) show very decidedly what is taking place. The observations for the polarization are given in the curves by small circles, and the observations for recovery are shown by small squares. It is apparent that the rapid fall of electromotive force at the commencement of a run is to be attributed to polarization at the carbon electrode. Furthermore, the polarization at the zinc electrode cannot be left out of account. The curve shows that the zinc polarization increases uniformly. When the cell is thrown upon open circuit the phenomena are almost exactly reversed. In the first minute of recovery, 18 per cent of the entire loss in electromotive force is regained, and 85 per cent of this gain occurs at the zinc electrode. In five minutes recovery the polarization at the zinc electrode had vanished. The recovery of total electromotive force for the rest of the thirty minutes is due to recovery from polarization at the main carbon electrode. The recovery at the carbon electrode is nearly uniform for thirty minutes. However, the observations are not so decisive as the observations upon the increase of polarization at the zinc electrode. The sum of the observed electromotive force and of the polarization at the two electrodes should be constant and equal to the original electromotive force. We have designated the polarization at the zinc and carbon electrodes by π_{zn} and π_c, respectively, and have drawn a curve showing $E + \pi_{zn} + \pi_c$. This curve is as near a straight line as could be expected. As is apparent, E_s should equal E plus E_c. The curve designated $E_s - (E + E_e)$ shows, however, a constant value equal to 0.054 volts.

Upon the theory of Nernst, we can understand the phenomena here noted. The electromotive force at the zinc electrode is expressed by $\frac{RT}{2c} \log_e \frac{P}{p}$, where R, T, c, P, and p represent the gas constant, absolute temperature, current required to give the one gram-equivalent of the metals, solution pressure, and osmotic pressure of the zinc ions, respectively. Before a current is generated the quantity p is very small. During the action of the cell, the pressure p of the zinc ions would increase at the zinc electrode, and therefore the electromotive force would fall. The ions would of course be more concentrated in the strata nearest to the zinc electrode. The ion first passing into solution would, in a period of thirty minutes, be less than 2 mm. from the electrode. When the cell is thrown upon open circuit, diffusion of the zinc ions occurs. From the formula it is apparent that the recovery curve at the zinc electrode should be exponential in character. The observations show that this is probably the case, but the readings of the galvanometer are neither large enough nor frequent enough to demonstrate this character beyond a doubt. At the carbon electrode the gases liberated form a gas battery in opposition to the main circuit. The back electromotive force would increase rather rapidly at first, and later more uniformly. Upon open circuit gas absorption would still continue, and the solution pressure of the gas slowly decrease. It has been shown that the solution pressure of a gas can be expressed by the Nernst formula, and it would scarcely be expected to be as uniform as here noted. However, after standing one hour upon open circuit, the polarization at the main carbon electrode is still apparent, and its magnitude shows that the recovery curve cannot be considered a straight line.

Conclusions.

1. Variation in kathode resistance with current strength does not account for the differences in internal resistance noted in this experiment.
2. Polarization occurs at both electrodes, but is more marked at the carbon electrode.
3. On closing the circuit the rapid fall of electromotive force is due principally to polarization at the carbon electrode.
4. The rapid rise in electromotive force, when a cell is first thrown upon open circuit, is largely due to recovery from polarization at the zinc electrode.

University of Illinois, July, 1896.

THE TRACE OF THE GYROSCOPIC PENDULUM.

By Ernest Merritt.

THE gyroscope, in its numerous more or less modified forms, has long been an object of interest to the student of physics. Its erratic movements when disturbed, and its seeming defiance to the laws of gravitation, cannot fail indeed to arouse the curiosity of all who observe it. For this reason, if for no other, the theory of its behavior deserves a clearer and more thorough treatment in works on mechanics than it usually receives.

The principles on which the action of the gyroscope depends find extensive application also in all branches of physics, both pure and applied. In practical engineering they are of importance in connection with high-speed machinery. In the theory of light and magnetism they have led to Kelvin's hypothesis of a gyroscopic ether, and to an explanation of the magnetic rotation of the plane of polarization. In pure mechanics the gyroscope may be made to give numerous excellent illustrations of general dynamical principles and methods.

Elementary explanations of the behavior of the gyroscope are usually unsatisfactory, either because they are lacking in clearness and completeness, or because they are unsound. Professor Perry's little volume on "Spinning Tops" forms, however, a notable exception to this general statement. The treatment there given, although quite elementary, is so clear and direct, and so suggestive, that it may well be recommended even to advanced students. Much can be accomplished by the straightforward, physical methods adopted by Professor Perry, even without the use of a single formula or mathematical expression; and I think all will agree that such methods, when applicable, are to be preferred to a discussion in which the physical truths are largely disguised by their analytical clothing. For a complete and quantitative discussion of the theory of the gyroscope, recourse must however be had to the methods of analysis. Such a discussion is given by Routh, Thomson and Tait, and others. The only objection to the treatment given to the subject by these authors is that it is unnecessarily hard to read.

It has often been suggested that an advantageous method of explaining the precessional movements of a gyroscope is to consider the latter as a special case of the gyroscopic pendulum. In the ordinary form of gyroscope shown in Fig. 1, the supports serve merely to maintain the center of figure fixed in position. The body may be looked upon as free to move in any manner consistent with the condition that this point shall remain fixed. If everything is perfectly balanced, its center of gravity coincides with the fixed point, and there is no tendency for the axis of rotation to change. But if a weight is attached at the end of the shaft, the center of gravity is displaced, and the instrument becomes a pendulum, whose point of suspension is the center of figure, and whose length depends upon the magnitude of the attached weight. If now a motion of rotation is given to the fly wheel we have a gyroscopic pendulum, whose length in most cases is quite small; and, as soon as the weight is released, the motions of precession and oscillation peculiar to such instruments begin.

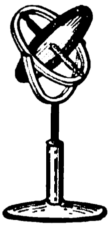
Fig. 1.

When the speed of rotation is high it is usually not possible to observe the oscillations of the axis, even when the attention is directed especially to this phenomenon. The theory of the motion shows in fact that these oscillations must be quite rapid as well as of extremely small amplitude. Another fact which makes the oscillations difficult to observe, even by indirect means, is the unavoidable presence of friction in the supports of the instrument. The frictional damping is in most cases sufficient to reduce the motion to a steady precession within a few seconds after the weight has been released. It is only by using a small speed of rotation, in order to make the gyroscopic action comparatively insignificant, that the true nature of the motion can be seen. But, with suitable conditions, it is possible to show not only the oscillatory character of the motion, but also the fact that the vibrations become more rapid, and die out more quickly, as the speed of the fly wheel is made greater.

If a gyroscopic pendulum of the form shown in Fig. 2 is used, the motion can be more readily observed and studied. The oscillations toward the vertical and back again are of considerable amplitude, while the period may be made so great that the vibrations can be followed by the eye. If such a pendulum is made adjustable in length, it is possible to observe the progressively increasing influence of the rotating fly wheel from the case of a long pendulum, whose behavior is scarcely influenced at all by the gyroscopic action, to the case of an extremely short pendulum, which differs only slightly in its action from the gyroscope shown in Fig. 1. While presenting the theory of the gyroscope to an advanced class during the past year, I constructed such a pendulum by merely attaching the rotating part of an ordinary gyroscope to the end of a brass rod, and suspending the latter by means of a universal joint (*P*, Fig. 2). This simple instrument was found to serve its purpose most satisfactorily. The gyroscopic action could be varied through a considerable range by altering the speed of the fly wheel, while still further variation was obtained by changing the length. The general characteristics of the motion, including the effect of various starting conditions, etc., could be studied by direct observation.

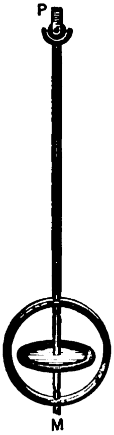

Fig. 2.

With an extremely short pendulum, however, and a high speed of the fly wheel, the motion became too rapid to be readily followed. To obtain a record of the curve described by the pendulum in such cases the photographic traces shown in the accompanying plate were made. I am led to present these results here by the fact that such traces are rarely found in the text-books and treatises.[1] The method of obtaining them was as follows: At the end (*M*) of the pendulum a bit of mirror, about 1 mm. square, was attached by means of wax so as to make an angle of 45° with the vertical. The pendulum was suspended in such a position that during the vibrations of the instrument this mirror should remain within the beam of parallel rays sent out by a large projecting lantern. A small pencil of light was thus reflected by the mirror in the direction of the axis of the gyroscope,[1] and left its trace upon a horizontal photographic plate immediately underneath. The work was of course done in a dark room, and screens were arranged so that light was received only by the mirror, and not by the gyroscope itself. Any fogging which may be observed in the photographs is due to an imperfect adjustment of these screens. In making the exposures the plate was protected by a piece of cardboard until the pendulum had been properly started. The cardboard was then removed, and replaced again after the pendulum had completed five or six oscillations. Whether or not a symmetrical figure would be obtained, such, for example, as that shown in Fig. 4, was largely a matter of chance.

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[1] I have been able to find only two such traces, viz., in Thomson and Tait, Vol. 1, Art. 74, and in Perry's "Spinning Tops," p. 57. In both cases, the trace is such as would be obtained when an initial negative precession is imparted. Compare Fig. 6.

Before discussing in detail the various traces shown in the plate, it may be well to refer briefly to the analytical discussion of the gyroscopic pendulum. Let the position of the pendulum be specified by the two angles θ and ϕ, θ being the angle between OY and PQ, while ϕ is the angle between the planes YOZ and OPQ. Let the three principal moments of inertia of the pendulum be B about the axis PQ, and A and C about axes perpendicular to PQ and to each other. Since the instrument is symmetrical about PQ, $A = C$.

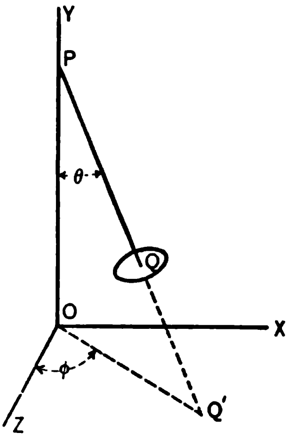

Fig. 3.

We may now apply the general principle that the rate of change of the moment of momentum about any fixed axis is equal to the moment, taken about the same axis, of the force producing this change. Neglecting frictional forces for the present, the only forces to consider are gravity, acting directly downward through the center of mass, and the constraining reactions at the point of support P. Neither of these forces has a moment about either PQ or OY. We conclude therefore (1) that the moment of momentum about OY will be constant, (2) that the angular velocity ω_2 about PQ will also remain unchanged.[2] The first conclusion may be put into mathematical form by making use of the fact that moments of momenta, as well as angular velocities, are vector quantities, which can be resolved and combined in the same manner as velocities, forces, etc. The component moments of momenta of the pendulum about the axes PA (in the plane OPQ and perpendicular to PQ), PQ, and PC (perpendicular to the plane OPQ) are readily seen to be: $A\dot{\phi}\sin\theta$, $B\omega_2$, and $A\dot{\theta}$ respectively.[1] The moment of momentum about OY is therefore

---

[1] Owing to the change in direction of PM, the reflected beam does not lie *exactly* in the prolongation of the axis. This fact, together with the changing plane of vibration, has led to the elongated, or elliptical, appearance of some of the traces, *e.g.* Figs. 4 and 9. Since only qualitative results were sought, this distortion seems of small importance.

[2] This conclusion is also dependent on the fact that the inertia axes A and C are equal.

$$M_Y = \text{const.} = A\dot{\phi}\sin^2\theta - B\omega_2\cos\theta. \qquad (1)$$

By expressing the fact that the rate of change of the angular momentum about PC (not shown in the figure) is equal to the moment of the weight of the pendulum about that axis, we obtain the equation

$$-mgr\sin\theta = A\ddot{\theta} - A\dot{\phi}^2\sin\theta\cos\theta - B\omega_2\dot{\phi}\sin\theta, \qquad (2)$$

where r represents the distance between P and the center of gravity.

The derivation of equation (2) by the method indicated is unfortunately complicated by the fact that the axis PC is not fixed. All the necessary equations in fact can be obtained more readily by treating the problem as a case for the application of Lagrange's equations and generalized coördinates.

If ω_1, ω_2, ω_3 represent the component angular velocities about the three principal axes, PA, PB, PC, the kinetic energy is

$$T = \tfrac{1}{2}(A\omega_1^2 + B\omega_2^2 + A\omega_3^2). \qquad (3)$$

Although the angular velocities ω_1, ω_2, ω_3 are sufficient to determine the value of T, they do not completely define the *position* of the body, and are therefore not suitable for use as generalized coördinates. Let θ, ϕ, and ψ be chosen as coördinates, θ and ϕ having the same significance as before, while ψ represents the angle between the plane OPQ and some plane fixed in the pendulum and passing through the axis PQ. We then have[2]

$$\left.\begin{aligned} \omega_1 &= -\dot{\phi}\sin\theta\cos\psi + \dot{\theta}\sin\psi \\ \omega_2 &= -\dot{\phi}\cos\theta - \dot{\psi} \\ \omega_3 &= \dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\psi \end{aligned}\right\}. \qquad (4)$$

---

[1] A dot placed above a symbol represents differentiation with respect to the time; *e.g.*

$$\dot{\theta} = \frac{d\theta}{dt}, \quad \ddot{\theta} = \frac{d^2\theta}{dt^2}.$$

Note also that a "right-handed" system of axes is used; *i.e.* positive rotation about any axis is right handed to an observer looking along the axis in the positive direction.

[2] "Euler's Geometrical Equations": Routh, Elementary Rigid Dynamics, p. 216.

Equation (3) now becomes

$$T = \tfrac{1}{2}A(\dot{\phi}^2 \sin^2\theta + \dot{\theta}^2) + \tfrac{1}{2}B(\dot{\phi}\cos\theta + \dot{\psi})^2, \tag{5}$$

while the potential energy U is

$$U = mgr(1 - \cos\theta). \tag{6}$$

The Lagrangian equations of motion may now be written in the form

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\phi}} - \frac{\partial T}{\partial \phi} + \frac{\partial U}{\partial \phi} = 0. \tag{7}$$

Substituting the values of T and U in this equation, and in the corresponding equations for θ and ψ, we obtain:

$$\frac{d}{dt}[A\dot{\phi}\sin^2\theta + B\cos\theta(\dot{\phi}\cos\theta + \dot{\psi})] = 0; \tag{8}$$

$$A\ddot{\theta} - A\dot{\phi}^2\sin\theta\cos\theta + B(\dot{\phi}\cos\theta + \dot{\psi})\sin\theta + mgr\sin\theta = 0; \tag{9}$$

$$\frac{d}{dt}B(\dot{\phi}\cos\theta + \dot{\psi}) = 0. \tag{10}$$

From (10), we see that

$$\dot{\phi}\cos\theta + \dot{\psi} = -\omega_2 = \text{const.}$$

Writing $-\omega_2$ in place of $\dot{\phi}\cos\theta + \dot{\psi}$ in (8) and (9), these equations become:

$$B\omega_2\cos\theta - A\dot{\phi}\sin^2\theta = \text{const.}; \tag{11}$$

$$A\ddot{\theta} - A\dot{\phi}^2\sin\theta\cos\theta - B\omega_2\dot{\phi}\sin\theta + mgr\sin\theta = 0; \tag{12}$$

which are the same as equations (1) and (2) above.

The solution of equations (11) and (12) is not easy, even if we confine ourselves to small oscillations about the vertical. But it is possible to determine the main characteristics of the motion without actually solving the equations.

I shall consider, first, the case of zero initial velocity; *i.e.* the pendulum is displaced from the vertical through the angle θ_0, and is then set free. We have, initially, $\dot{\theta} = 0 = \dot{\phi}$, and $\theta = \theta_0$. Equation (11) now becomes:

$$B\omega_2(\cos\theta - \cos\theta_0) - A\dot{\phi}\sin^2\theta = 0. \tag{13}$$

The initial value of $\ddot{\theta}$ is seen from (12) to be the same as it would be for an ordinary pendulum. The pendulum therefore *starts* to fall toward the vertical. But as soon as θ begins to differ from θ_0, $\dot{\phi}$ acquires a finite value, which will be positive or negative according to the sign of ω_2. Equation (13) shows that $\dot{\phi}$ and ω_2 always have the same sign; *i.e.* the direction of precession is opposite to the direction of rotation of the fly wheel. As the pendulum approaches the vertical, the velocity of precession ($\dot{\phi}$) becomes greater. We see, however, that the pendulum can never pass *through* the vertical, for (13) cannot be satisfied by $\theta = 0$ unless $\omega_2 = 0$. When θ has reached its minimum value it begins to increase again, and, in the absence of friction, will finally reach its original value θ_0. This fact is indicated by equation (12), which shows that, with decreasing θ and increasing $\dot{\phi}$, the acceleration $\ddot{\theta}$ passes through 0 and becomes negative, only to retrace its course when θ has passed its minimum

We may therefore conclude that the motion will consist of a series of oscillations toward the vertical, and back again, accompanied by a variable motion of precession. The greater the velocity of rotation of the fly wheel, the more rapidly will these oscillations take place, as shown by the presence of the term $B\omega_2\dot{\phi}\sin\theta$ in (12). The precessional velocity is zero at the beginning of each oscillation, and reaches a maximum value when θ is least.

The type of motion just considered is illustrated by the traces shown in Figs. 4 and 8, the former trace being taken with a long pendulum, while the latter was made with a pendulum as short as could conveniently be used. The increased influence of the gyroscopic action in the latter case, as indicated by the small amplitude of the vibrations and the short distance between cusps, is well marked. One can readily see that, with a still shorter pendulum, the vibrations might be so small as not to be directly observable. Theory indicates that when ω_2 is large, or r small, the curve should consist of a series of infinitesimal cycloids. The approach to this form is clearly indicated in Fig. 8.

In making the traces shown in Figs. 4 and 8, great care was used to avoid giving any initial velocity to the pendulum. The sharply defined cusps show that the condition of zero initial velocity was very nearly fulfilled. In Fig. 8 the effect of friction, in damping out the vibrations and reducing the motion to one of steady precession, is clearly seen.

If the initial value of $\dot{\phi}$, instead of being zero, is *positive*, the velocity of precession can never become zero. Instead of cusps at the end of each vibration, we shall have a smooth, continuous curve, as shown in Fig. 5. This trace was taken with a rather long pendulum.

The greatest variety of traces may be obtained by making the initial value of $\dot{\phi}$ *negative*. If $\dot{\phi}_0$ has a small negative value, a series of loops is obtained, as shown in Fig. 9. With a somewhat larger value of $\dot{\phi}_0$, these loops are more marked, and the pendulum may even pass through the vertical. Figure 6 shows such a case. If a large negative value is given to $\dot{\phi}_0$, the precession will always remain negative (unless modified by friction), and the trace obtained, like that of Fig. 7, is almost too complicated to follow.

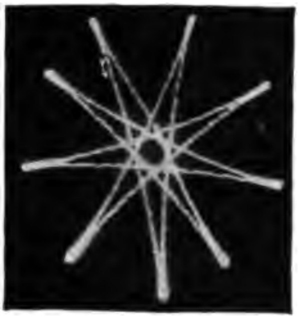

Fig. 4.

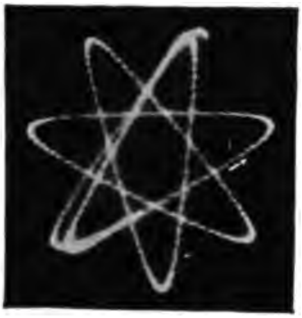

Fig. 5.

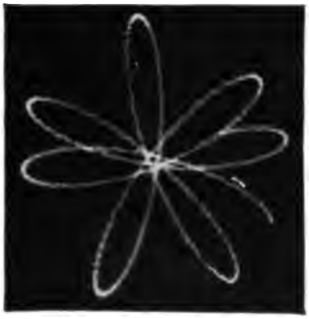

Fig. 6.

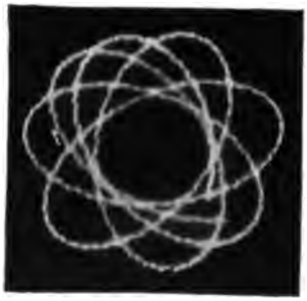

Fig. 7.

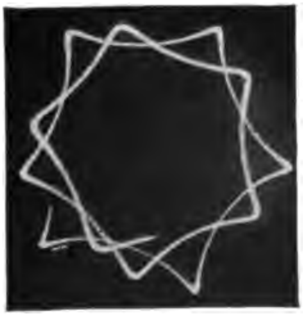

Fig. 8.

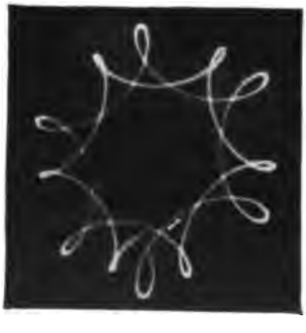

Fig. 9.

MERRITT: GYROSCOPIC PENDULUM.

The effect of friction is shown in Fig. 9, as well as in Fig. 6. In the former curve the loops, at first well marked, gradually become smaller, until finally they are almost reduced to cusps. Doubtless if the trace had been followed still further, these cusps would in turn have been replaced by a smooth curve such as that of Fig. 5. In all cases the effect of friction is to reduce the motion to one of steady precession. This result is accomplished by an *increase* in the velocity of precession. For example, in Fig. 9, the initial negative precession is destroyed, and a positive precessional velocity takes its place. This paradoxical conclusion, that friction may produce an increased velocity, represents only another of the puzzling peculiarities of the gyroscope. The truth of the conclusion can readily be shown if frictional forces are introduced into the equations. But it is not necessary to enter into the analytical treatment in order to reach the result. Bearing in mind the fact that the oscillatory motion is quite rapid as compared with the motion of precession, we see that frictional forces are much more important in the former case than in the latter. The effect of friction in damping out vibrations will be to make the average value of θ smaller; *i.e.* the pendulum approaches the vertical. But when θ diminishes, $\dot{\phi}$ must increase, as shown by equation (11), in order that the moment of momentum about the vertical shall remain constant. So long as the opposition to precession is small compared to the other frictional forces, the results will therefore be as stated above.

Many other features of gyroscopic motion may be studied to advantage by means of traces such as those here shown. If more care were used in obtaining them, I have no doubt that these traces could be made to give a quantitative verification of the theory of such motion. But it is chiefly on account of their value in helping to give a clear and definite interpretation of theoretical conclusions that I have thought it worth while to publish the results contained in this article.

NOTE ON DIFFERENT FORMS OF THE ENTROPY FUNCTION.

BY W. F. DURAND.

IT is well known that the heat $\int dQ$ involved in any reversible change in a given substance or system is not independent of the path followed, or in other words of the intermediate conditions passed through. It follows that dQ is not a complete differential, and that $\int dQ$ cannot be expressed as a function of the initial and terminal conditions. As is well known, however, it is possible by means of an integrating factor to reduce dQ to a complete differential, and hence to express its integral thus transformed, as a function of the end conditions. The factor commonly used for this purpose is the reciprocal of the absolute temperature T. We have thus: —

$$\frac{dQ}{T} = d\eta,$$

in which $d\eta$ is a complete differential. The integral η is then termed the entropy, any convenient set of initial conditions being taken from which to measure its value. It is also well known that where one integrating factor, as $\frac{1}{T}$, exists, there will also exist an indefinite number of other factors, leading to a corresponding indefinite series of values of the complete integral.

It is the purpose of the present note to show some of the forms which the integrating factors may take in the case of a perfect gas, as well as the resultant forms of the entropy function, η.

To this end let β denote any integrating factor, and η the corresponding value of the entropy function. We shall then have

$$\int \beta dQ = \eta.$$

It will be noted that the function η thus defined is the general value for any integrating factor β, the common value resulting when $\beta = 1 \div T$. We here, therefore, use the symbol in a general sense, of which the common use is a particular instance.

We shall also find it convenient to assume an auxiliary function P which shall be constant in value for a change involving no inflow or outflow of heat. The function P is therefore constant under the same conditions as η, and each may therefore be expressed as a function of the other. For the purpose in view, P may be entirely unlimited so long as the above condition is fulfilled. In order, however, to fix our ideas, and with no loss of generality, we may take P as a function of the independent variables defining the condition of the system, such that $P = constant$ shall be the general equation to the family of isentropic loci, such equation being expressed in its simplest form. This is still indefinite since many forms may be given to this equation with but slight difference in simplicity. Any one form, however, will answer, and no matter what form be chosen, the general result will be the same. With the language used above, the definition of P holds for any number of independent variables. Where they are more than two, however, space of more than three dimensions is required, and hypergeometrical concepts and terminology must be introduced. For the present purpose, however, it will be quite sufficient to limit our consideration to two independent variables which we shall prefer to take as pressure and volume, denoted by p and v.

Using the auxiliary variable P, we may put in general for any reversible change —

$$dQ = \frac{\partial Q}{\partial P} dP. \tag{1}$$

But $\frac{\partial Q}{\partial P}$ is completely determinate at each point and depends on the properties of the body. It must, therefore, be expressible as a function of p and v. Let us for convenience, as will be seen later, put $\frac{\partial Q}{\partial P}$ in the form

$$\frac{\partial Q}{\partial P} = f(pv)F(P). \tag{2}$$

In this expression the form of the function F is entirely unlimited, while that of the function f will result from that taken for F. We shall have, therefore, from (1) and (2) —

$$\frac{dQ}{f(pv)} = F(P)dP. \tag{3}$$

Now $d\eta$ is by definition a complete differential, and from the relation between P and η, dP must be such likewise. Hence $F(P)dP$ will be a complete differential, and hence, likewise, $\frac{dQ}{f(pv)}$. The same conclusion might also be reached by remembering that P may be defined simply as a function of the coördinates fulfilling certain conditions as above specified, and hence $F(P)dP$ must be a complete differential.

It follows that $\frac{1}{f(pv)}$ is an integrating factor, and we may put —

$$\frac{dQ}{f(pv)} = F(P)dP = d\eta. \tag{4}$$

Remembering that the function F is unlimited in form, it is plain that $\frac{1}{f(pv)}$, $d\eta$, and η may have an indefinite number of values, and in particular that multiplication of the first two members of (4) by P, or by any function of P, will not affect the existence of the general entropy function η. Hence if any particular quantity represented by $\frac{1}{f(pv)}$ or by β is an integrating factor, then the same quantity multiplied by P, by any function of P, by η, or by any function of η, will be an integrating factor also.

From (4) we have for the general value of the series of integrating factors —

$$\beta = F(P)\frac{\partial P}{\partial Q}. \tag{5}$$

While for the similar general value of the series of entropy functions we have —

$$\eta = \int F(P)\,dP. \tag{6}$$

In both of these it will be remembered that the function F is entirely unlimited in form.

It follows that we may also write —

$$\eta = \theta(P), \tag{7}$$

where θ is any function unlimited in form. Again, P may be taken as a special value of η which we may denote by η_1. We may therefore write —

$$\eta = \theta(\eta_1). \tag{8}$$

Let us now express the above general value of β in terms of other differential coefficients. We first note that at any given point, or in any given condition, $\frac{\partial Q}{\partial P}$ is independent of the path or direction of change.

Let us then, as a matter of convenience, take $\frac{\partial Q}{\partial P}$ for constant pressure. Denoting this by $\left.\frac{\partial Q}{\partial P}\right)_p$ and others similarly we have —

$$\left.\frac{\partial Q}{\partial P}\right)_p = \left.\frac{\partial Q}{\partial T}\right)_p \left.\frac{\partial T}{\partial v}\right)_p \left.\frac{\partial v}{\partial P}\right)_p = f(pv)F(P),$$

or

$$\beta = \frac{F(P)}{\left.\frac{\partial Q}{\partial T}\right)_p \left.\frac{\partial T}{\partial v}\right)_p \left.\frac{\partial v}{\partial P}\right)_p}.$$

Now let us apply these expressions to a perfect gas, defined as a substance fulfilling the condition —

$$\frac{pv}{T} = \text{constant} = R.$$

For the function P in this case we shall have pv^γ, the well-known equation to the isentropic lines being $pv^\gamma = \text{constant}$. Hence —

$$\left.\frac{\partial Q}{\partial T}\right)_p = \text{specific heat at constant pressure} = \sigma_p,$$

$$\left.\frac{\partial T}{\partial v}\right)_p = \frac{p}{R},$$

$$\left.\frac{\partial v}{\partial P}\right)_p = \frac{1}{\gamma p v^{\gamma-1}}.$$

Hence, reducing, we have —

$$\frac{\partial Q}{\partial P} = \frac{\sigma_v T}{P} \text{ and } \beta = \frac{PF(P)}{\sigma_v T}.$$

This is the general form of the integrating factor for a perfect gas with this particular value of P. For the general value of η we have as in (6),

$$\eta = \int F(P)\,dP.$$

We may now briefly discuss these values.

Let $F(P) = \frac{\sigma_v}{P}$. Then $\beta = \frac{1}{T}$, and we have the usual integrating factor. For the entropy we have —

$$\eta = \int \sigma_v \frac{dP}{P} = \sigma_v \log P = \sigma_v \log (pv^\gamma).$$

Next let $$F(P) = \frac{\sigma_v}{R}.$$

Then $$\beta = v^{\gamma-1}, \text{ and } \eta = \frac{\sigma_v}{R} P = T\sigma_v v^{\gamma-1}.$$

Again let $$F(P) = \frac{\sigma_v P^{\left(\frac{1}{\gamma}-1\right)}}{R}.$$

Then $$\beta = p^{\frac{1}{\gamma}-1},$$

and $$\eta = \frac{\gamma\sigma_v P^{\frac{1}{\gamma}}}{R} = \gamma T\sigma_v p^{\left(\frac{1}{\gamma}-1\right)}.$$

The second and third forms are of interest as showing that an integrating factor may be found expressed simply as a function of v, and one expressed simply as a function of p.

It will be noted that $v^{\gamma-1}$ and $p^{\frac{1}{\gamma}-1}$ are integrating factors for a perfect gas only, and that for other substances, or systems, factors similarly found would have different values. In this connection it may not be without interest to note in closing that $\frac{1}{T}$ is an integrating factor for all substances or systems simply because it is so defined, and it is possible thus to have an integrating factor common in form for all substances, because the quantitative definition of temperature is not previously required, and we are therefore at liberty to choose the nature of the unit and scale in accordance with this purpose. We cannot have $v^{\gamma-1}$, or any other function of v, an integrating factor for all substances, because we are not at liberty to choose an arbitrary scale of volume such that this condition shall be fulfilled. Volume and its measure are geometrical concepts, and are fixed by considerations antecedent to the problems of thermodynamics. Similar considerations show that no function of p alone can be an integrating factor for all substances. It may also be noted that while $1 \div T$ is an integrating factor for all substances, the expression of its value as a function of p and v will vary according to the substance.

NEW BOOKS.

Physikalische Technik. By F. Frick. Sixth edition, revised by O. Lehmann. Two volumes, large 8vo, pp. 725 and 1054, respectively. Braunschweig, Viewig und Sohn, 1895.

Frick's *Physikalische Technik* is one of the earliest of the books intended to assist the lecturer on physics in the preparation of illustrative experiments. Written by the director of the Hochschule of Freiburg about the middle of this century, at a time when the value of experimental work and concrete methods of instruction was only beginning to be generally appreciated, its usefulness was quickly recognized, and in 1862 the second edition was translated into English. Since that time the same field has been covered, in whole or in part, by such books as Weinhold's *Physikalische Demonstrationen*, and Lehmann's *Physikalische Technik*, in Germany, and Dolbear's *The Art of Projection*, Wright's *Light*, and Tyndall's *Lectures*, in England and America.

The work before us, now in its sixth edition, still remains, however, the most ambitious, as well as the most complete, of its kind. While the general plan of the earlier editions has been retained, Dr. Lehmann has added greatly to the amount of material presented, and has subjected the whole to a thorough revision. Instead of a book of less than 500 pages we now have two large volumes, one of which alone is twice as large as the original work.

The first volume begins with a consideration of the best arrangement for the lecture room, apparatus cabinets, and work shop. In this chapter various fixed accessories are also discussed. Then follow directions for cleaning and repairing apparatus, for lacquering, cementing, blowing and cutting glass, and for working such materials as cork and rubber. This chapter, which occupies about 80 pages, contains a vast amount of useful information about mechanical and physical manipulations of all kinds.

The detailed description of lecture experiments begins with the second part of Vol. I., which takes up the subjects of statics (solids, liquids, and gases), heat, and dynamics and thermodynamics, in the order here given. The experiments are almost without exception of an illustrative character consistent with the general plan of the book, and are described in a direct and practical manner. The arrangement of subjects and experiments, however, leaves much to be desired. In most cases the number of experiments under each head is far in excess of what could be presented during any ordinary lecture course. This is not true, however, in the case of surface tension and capillarity. These two subjects, which Boys has shown to be capable of such fascinating experimental illustration, are treated with a briefness quite out of keeping with the rest of the book.

It is in the second volume, and especially in that part which deals with the subjects of electricity and magnetism, that the changes made by Dr. Lehmann are most radical. In the new material added we find references to the most recent developments of electrical science: for example, to such subjects as Tesla's high frequency phenomena, Hertz waves, the discharge of negative electricity by light, kathode rays, etc. It is not possible to treat all of these subjects in detail, even in a book as large as Frick's *Physikalische Technik.* In fact, the directions given for the preparation of experiments are only in a few cases sufficient for actual use. They will nevertheless serve as valuable suggestions, while the references to original articles, which are in most cases given, will help in the preparation of special lectures.

From the American standpoint the editor's attempt to modernize the chapters dealing with electricity can hardly be looked upon as successful. The experiments relating to dynamo electric machinery, and the discussion of electrical measuring instruments for lecture purposes, are especially unsatisfactory. In some cases the methods and apparatus might almost be referred to as antiquated. This is perhaps to be explained by the fact that in this country, much more than in Germany, the teaching of electricity as a branch of physics, and its presentation as a basis for the education of the electrical engineer, have gone hand in hand. Both the science and the art have benefited by this arrangement. The teacher and the investigator have gained the use of instruments and methods perfected by the engineer; while they in turn have contributed to the general stock of engineering data the results of investigations of the highest practical importance. There is little evidence of this union of the pure and applied science in the work before us. In the chapters on current electricity and magnetism the methods and apparatus are practically those of ten or fifteen years ago. It is hardly to be expected, therefore, that this portion of the book will prove of much value in this country. On the other hand those chapters which deal with statical electricity, and with other branches of the subject not yet generally applied in practice, are more extensive, as well as more satisfactory, than any similar treatment in English. Dozens of interesting and important experiments are here described which rarely find their way into our English or American text-books. It can hardly be denied that the tendency in this country is often to give prominence to the practical side of electricity at the expense of those portions of the subject whose practical utility can now be only dimly seen. There results a distorted presentation of the subject which is as harmful to the engineer as it is to the development of the science. The work of Dr. Frick offers suggestions in regard to these neglected branches which will be of considerable assistance to a lecturer desirous of presenting a well-rounded course.

Throughout the work, and especially in the second volume, considerable space is given to a presentation of general theory. Such discussions, of necessity incomplete, seem to me entirely out of place in a work intended primarily as a help in the preparation of experimental lectures. For the lecturer who knows his subject, they are unnecessary; for one whose preparation is insufficient, they are too brief to be of any assistance. In my opinion such discussions serve merely to increase the bulkiness of the work, and thus to diminish its usefulness.

Like so many of the recent books on scientific subjects, this new edition of the *Physikalische Technik* shows unmistakable evidence of having been written with the shears and paste pot. From the very nature of the task we can expect nothing else. The gathering together of such a mass of useful material from its concealment in the enormous periodical literature of physics, is a task that would well deserve our thanks, even if the author and editor had not contributed a single new sentence. But the reader has a right to expect a logical and convenient arrangement of the materials; and this is *not* found in the work before us. The arrangement of subjects and experiments in the second volume is simply bad beyond all description. A fairly complete index does indeed make it possible to find the description of an instrument whose exact name is known, or of some branch of physics that can be specified by a definite title. But the reader who hopes to find the experiments dealing with related subjects grouped together, so that suggestions may be quickly gained by merely glancing them over, will be grievously disappointed. For example: A lecturer desiring information regarding the Lichtenberg figures and allied phenomena would, I think, naturally look in Section O of Volume 2, which is entitled "Elektrische Staub- und Lichtfiguren." His search for Lichtenberg's figures would indeed be successful, but the related phenomena of dust figures and breath phantoms would escape him; for these subjects, widely separated from one another, are included in a preceding chapter on the "Durchgang von Elektricität durch schlechte Leiter." The subject of breath phantoms is especially well concealed, being directly preceded by the description of tubes for the development of ozone, and followed by a discussion of oil condensers. Except the one topic, Lichtenberg figures, Section O, contains nothing that could properly be classified under the title "Staub- und Lichtfiguren." Among the subjects that it *does* contain may be mentioned the action of a magnetic field on the spark discharge, the electric furnace, Crookes' tubes, and the electric arc. Similar instances of bad arrangement might be cited almost without limit. The material contained in the book will undoubtedly prove of great value, in spite of the manner in which it is arranged. But it is unfortunate that the opportunity of accomplishing much more than this has not been utilized.

Ernest Merritt.

Submarine Cable Laying and Repairing. By H. D. Wilkinson, M.I.E.E. pp. 406. Just published. New York, D. Van Nostrand Company, 23 Murray Street. London, Electrician Printing and Publishing Company, Limited.

This book contains a very full description of methods of laying and repairing submarine cables, and the author has evidently added a number of sketches and photographs of his own, taken during such operations. The book is divided into four main chapters, the first of which is devoted to the operations performed by cable ships, the second to cable ships themselves, the third to methods of localizing faults in submarine cables, and the fourth to the laying of submarine cables. The book abounds in just such practical information upon this interesting and important subject as it is almost impossible to find in other publications with which we are acquainted. The subject-matter is well illustrated by 219 cuts, and the practical side of the subject has received very full treatment. An excellent index is supplied for the illustrations as well as for the topics discussed. The book is addressed to all those who are interested in the subject of submarine telegraphy, and as such it is well calculated for use in general libraries. It is also designed to be of great use to those who are interested in submarine telegraphy from a technical point of view, since it not only contains much information which is very useful for reference, but also gives the student a clear insight into the leading principles dealt with. We believe the book will be very cordially received, as it is the only one we know which deals with its subject in a simple, interesting, and thorough way.

A. E. K.

Physikalische-Chemische Propaedeutik unter besonderer Berücksichtigung der medicinischen Wissenschaften, und mit historischen und biographischen Angaben. By Dr. H. Griesbach. Vols. I. and II. Leipzig, Wilhelm Engelmann, 1895–96.

This book attempts the difficult task of bringing together the essential truths in our knowledge of nature on its animate as well as its inanimate side. In other words, it professes to be a compendium of all that is funda-

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be grievously disappointed. For example: A lecturer desiring
regarding the Lichtenberg figures and allied phenomena would
naturally look in Section O of Volume 2, which is entitled "El
Blitz- und Lichtfiguren." His search for Lichtenberg's figures would
be successful, but the related phenomena of dust figures and breath
tions would escape him; for these subjects, widely separated fro
another, are included in a preceding chapter on the "Durchgang
Elektricität durch schlechte Leiter." The subject of breath figures
especially well concealed, being directly preceded by the descript
tubes for the development of ozone, and followed by a discuss
oil condensers. Except the one topic, Lichtenberg figures,
contains nothing that could properly be classified under the head
und Lichtfiguren." Among the subjects that it
tioned the action of a magnetic field on the
furnace, Crookes' tubes, and the electric

arrangement might be cited almost without limit. The material containe in the book will undoubtedly prove of great value, in spite of the manne in which it is arranged. But it is unfortunate that the opportunity o accomplishing much more than this has not been utilized.

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Physikalisch[illegible]mische P[illegible] [illegible]inischen [illegible] unter besonderer Berück-[illegible]en. [illegible]u, und mit historischen und [illegible] [illegible]RIESBACH. Vols. I. and II.

[illegible] bringing together the essential [illegible]nimate as well as its inanimate [illegible] compendium of all that is fundamentally important in the physical, chemical, and biological sciences. The author has evidently read widely in the entire literature of the natural sciences, and has succeeded in condensing into small compass a great amount of useful and interesting information which in the main seems to be reliable. The special aim of the book apparently is to furnish students of the biological or medical sciences with brief descriptions of physical conceptions in regard to the nature and properties of matter and energy, and so far as possible to illustrate the value of these conceptions in the study of living processes. It contains many concise definitions which will be useful to those whose knowledge of physics is incomplete. For the most part, also, the descriptions are not technical and may be followed easily by the non-scientific reader. The book seems therefore to be well adapted to the needs of those who desire simply intelligible statements of accepted scientific beliefs, and do not care for elaborate expositions or scientific proofs. Necessarily it covers a very wide range of subjects. Indeed the extremely miscellaneous character of the subjects treated prevents the book from being readable as a whole, and lays it open to the objection of being merely an *omnium gatherum* of things scientific without any very serious educational purpose. For purposes of reference it may be regarded as a handy volume, particularly for students of biology or medicine, and also, as the author suggests in his announcement, for lawyers who come into occasional professional contact with scientific matters. A valuable feature for the specialist is the long list of references to standard treatises, and the numerous biographical sketches scattered throughout the book. It is to be hoped that when completed the book will contain an index that shall enable the reader to avail himself readily of this material.

W. H. HOWELL.

Physical Measurements. By FRANK C. WELDON. 8vo. pp. 232. London, Gill & Sons, 1895.

This little book is intended as a manual of laboratory practice for students in the secondary schools. The experiments described in this volume deal with the mechanics of solids, liquids, and gases, a great deal of space being devoted to simple geometrical measurements, such as the determination of area and volume. Most of the apparatus required is of extreme simplicity, and the experiments are of the elementary character suited to the grade of students for which the book is intended. The author's plan of instruction will be best stated by a quotation from the preface: "The pupil should be led to regard the work in the physical laboratory as a systematic series of experiments by means of which he will discover for himself scientific truths."

E. M.

Volume IV. *March–April, 1897.* *Number 5.*

THE

PHYSICAL REVIEW.

THE LEAD CELL.

By B. E. Moore.

Part I.

Theoretical.

WE owe the discovery of the lead cell to Planté. The cell has undergone many practical variations, but the essential features have remained about the same. To Planté is also due the first explanation of the action of the cell. He attributed its source of energy to occluded gases. He recognized the formation of sulphate, but regarded it as a secondary phenomenon. Occlusion — a phenomenon very familiar in platinum and palladium electrodes — had been impressed strongly upon him, and the evolution of gases seemed to favor that opinion. Had this phenomenon not been very strongly impressed upon him, it seems probable that Planté might have given us the lead sulphate theory. The occlusion theory satisfied most of the scientific world for a long time. Gladstone and Tribe made a chemical study of the cell in 1882,[1] and showed that the formation of lead sulphate from lead peroxide was the principal source of the energy. Views once accredited are hard to eradicate, and the new intruder did not supplant the orthodox view without a protest. The strongest opponent of the new view was, perhaps, Dr. Oliver J. Lodge.[2]

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[1] Electrician, Vol. VIII., p. 309, and subsequent numbers.

[2] Electrician, Vol. IX., p. 92, and subsequent numbers.

Other experiments followed, which showed that the occluded gases could only yield a minute quantity of energy, and the principal reaction of Gladstone and Tribe has come to be generally accepted. The Kelvin law, which demands the equivalence of heat and electrical energy, has added confidence to the chemical evidence. This treatment is a thermodynamical one, and some assert that it affords the strongest support of the sulphate theory. This law of Kelvin and v. Helmholtz was, however, shown later by v. Helmholtz to be wrong, and has been supplanted by the later v. Helmholtz equation. In this particular case it happens that the Kelvin law nearly expresses the facts. The right formula, however, can as easily be used, and the greater error of the equivalence of heat energy of a chemical reaction, and electrical energy of a similar reaction, would not be continually thrusting itself upon us.

The ideas of the new physical and electrochemistry met, at first, *much severer* attacks than the work of Gladstone and Tribe.

Discussion ought to bring out the truth, but in this case healthy and pertinent discussions soon waned, and then ceased. The adherents of the new ideas pushed forward with research and investigation, putting old facts in new light, and opening up entirely new fields, until at last the triumph is fully recognized, and a new theoretical chemistry is springing up in all of our better institutions. It would, therefore, seem desirable to have the lead storage cell explained upon the present hypotheses of electrochemistry. Le Blanc[1] has devoted a few words to the theory of the lead cell, and writes from the above standpoint. Very little experimental work bearing upon the theory of the cell has appeared in the last few years. The most recent contributions upon the theory, besides the work of Le Blanc, have been by Wade[2] and Barnett.[3] Both of these writers give an excellent historic review of the subject. Both give good discussions of the data at hand. Both ignore the recent developments in electrochemistry. Barnett even writes thermochemical equations which a decade ago might have been accepted unhesitatingly, but to-day would be questioned by the very conservative.

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[1] Lehrbuch der Electrochemie, p. 222.

[2] Electrician, Vol. XXXIII., p. 603, and subsequent numbers.

[3] Electrical World, Vol. XXVII., pp. 403, 455, 480.

The theory of Le Blanc will here be enlarged upon, and certain experiments will be given which have been suggested by the theory. The fundamental principles of Le Blanc are the theory of free ions, solution tension, and osmotic pressure, — principles, indeed, upon which rests the Nernst theory of electromotive force. Mathematic treatment, however, did not wait until experiment had matured these ideas, but appeared almost contemporaneously with them. The work of Max Planck is especially commendable. His treatment is based upon the second law of thermodynamics. His method can be applied to the storage cell. This method, *Industries* in 1891 maintained, must be applied for a proper understanding of the subject. This method of treatment seems more satisfactory to many persons, and agrees fully with the Nernst theory.

From the v. Helmholtz formula under assumption of a zero temperature coefficient, assuming the reaction,

$$PbO_2 + \text{water} + H_2SO_4 + Pb = 2\ PbSO_4 + \text{water} + 87{,}000 \text{ calories},$$

we have a voltage of 1.885. This is based upon the investigations of Streintz,[1] and agrees well enough with the value of 1.9 volts found with dilute acid. As before mentioned, these thermochemical reasons are confirmed by the chemical researches of Gladstone and Tribe. Observations upon electromotive force, with varying concentration of acid, have been carried on by Gladstone and Hibbard.[2] The observed and calculated electromotive force differs seldom more than one per cent, which would indicate in all cases a small temperature coefficient in the v. Helmholtz equation. Gladstone and Hibbard's thermochemical calculations are very abbreviated, and with such calculations grave doubts frequently arise. It would seem that a determination of the electrical output and the temperature coefficient of the cell would be a more satisfactory method of determining the "heat-toning" in such cases. However, the latter factor is difficult to determine. The heating

[1] Wien. Akad. Ber., 103, Jan., 1894.

[2] Electrician, Vol. XXIX., pp. 67, 95, 119.

effects noted by Professor Ayrton during the charging process, and the cooling of the cell noted by Dr. Duncan during discharge are good indications of a very appreciable temperature coefficient.

We will assume the battery charged and ready for use. In this condition the positive plate is covered with lead peroxide and the negative plate with spongy lead. The solution consists of sulphuric acid. For equilibrium, when solids and liquids are in contact, particles of the one must be contained within the other. At one electrode we should have lead peroxide in the solution; at the other we should have spongy lead particles in solution. Thermodynamically considered when such a substance as PbO_2 passes into solution, we can have equilibrium only upon partial dissociation of the substance. For the substance in question this follows under the formation of tetravalent $\overset{++}{\overset{++}{\mathrm{Pb}}}$ ions and $\overset{--}{\overset{--}{(\mathrm{OH})}}_4$ ions. The product of the concentration of the ions in a definite volume being constant, and the number of the $\overset{-}{\mathrm{OH}}$ ions being determined by the dissociation of H_2O, the concentration of tetravalent $\overset{++}{\overset{++}{\mathrm{Pb}}}$ ions in solution is the fourth root of the dissociated water molecules. The number of lead ions would therefore be extremely small. A theory resting upon such small quantities for carrying large currents would seem to be weak. However, in current conduction these products need penetrate only the first layers of the solution, and could, therefore, be easily supplied by the peroxide. The tetravalent $\overset{++}{\overset{++}{\mathrm{Pb}}}$ ions pass over into bivalent $\overset{++}{\mathrm{Pb}}$ ions during discharge, and *this is the principal source of the electromotive force of the lead cell.* The product of the concentration of the bivalent $\overset{++}{\mathrm{Pb}}$ ions and the $\overset{--}{\mathrm{SO}}_4$ ions, though having a larger value than the $\overset{++}{\overset{++}{\mathrm{Pb}}}$ and $\overset{--}{\overset{--}{(\mathrm{OH})}}_4$, is still small. As a result they combine, forming insoluble lead sulphate, which precipitates upon the electrode. This process follows under abstraction of sulphuric acid and formation of water. At the anode the bivalent $\overset{++}{\mathrm{Pb}}$ ions unite with $\overset{--}{\mathrm{SO}}_4$ ions, and the liberated $\overset{+}{\mathrm{H}}$ ions migrate through the solution. This process must follow with no very appreciable potential difference at the electrode. When the cell is discharged bivalent

$\overset{++}{\mathrm{Pb}}$ ions and the corresponding $\overset{--}{\mathrm{SO}_4}$ ions are formed at each electrode.

In the charging process the bivalent $\overset{++}{\mathrm{Pb}}$ ions go over into tetravalent $\overset{\substack{++\\++}}{\mathrm{Pb}}$ ions at the positive electrode, giving $\overset{\substack{++\\++}}{\mathrm{Pb}}$ and $\overset{\substack{--\\--}}{(\mathrm{OH})_4}$ ions. The low solubility of this product yields PbO_2 at the electrode. Since the solution is already saturated with $\overset{++}{\mathrm{H}_2}$ and $\overset{--}{\mathrm{SO}_4}$ ions, the ions liberated form $\mathrm{H_2SO_4}$, and the remaining $\overset{++}{\mathrm{H}_2}$ ions wander through the solution in the direction of the current to appear ultimately at the other electrode.

At the negative electrode the $\overset{++}{\mathrm{Pb}}$ and $\overset{--}{\mathrm{SO}_4}$ ions yield spongy lead at the plate and form sulphuric acid with the oncoming $\overset{++}{\mathrm{H}_2}$ ions.

The process, thus far considered, gives the formation of sulphuric acid at both electrodes, and the abstraction of water at the electrode where water had been formed in the discharging process. The process is made more complicated by the more rapid rate of migration of the $\overset{+}{\mathrm{H}}$ ions. Conduction through the solution is thus made to depend almost entirely upon the $\overset{++}{\mathrm{H}_2}$ and $\overset{--}{\mathrm{SO}_4}$ ions. Le Blanc states that ions take part in current conduction in proportion to their relative concentration, but at the electrode the easiest liberation product is set free. While lead and the peroxide may exist in the stratæ next to the electrode, there would scarcely be a trace in the body of the electrolyte.

If the theory were dropped at this point it would offer no suggestions as to the electrical behavior of the cell. To gain such insight, we must make use of thermodynamics or the Nernst theory of electromotive force. Upon the latter theory, the electromotive force of a cell may be expressed by the equation

$$E = \frac{RT}{2C_0}\left(\log_e \frac{P_+}{p_+} - \log_e \frac{P_-}{p_-}\right),$$

where P and p represent solution tension of the electrode substance and osmotic pressure of the ions respectively, R the gas constant, T absolute temperature, C_0 the current generated by the liberation of one gram-equivalent of any substance, and E the electromotive force. The + and − signs refer to the electrodes. Applying this formula to the lead cell, we see at once that the equation becomes one of four variables. The quantities P change, since in the charging or discharging process the character of the electrode is changing. The magnitude of p must also change. In the discharging process the bivalent $\overset{++}{\mathrm{Pb}}$ rapidly replaces the tetravalent $\overset{++}{\overset{++}{\mathrm{Pb}}}$. The solubility of the former is much greater than that of the latter and an increase in the value of p occurs. The reverse occurs upon charging the cell. The precipitation of a salt from solution, or the formation of a solution, occupies a certain amount of time; from which it follows, that a rapid increase in p would occur during the first few moments of a discharge, and a corresponding decrease during the charging process. Equilibrium is obtained when the current removes ions from or carries them into the solution, at the same rate at which the substance is dissolved or precipitated. This action of the current produces what one may call undersaturation and supersaturation. This action lowers the efficiency of the cell, and the stronger the current the lower the efficiency. Although there are four variables, the latter two are of little moment, and they vary in the same way as the first two. The effect upon the electromotive force of any change in the magnitude of P or p is noticeable by a simple inspection of the formula. Beside the initially rapid changes in P and p bringing about a new condition of equilibrium, there would be a slower change in the material of the electrode, and consequently slow changes in P and p.

Another series may arise in practice, which is designated as a concentration series. The following is an example: —

$$\mathrm{Ag} \mid \mathrm{n.\ AgNO_3} \mid \tfrac{1}{10}\ \mathrm{n.\ AgNO_3} \mid \mathrm{Ag}.$$

The terminals are of the same material, and the only electromotive force possible arises at the contact of the solutions of different concentrations. The electromotive force of this series is expressed by

$$\frac{RT}{C_0}\log_e\frac{p}{p_1}, \text{ or } \frac{RT}{C_0}\log_e\frac{c}{c_1}.$$

c and c_1 are the concentrations of the solutions corresponding to the osmotic pressures p and p_1. The electromotive force of the above series is evidently 0.029 volt. The character of the series would not be changed were it written in the form

$$\mathrm{Ag} \mid \mathrm{n.\ AgNO_3} \mid \mathrm{2\ n.\ AgNO_3} \mid \tfrac{1}{10}\ \mathrm{n.\ AgNO_3} \mid \mathrm{Ag}.$$

In a two-liquid cell, a concentration series generally exists. In a storage cell at rest, there should be no trace of such a series; but in a storage cell in action, a change in the concentration at the electrodes occurs, and that very rapidly. The amount of change is dependent upon the strength of the current. We then have a concentration series of the last form, *i.e.* one with an intervening solution. The direction of the electromotive force of such a series is always given by the more rapidly wandering ion. In the lead cell it will always oppose the current, whether it be a charging or discharging one. It also always acts to lower the efficiency of the cell. The magnitude of this electromotive force may be overestimated. When the ratio of the end concentrations is at 1000 to 1, its value is only 0.09 volt. One would scarcely think this ratio could be exceeded, even in the discharging process, and we must therefore attribute the greater part of the variation in electromotive force to changes in the factors P and p.

Thermodynamical Considerations.

If a current is pressed through a storage cell or taken from the same, a change in the polarization occurs, which is determined by the variation in available energy. Upon the ionic theory, this is caused by a passage of ions from the solution to the electrode, or *vice versa.* If the temperature remains constant, the total variation in available energy is

$$\delta U - T\delta S,$$

where U represents the total energy; S, the entropy; and T, the absolute temperature.

The dependence of entropy and concentration has been treated mathematically by Planck, and his results may be applied to solutions. The electrodes or electrode surfaces may be looked upon as solid solutions containing ions.

In expanding this method of treatment we will use the following designations:[1] —

S_{mk}, C_{mk} = Entropy and concentration, respectively, of the neutral molecules in the kathode.

S_{ma}, C_{ma} = Entropy and concentration, respectively, of the neutral molecules in the anode.

S_k, C_k = Entropy and concentration, respectively, of the kation in kathode.

S_a, C_a = Entropy and concentration, respectively, of the anion in anode.

S_{wk}, C_{wk} = Entropy and concentration, respectively, of the water in kathode.

σ_m, c_m = Entropy and concentration, respectively, of the neutral molecule in solution.

σ_k, c_k = Entropy and concentration, respectively, of the kation in solution.

σ_a, c_a = Entropy and concentration, respectively, of the anion in solution.

σ_w, c_w = Entropy and concentration, respectively, of the water in solution.

n, ν = Molecule and ions, respectively.

We accordingly have for the entropy of the kathode: —

$$n_{mk}(S_{mk} - \log C_{mk}) + n_k(S_k - \log C_k) + n_{wk}(S_{wk} - \log C_{wk}),$$

and similarly, for the anode, we may write: —

$$n_{ma}(S_{ma} - \log C_{ma}) + n_a(S_a - \log C_a) + n_{wa}(S_{wa} - \log C_{wa}).$$

Finally, for the entropy of the solution, we have: —

$$\nu_m(\sigma_m - \log c_m) + \nu_k(\sigma_k - \log c_k) + \nu_a(\sigma_a - \log c_a) + \nu_w(\sigma_w - \log c_w).$$

If, under the influence of a current, a quantity, *dn*, of the kation passes from the solution to the kathode, we should have, as a first approximation for the change in entropy of the solution, at the kathode,

$$-(\sigma_k - \log c_k)dn,$$

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[1] Dr. Jahn, in his Grundriss der Electrochemie (see this JOURNAL, Vol. IV., p. 76), makes many similar applications of Planck's method.

and for the change in entropy of the kathode,

$$(S_{mk} - \log C_{mk})dn,$$

and accordingly

$$\delta U - T(S_{mk} - \sigma_k - \log C_{mk} + \log c_k)dn$$

is the expression for the change in the kathode energy or the kathode polarization. The anode polarization is evidently

$$\delta U' - T(S_{ma} - \sigma_a - \log C_{ma} + \log c_a)dn.$$

Any increase in C_{mk} or C_{ma} increases the polarization of the cell, *i.e.* causes an increase in the available electromotive force. An increase in C_k or C_a would have the opposite effect.

For a more general case we have for the variation of entropy at the kathode

$$(S_{mk} - \log C_{mk})dn_{mk} + (S_k - \log C_k)dn_k + (S_{wk} - \log c_{wk})dn_{wk}.$$

The second term may be legitimately dropped, as the kations all pass over into neutral molecules at the electrodes when on closed circuit. The concentration of the water within the electrode may be as conveniently regarded as the concentration at the contact surface of the electrode. Neither would the argument be affected were we to speak of the concentration of the sulphuric acid instead of the concentration of the water. Both quantities are mutually dependent. We can then write the change in entropy at the kathode without much error in the following form: —

$$(S_{mk} + S_{wk} - \log C_{mk} - \log c_{wk})\,dn.$$

The corresponding change in entropy of the solution upon the kathode side is

$$-\{(\sigma_m - \log c_m)\,dv_m + (\sigma_k - \log c_k)\,dv_k + (\sigma_w - \log c_w)\,dv_w\},$$

which we may write

$$-(\sigma_m + \sigma_k + \sigma_w - \log c_m - \log c_k - \log c_w)\,dn,$$

or the entire change in entropy for kathode and solution

$$\delta U'' - T(S_{mk} + S_{wk} - \sigma_w - \sigma_k - \sigma_m - \log C - \log c_{wk} \\ + \log c_m + \log c_k + \log c_w)\,dn.$$

This may be written in the form:—

$$\delta U'' - T\left(S_{mk} + S_{wk} - \sigma_w - \sigma_k - \sigma_m - \log\frac{C_{mk}}{c_k c_m} - \log\frac{c_{wk}}{c_w}\right)dn.$$

c_k is in all cases affected by the magnitude of c_m. When c_m is large there is relatively little change in c_k, but when c_k is small and c_w large, the tendency would be for c_k to vary rapidly under the influence of the current, and therefore cause rapid changes in the electromotive force of the cell. c_m is always small in the lead cell, and c_k must therefore be still smaller.

Instead of the product $c_k c_m$, a single factor could be written, owing to their mutual dependence. The exact analogy of the expression, $-\log\frac{C_{mk}}{c_k c_m}$, to the expression $\log\frac{P}{p}$ found in the Nernst formula, all will at once recognize. Similarly, $-\log\frac{c_{wk}}{c_w}$ represents the concentration series. But it would be more apparent if the expression for the anode polarization were written. The corresponding expression at the anode is $\log\frac{c_{wa}}{c_w}$, from which we obtain the true concentration series $\log\frac{c_{wk}}{c_{wa}}$. We have then practically the same form as the Nernst formula, and the conclusions to be drawn would therefore be the same.

To obtain the effects of change in temperature upon the electromotive force, the total thermodynamical expression for electromotive force must be differentiated as to T. This expression is so very long that it may be omitted. It reduces, however, to the v. Helmholtz equation, showing that the "heat-toning" and electrical energy are only equal at absolute zero, or when the temperature coefficient is zero.

PART II.

Experimental.[1]

In our observations we aimed to take readings for total and terminal electromotive force, and internal resistance at different times during the charging and discharging process. The cells were generally allowed to rest for a period equal to the time of action. From the standpoint of the theory we were particularly anxious to note the changes which took place in the cells during the first few seconds of action, or of recovery from action, both charging and discharging. We therefore had recourse to condenser readings. By means of a pendulum apparatus,[1] we were enabled to maintain constancy in the period of charge, and reduce the time of charge to about 0.1 second. With a dead-beat galvanometer we had no trouble in taking readings every five seconds when necessary. We generally alternated readings with total and terminal electromotive force. The internal resistance could then be calculated from these readings, and either a known external resistance or the known current.

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[1] The experimental work was done at University of Illinois, in June, 1896, with the kind assistance of Mr. H. V. Carpenter, student in electrical engineering.

We frequently had both of these quantities. In the charging process, the total electromotive force was obtained by breaking the circuit just before the pendulum reached the detents, the overturning of which charged the condenser, and then discharged it through the galvanometer. A hand break had to be resorted to for strong currents; and breaking the circuit 0.1 second before the pendulum reached its proper place, made an appreciable difference in the readings. With little practice, the circuit can be broken surprisingly near to the instant the pendulum meets the first detent. The circuit need not be broken in the charging process for terminal potential readings. This latter value should exceed the electromotive force of the cell by the fall of potential over the ohmic resistance of the cell. However, the impression is current that the electromotive force of the cell is far below the difference between the charging electromotive force and the fall of potential over the ohmic resistance.

To make our observations doubly sure, we threw the charging cell momentarily upon a discharging circuit in which the external resistance was adjusted to give us a current of about the same magnitude as the charging current. The results gave the same internal resistance in both cases, and showed in all cases the charging electromotive force to be equal to the sum of the electromotive force of the cell and fall of the potential over the ohmic resistance of the same. Our observations were upon a 10, 20, and 40 ampere current discharge, and recovery. The charging currents were 10, 20, and 15 amperes. The latter started at 20 amperes, but we found that the machine used for charging would not give that current after the counter electromotive force of the cell reached a certain limit. We accordingly dropped to 15 amperes, which makes the curves fully as instructive. The current was regulated by a carbon and a water rheostat in series.

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[1] Physical Review, Vol. II., p. 392.

Tables of Results.

In the following tables, *t* and *E* denote times and electromotive force, respectively. Time is reckoned from the moment the circuit is closed or thrown open as zero. The first part of each column represents the closed circuit, and the last part of each column, the open circuit.

Table I.

TEN-AMPERE DISCHARGE AND RECOVERY.

I.		II.		III.	
t	*E*	*t*	*E*	*t*	*E*
min. sec.		min. sec.		min. sec.	
0 0	2.065	0 5	2.041	0 5	1.920
0 5	2.041	2 10	2.031	0 15	1.808
3 0	2.025	3 0	2.020	2 10	1.726
6 0	2.020	30 0	2.020	3 0	1.667
11 0	2.020	90 0	1.990	4 0	1.566
30 0	2.020	150 0	1.535	7 0	1.557
				17 0	1.530
				20 0	1.404
0 5	2.031	0 5	1.576	0 5	1.454
0 45	2.044	1 30	1.646	0 30	1.485
2 0	2.050	3 0	1.687	2 0	1.530
12 0	2.060	8 30	1.808	4 30	1.606
20 0	2.060	12 0	1.848	8 0	1.667
		52 0	1.950	25 0	1.778
				31 0	1.828
				52 0	1.898
				68 30	1.924

Table II.

TEN-AMPERE CHARGE AND RECOVERY.

I.		II.		III.	
t	*E*	*t*	*E*	*t*	*E*
min. sec.		min. sec.		min. sec.	
0 0	1.950	0 10	2.081	0 0	2.080
0 5	1.980	1 0	2.101	0 10	2.110
2 15	2.020	2 0	2.111	1 0	2.139
6 30	2.071	3 0	2.117	2 0	2.155
8 0	2.091	7 0	2.121	5 0	2.181
12 0	2.101	25 0	2.141	15 0	2.190
19 0	2.127	30 0	2.141	30 0	2.205
20 0					
0 15	2.106	0 5	2.121	0 10	2.170
1 0	2.096	1 10	2.101	1 30	2.149
3 0	2.091	4 0	2.091	3 0	2.135
4 0	2.085	30 0	2.076	10 0	2.125
9 0	2.066			30 0	2.085
20 0	2.060			990 0	2.065

Table III.

TWENTY-AMPERE DISCHARGE AND RECOVERY.

I.		III.		IV.		V.		VIII.	
t	*E*	*t*	*E*	*t*	*E*	*t*	*E*	*t*	*E*
min. sec.		min. sec.		min. sec.		min. sec.		min. sec.	
0 0	2.080	0 0	2.050	0 0	1.970	0 5	1.800	0 0	1.818
0 10	2.045	0 5	2.015	0 10	1.864	0 10	1.784	0 5	1.773
2 0	2.023	0 10	2.000	0 40	1.787	1 0	1.765	0 10	1.750
15 0	2.015	4 0	1.985	2 0	1.765	3 0	1.754	0 50	1.735
20 0	2.015	7 0	1.985	3 30	1.757	18 30	1.727	2 0	1.670
		9 30	1.875	12 0	1.740	20 0	—	4 0	1.623
		15 0	1.864	20 0	1.734			6 0	1.610
		18 0	1.777					11 0	1.584
		30 0	1.770					17 0	1.579
								19 0	1.579
								20 0	—
0 10	2.037	0 5	1.795	0 10	1.773	0 5	1.765	0 5	1.630
1 0	2.050	0 40	1.803	2 0	1.803	0 10	1.784	0 10	1.653
7 0	2.061	2 0	1.856	7 0	1.848	1 0	1.800	1 0	1.697
12 0	2.061	4 0	1.925	10 0	1.856	3 0	1.811	5 0	1.742
20 0	2.068	9 0	1.962	13 0	1.864	6 0	1.818	13 0	1.788
	2.068	20 0	1.970	20 0	1.879	15 30	1.826	20 0	1.788
						20 0	1.833	1020 0	2.004

Table IV.

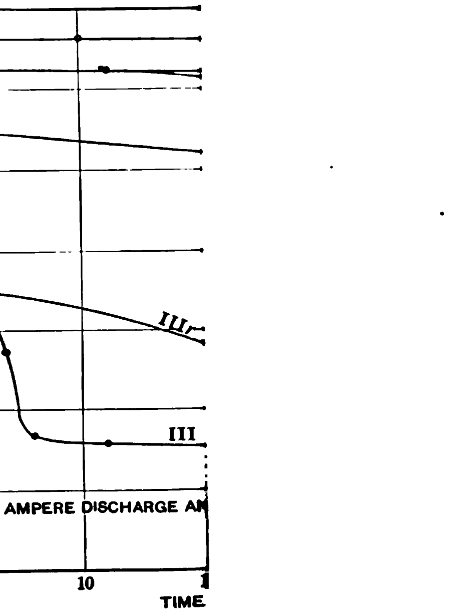
TWENTY AND FIFTEEN AMPERE CHARGE AND RECOVERY.

I.		II.		III.		IV.	
t	*E*	*t*	*E*	*t*	*E*	*t*	*E*
min. sec.		min. sec.		min. sec.		min. sec.	
0 0	2.004	0 25	2.125	0 0	2.060	0 0	2.098
0 5	2.037	8 30	2.197	0 3	2.091	0 5	2.184
0 10	2.067	18 0	2.205	0 15	2.174	0 20	2.257
1 0	2.106	20 0	2.219	2 0	2.287	2 0	2.371
1 30	2.114			8 0	2.394	6 0	2.490
3 30	2.151			13 0	2.424	11 0	2.492
5 0	2.166			14 30	2.432	14 30	2.500
12 0	2.197			20 0	2.440	20 0	2.500
19 50	2.201						
20 0	—						
0 5	2.166	0 5	2.174	0 10	2.311	0 5	2.424
0 40	2.106	0 30	2.122	0 20	2.212	0 10	2.364
1 20	2.098	1 0	2.106	1 0	2.174	1 0	2.235
2 0	2.087	2 0	2.091	6 0	2.136	4 0	2.182
10 0	2.057	14 0	2.045	7 30	2.129	6 30	2.166
19 0	2.045	20 0	2.045	19 0	2.091	20 0	2.130
20 0	—			20 0	—		

Table V.

FORTY-AMPERE DISCHARGE AND RECOVERY.

I.		II.	
t	*E*	*t*	*E*
min. sec.		min. sec.	
0 0	2.115	0 5	2.000
0 5	2.050	1 10	1.971
1 0	1.998	2 0	1.970
2 0	1.985	12 30	1.965
10 0	1.950	16 0	1.948
20 0	1.946	19 0	1.838
		20 0	—
0 5	1.985	0 5	1.880
1 0	2.055	0 30	1.900
10 0	2.081	1 30	2.000
19 0	2.090	2 10	2.017
20 0	2.091	5 30	2.029
		7 0	2.032
		9 0	2.041
		17 0	2.047
		20 0	2.050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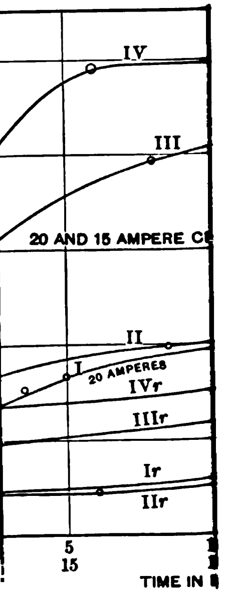
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. E. MOORE: T

Curves.

In all curves, the recovery was plotted in the inverse order. This enables one to compare with greater ease the successive runs. The discharges are marked in the order of their occurrence; and the corresponding recovery curves are similarly marked, but with the subscript r added. In the 10-ampere charge between II. and III., is a period of charge equal to forty ampere hours. In the 20-ampere discharge, II. is nearly the duplicate of I., and neither data nor curves are inserted; and VI. and VII. were omitted, to leave the curves less cumbersome. The value which VIII. obtained after 1020 minutes is indicated on the curve by an x. In the curves marked 20 and 15 ampere charge, the first only is a 20-ampere charge; and during the last five minutes, it sank below 19 amperes. Between II. and III., there is a period of charge equal to seventy ampere hours, after which the cells rested one hour and forty-five minutes, before the charging was again begun. Between III. and IV. was a rest over night, followed by twenty-five ampere hours' charge, and then a period of two hours' recovery. The observations on 40-ampere discharge were continuous.

The curves in general show great regularity, and their character is in harmony with the theory. The drop in electromotive force on closing the discharging, or the rise in electromotive force on closing the charging, current does not by any means take place instantaneously. In passing from charging to discharging, the curve of electromotive force would not be discontinuous. Let us notice the 20-ampere discharge and recovery curves. The cells, to begin with, were not charged to their full capacity, but the electromotive force in zero time, curve I., is about the normal value. In one minute, a marked change has occurred. It would not take a great stretch of the imagination to see that, if observations were frequent and delicate enough, the character of this curve could be logarithmic, as demanded by the Nernst formula in establishing a concentration series, and in producing new value of p, the osmotic pressure of the ions, brought about by bringing lead sulphate into solution more rapidly than it is precipitated from the same. The new conditions arrive at equilibrium in about one minute, and the change in electromotive force is about constant for the remaining nineteen minutes. Upon throwing the circuit open, the reverse action occurs. The concentrated acid diffuses and changes in ionic pressure, also return to something near the original value. Were the series purely a concentration series, the thickness of the denser strata could be calculated when the time of diffusion is one minute. The strata of the more dilute acid would then not exceed 0.1 mm., and assuming ionic changes, the thickness would be less. The point to which I_r recovers at the end of twenty minutes shows approximately the change in electromotive force due to change in P, or electrode substance, and to change in p, due to increased bivalent $\overset{\overset{++}{++}}{\text{Pb}}$ ions. The influence of the lead sulphate upon the plate until after fifteen ampere hours have been discharged is small. We think the character of curve III. is probably more uniform than noted, and that the error arose from inaccurate adjustment of the current. The thickness of a layer of lead required for fifteen ampere hours' discharge in this cell is 0.01 mm. The sulphate deposited would naturally be thicker, but that is more than doubly compensated by the porosity of the plate. Whatever the thickness or the thinness of the sulphate, it apparently begins, in this case, to exert its influence upon the solution pressure of the peroxide after about fifteen ampere hours have been discharged; and as the deposit grows, to leave a more marked effect upon the later discharges. More or less diffusion of peroxide through the soft and thin layer of lead sulphate might take place. The deposit need not be perfectly uniform. It shows great porosity, is filled with dilute acid, all combining to aid the peroxide in coming in contact with the solution. It is a well-known fact that mixed electrodes act as electrodes of a single metal as long as that metal is present in considerable mass and has a larger solution pressure than the other elements. Such, for example, is the zinc-mercury amalgam electrode used in the Clarke cell. We are in all cases concerned only with the electrode surface, not with the interior of the electrode. A strong current taken from a Clarke cell removes the surface zinc more rapidly than it can be supplied from within to restore equilibrium. But upon resting, the electromotive force regains its value. This process may be continued until a large part of the zinc is exhausted, and the cells still show the solution pressure of the zinc. The cell, aided by the zinc sulphate crystals, shows no change in *p*. Were these crystals to harden over the amalgam and prevent liquid contact, little could be expected of the cell. This is somewhat analogous to the lead cell.

The lead sulphate is incrusted upon the plate, is very thin, and is generally found well saturated with the peroxide. If the electromotive force at any instant is given by the PbO_2 in contact with the solution, and the number of ions in the solution, the drop would be greater when a coating begins to form over the peroxide. The current would use up the peroxide more rapidly than the fresh peroxide is able to come in contact with the solution. This effect would be magnified as the coating grew. During rest, the peroxide would still continue to diffuse as in the zinc amalgam electrode, the solution pressure would continue to rise, and when the cell is well discharged, would take hours to reach near the original value. The x on the curve sheet under discussion shows the value to which VIII_r has attained after seventeen hours.

The presence of the sulphate is also attested by the increased internal resistance, though its effect is much less than what would be expected of pure sulphate. But it must be remembered how thin the stratum of sulphate is, and that it is intermixed with peroxide, soft and thoroughly wet with dilute acid. Gladstone and Hibbard maintain that the pores are partly shut up by the sulphate during the last stages of the discharge, and the gases formed cannot escape, which causes an increase in internal resistance. To this, it must be said that gases are not occluded at an electric pressure as high as that required for evolution of the same. According to the second law of thermodynamics, gases could no more pass from a lower to a higher pressure of themselves than heat could pass of itself from a lower to a higher temperature. Occluded gases could not, therefore, reach the gaseous state during discharge, but must pass directly from the occluded to the liquid condition at the same electromotive force at which they were formed. No doubt H and O are, to a slight extent, present, and would behave as here noted, the process being a reversible one. The formation of a free gas, however, is irreversible, and belongs to a much higher electrical pressure. With black platinum electrodes, the former occurs at 1.07 volts, and the latter at 1.70 volts. What their respective magnitudes are in the lead cell cannot be definitely stated; but, for reason mentioned later, the irreversible evolution of H occurs at a much higher value, and it is probable that the occlusion point is also higher. The following consideration also refutes the claim of Gladstone and Hibbard. If the gases were the cause of increased resistance during discharge, the slow escape and reabsorption of the same during a period of rest would produce a diminution in the resistance of well-discharged cells. The opposite effect we have found to take place. The internal resistance actually rises and becomes nearly double after long periods of rest, in spite of the diffusion of the acid at the electrodes, which would lower slightly the resistance. This latter effect may be too small to be appreciable. This explanation offers itself to the writer: the sulphate, on standing, becomes more compact, and therefore would have very little acid distributed through it.

Ayrton has shown that the rate of consumption of the PbO_2 upon the plate is uniform. Gladstone and Hibbard, basing their conclusion upon Ayrton's work, maintain that it is shown conclusively that the fall in electromotive force during discharge cannot be due to the amount of PbO_2, our P_+ factor, upon the plate. With this conclusion the writer cannot agree. A constant decrease in PbO_2 at the electrode surface would mean a logarithmic decrease in the electromotive force of the cell, did not other factors before mentioned enter into consideration. A cell discharged continuously at constant current shows that the curve, debarring the initial drop, is more nearly a logarithmic one than any other through a wide range. The question, however, is not how much PbO_2 one may determine to be upon the plate, but how much is in actual contact with the solution. Gladstone and Hibbard carried on some valuable experiments upon the effects of the concentration of the acid upon the electromotive force of the cell. The concentration of the acid certainly must modify the *factors* which are *effective* in producing an electromotive force, and in so far the electromotive force may be said to depend upon the concentration of the acid. But to regard the acid as the cause of the electromotive force is a step too far. Wade, in his criticism of Gladstone and Hibbard's work, — and Barnett endorses his objection, — says concerning their theory, "While extremely plausible, it is hardly consistent with the whole facts of the case. If it were correct, the local weakening of the acid on discharge ought to be equally marked, provided the current in both cases is the same, and the accompanying electromotive force as much too low as the other is too high. Instead of this, it will be always found that, throughout the greater part of the discharge, the electromotive force is but slightly below what is required to correspond to the strength of the acid indicated by a hydrometer inserted in the main body of the liquid." This objection seems well founded from the standpoint of Gladstone and Hibbard, and a hydrometer may, when the cell is at rest and readings are taken with a condenser, indicate the electromotive force, but when the cell is in action it cannot do so. The question is, "whether the accompanying electromotive force is as much too low, upon discharge, as the other is too high." The charging electromotive force Gladstone and Hibbard recognize as being too high. Our discharge curve shows that inside one minute, values are reached far too low, and simple inspection of the curves for the charging process shows that the behavior of the lead cell during the charging process is analogous to the behavior during discharge. The various factors change in the inverse order to that already noted, and need not be further discussed.

The claim is frequently made that by-products such as H_2O_2 and $H_2(SO_4)_2$ are formed during the charging process. Such formations may be quite possible, but why regard them as by-products? When the cell is well charged, and the charging process still continues, the sulphate is all, or nearly all, used up, the acid at the electrode surface is nearly pure, and the electromotive force necessary to overcome the electromotive force of the cell must be very large. The products of ionization at the electrodes are $\overset{++}{Pb}$ and $\overset{--}{SO_4}$, $\overset{++}{H_2}$ and $\overset{--}{SO_4}$, and $\overset{+}{H}$ and $\overset{-}{OH}$. The lead ions present are no longer supplied sufficiently rapidly to carry the current; the $\overset{+}{\mathrm{H}}$ ions corresponding to the $\overset{-}{\mathrm{O}}\mathrm{H}$ are always small; and the rapid removal of the principal positive ion, viz.: $\overset{+}{\mathrm{H}}$, means a supersaturation of $\overset{--}{\mathrm{SO}}_4$ and $\overset{-}{\mathrm{O}}\mathrm{H}$ ions at the electrodes. This condition cannot be maintained, and it is avoided if we assume the immediate and direct formation of $SO_4 - H - H - SO_4$ and of $OH - HO$. As a forced condition, the electromotive force necessary for the formation would be above the ordinary. The electrode products are liberated, and the process becomes irreversible. The gases so liberated are not formed at the same electromotive force at which gases are occluded. The latter is a reversible process, the former generally is not, and cannot be, under conditions existing in the ordinary lead cell. The energy consumed is largely lost. It only succeeds in tearing up the plate, *i.e.* in making it more porous, and in bringing a little more lead in contact with the liquid. This, of course, slightly increases the output of the cell. That such products as $H_2(SO_4)_2$ and H_2O_2 should break up under rapid evolution of heat seems quite plausible. However, it does not explain the heating at the electrodes, for we know that the products must have been formed under absorption of an equivalent amount of heat energy; and if, therefore, no such product had been formed at all, the amount of heat generated would have been exactly the same as if all of the products had broken up, instead of a small quantity remaining in the solution. Either case would lead us back to the v. Helmholtz equation for an explanation of the heat developed. We must accordingly assume a large divergence in the value of the "heat-toning" and of the electrical energy at this electromotive force.

The data and curves for terminal potential are not given. The internal resistance for curve I., 20-ampere discharge and recovery, was 0.006 ohm. At the end of one minute this value fell to 0.0048 ohm, which value it retained until curve III. is reached, when it again increases and reaches at the end of run VIII. 0.0072 ohm. After standing seventeen hours the resistance of this cell had increased to 0.01 ohm. A few readings were then taken at intervals of five to ten seconds, while we were adjusting the external resistance to give a 10-ampere current. The sixth reading showed an internal resistance of 0.0065 ohm. The intermediate readings show no great regularity, as neither time nor current was observed with accuracy. Periods of rest also existed between the few seconds' run. We had taken the readings chiefly to see if the galvanometer was working rightly, and the readings were accidentally preserved, and what had occurred dawned upon the writer after he had no further opportunity to repeat the observations. In every case, however, where the period of rest from discharge is longer than common, the internal resistance increases.

Now, if a small current, say of one or perhaps even five amperes, was taken from the cell after a period of rest, the drop in total electromotive force would be small for a short time, and the terminal potential readings would go up, owing to decreased internal resistance. This is perhaps what was really noted by those who state that the electromotive force rises when a current is first taken from a cell.

The internal resistance was generally smaller for strong currents, but the difference may be within the limits of experimental error. The lowest internal resistance noted was 0.0032 ohm, and was observed at the maximum of curve II., 15-ampere charge. This may probably be due to the increase in the concentration of the acid formed at the electrode and diminution of lead sulphate. At the maximum of curve IV. the terminal potential indicated an internal resistance of 0.0081 ohm. This increase in resistance may safely be attributed to liberated gases, and to the *very* concentrated acid. These wide variations need farther investigation.

Every curve incloses an area representing so much lost work, which, in the very nature of the cell, it is impossible to obviate. Both theory and observation show that these areas will be smaller with small currents, other conditions remaining the same. Curves IV. and IV$_r$., 15-ampere charge, embrace an area three times as large as the C^2R loss within the cell. This area, figured out in heat units, represents about 23 gram-calories, which would not cause a very appreciable rise in temperature, as the cell consisted of 92.4 Kg. of lead, and 61.6 Kg. of solution and glass. The other curves more generally show even smaller magnitude, and indicate the difficulty there would be encountered in trying to determine this loss in heat units. We are further at a loss to know how many heat units to expect; for the cell can work at only *one* electromotive force, according to the v. Helmholtz equation, at which there will be no absorption nor abstraction of heat. This electromotive force is the normal electromotive force of the cell if, as before mentioned, the temperature coefficient is zero. Of this we are not certain, and we shall therefore designate it by E_0. The normal electromotive force of the cells in question certainly lies somewhere between 2.0 volts and 2.1 volts. To charge above E_0 or discharge below it would, according to the equation, produce heat. The greater the divergence of the electromotive force from E_0, the greater the quantity of heat absorbed or generated. It is probably much easier to hold the charging electromotive force far above this value for a considerable period than to hold the discharging current below it, and hence the heating effects would be more easily noted in the former process. This difference could be made more apparent if E_0 were below the normal electromotive force. We could then have cooling during certain stages of the discharge of a cell. The inclosed area could only represent developed heat, if all or the great part lies below the horizontal line for E_0. This leaves the explanation of the disappearance of the lost energy in practically the same condition in which the author found it. Numerous attempts have been made to get some light upon the problem by determination of the temperature coefficient at widely different electromotive forces. All efforts were baffled, and it seems almost impossible to maintain sufficient constancy in the cell for such a determination.

It is commonly observed that much energy is wasted in discharging the cell below about 1.7 volts. It is equally noticeable that there is a corresponding loss when the charging is carried above 2.25 volts. Practically it is questionable whether one should charge at all with a 15-ampere current, unless time be a grave consideration.

When large currents, like 40 amperes or more, are taken from a cell, it is quite impossible to get a high efficiency, owing to the establishing of a concentration series and the large changes which p and P undergo. These are inherent difficulties too little recognized in the electric storage car problem.

University of Nebraska, November, 1896.

THE INFLUENCE OF A STATIC CHARGE OF ELECTRICITY UPON THE SURFACE TENSION OF WATER.

By Edward L. Nichols and John Anson Clark.

THE influence of a static charge upon the form and behavior of a water jet is well known. The phenomena formed a favorite subject among the earlier electricians.[1] It has commonly been ascribed to a change in the surface tension of the liquid. J. W. Draper described some experiments showing the influence of electrification upon capillary phenomena,[2] in 1845. In 1861 Tate proposed to use the rate of discharge of a siphon to measure the degree of electrification of the vessel with which it was connected.[3] In 1870 Van der Mensbrügghe published an account of his interesting experiments upon electrified liquid films, in verification of his views concerning surface tension.[4] So far as the present writers know, however, no direct measurements touching this point have been published.

It is the object of the investigation described in this paper to determine quantitatively the influence of an electric charge upon the surface tension of water. The method of determining surface tension consisted in weighing drops. The water dropper, the general construction of which is shown in Fig. 1, consisted of a burette of 100 c.c. capacity, which served as a reservoir for the liquid. This was provided with an orifice made from a piece of capillary tubing about 6 cm. long. One end of this was melted,

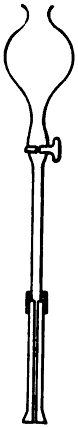

Fig. 1.

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[1] Nollet, Recherches sur les causes des phénomènes électriques (1749), p. 327; Beccaria, Dell' ellettricismo artificiale (1772); Carmoy, Journal de physique (1788); Singer, Elements of Electricity and of Electrochemistry (1814); Faraday, Experimental Researches (Art. 1571).

[2] Philosophical Magazine (3), Vol. 26, p. 185.

[3] Philosophical Magazine (4), Vol. 21, p. 452.

[4] Poggendorff's Annalen, Vol. 141, p. 287.

and was then "upset" until its diameter was increased to about 1 cm. The upset portion was then ground off so as to form a plane surface at right angles to the axis of the tube, and this surface was ground at the edges until a circular disk was obtained. The water after passing through the capillary tube floated out upon the ground-glass surface, where it remained until a sufficient amount had gathered to form a drop. Figure 2 shows this orifice with the drop clinging to it. With this arrangement, which was the result of many trials with other forms of water dropper, the drops were found to be very nearly of the same weight.

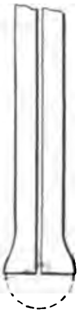

Fig. 2.

In order to determine the degree of accuracy with which surface tension could be measured with this apparatus, 16 series, consisting of 20 drops each, were collected and weighed successively. The results are given in Table I.

Table I.

DATA FOR THE CALIBRATION OF THE DROPPER.

Number of drops.	Weight of beaker and water.	Weight of beaker.	Weight of the water.	Deviation from the mean.
20	28.5951	25.8230	2.7721	+0.0015
20	28.5935	25.8230	2.7705	−0.0001
20	28.5942	25.8230	2.7712	+0.0006
20	28.5906	25.8230	2.7676	−0.0030
20	28.5924	25.8230	2.7694	−0.0012
20	28.5961	25.8230	2.7731	+0.0025
20	28.5912	25.8230	2.7782	+0.0076
20	28.5944	25.8230	2.7714	+0.0008
20	28.5943	25.8230	2.7713	+0.0007
20	28.5926	25.8230	2.7696	−0.0010
20	28.5933	25.8230	2.7703	−0.0003
20	28.5864	25.8230	2.7574	−0.0132
20	28.5935	25.8230	2.7705	−0.0001
20	28.5912	25.8230	2.7782	+0.0076
20	28.5932	25.8230	2.7702	−0.0004
20	28.5925	25.8230	2.7695	−0.0011
Mean			2.7706 g.	0.0026

It will be seen that the mean value of the weight of 20 drops is 2.7706 g., and that the average deviation from the mean is 0.0026 g., which is less than the tenth of one per cent.

Since the surface tension of water is a function of the temperature, it was necessary to reduce all weighings to the same temperature, for which purpose the temperature of each set of drops had to be determined. For this purpose a thermo-couple, consisting of a piece of No. 20 German silver wire and of copper wire of the same size, was used. One junction was placed in a glass tube about 12 cm. long and 1 cm. in diameter, which was closed at one end. The junction within the tube was surrounded with molten sulphur, which was allowed to cool. This tube was then suspended by an insulating rod in a glass beaker, which was also insulated. The beaker was filled with distilled water, which was kept boiling slowly by means of a Bunsen burner. The other junction was placed in such a position that just as each drop left the orifice, it completely surrounded the junction. In this way the temperatures of the drops were measured at the instant when they were detached from the orifice, a method which was found to give a much better result than any attempt to measure the temperature of the liquid after it had been collected for weighing.

The wires for this thermo-couple were supported by insulating glass rods, which carried them to a sensitive galvanometer with high resistance coils. The galvanometer itself was likewise well insulated. It was necessary to insulate this entire circuit, including the instrument, because the thermo-element was brought into contact with the charged water at the instant of dropping. The galvanometer needle was so heavy that the static charge had no appreciable effect upon its movements. By means of this arrangement, it was possible to read the temperature of the drops accurately to tenths of a degree. In order to open and close the electric circuit without discharging the system of which it formed a part, a porcelain cup of mercury was placed upon an insulating stand, and one terminal of the galvanometer was inserted permanently into the mercury. The other terminal was fastened to a copper disk furnished with a vulcanite rod 25 cm. long which served as a handle. This disk was allowed to float upon the surface of the mercury. By raising it the circuit was broken, and the swing of the galvanometer needle was observed.

The thermo-element was calibrated in the usual manner by placing the variable junction in a water bath, the temperature of which was determined by means of a thermometer reading to tenths of a degree. The successive readings were plotted, and a curve of calibration was obtained.

By means of the arrangement just described, the weight of a drop of water was determined as a function of the temperature for the range of temperatures through which it was necessary to work. The data for this temperature correction are given in Table II., and the results are shown graphically in Fig. 3.

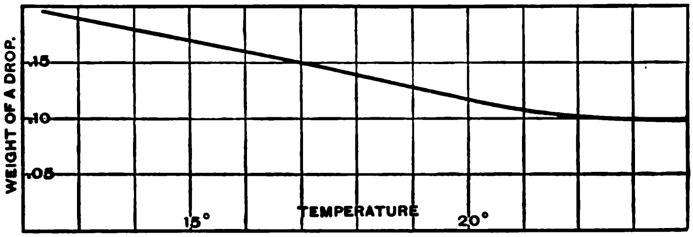

Fig. 3.

Table II.

DATA FOR THE TEMPERATURE CORRECTION CURVE OF THE DROPPER.

Galvanometer throw.	Temperature.	Weight of 20 drops.	Weight of 1 drop.
31.1	17.8	2.801	0.140
31.0	18.0	2.782	0.139
29.0	22.3	2.001	0.100
28.6	23.1	1.982	0.099
28.2	24.0	1.963	0.098
28.2	24.0	1.962	0.098
28.3	23.7	1.970	0.098
29.2	22.0	2.103	0.105
29.7	20.8	2.311	0.115
30.3	19.4	2.632	0.131
30.8	18.3	2.705	0.135
31.4	17.1	3.012	0.156
32.3	15.2	3.281	0.164
33.6	12.4	3.905	0.195

In order to obtain a more constant potential than could be secured with the Holtz machine already mentioned, a battery of 24 Leyden jars was placed in the static circuit. By the use of these jars, and by carefully regulating the speed of the motor, it was possible to obtain a fairly constant potential for considerable intervals of time. The potential was varied by varying the speed of the motor. The Holtz machine was placed in an adjoining room in order to avoid, as far as possible, disturbances due to induced charges.

For the measurement of the potential, an electrometer was devised in which the repellent forces between a charged ball and a vertical plate were balanced against the force of gravity. This principle, which is very old, having been employed by Henley in his quadrant electroscope in the last century, was applied in such a manner as to make it possible to compute potential in absolute measure with fair approximation. In its essential features, this instrument consisted of a metal ball suspended by means of a long wire, in such a manner that its center coincided with the axis of a vertical disk of metal. To obtain a suitable ball for the range of potential which it was desired to measure, large leaden shot were heavily plated with copper. The lead was then removed by fusion in the luminous flame of a Bunsen burner, leaving behind a hollow sphere of copper. This sphere was reduced in weight by further suspension in an electrolytic bath, in which the sphere was the anode. In this way several light and fairly accurate metal spheres were obtained. The suspension wire was of German silver No. 41, which had been straightened by heating it to a dull red under tension.

The dimensions of the electrometer were as follows:

Length of the suspension wire	73.750 cm.
Radius of disk	15.750 cm.
Mass of repelled ball	0.142 g.
Radius of the ball	0.436 cm.
The value of the force due to gravity was taken as	980.380 dynes.

The method of using this instrument consisted in connecting the plate and the ball, the latter by means of its suspension wire, with the charged system of which the water dropper forms a part.

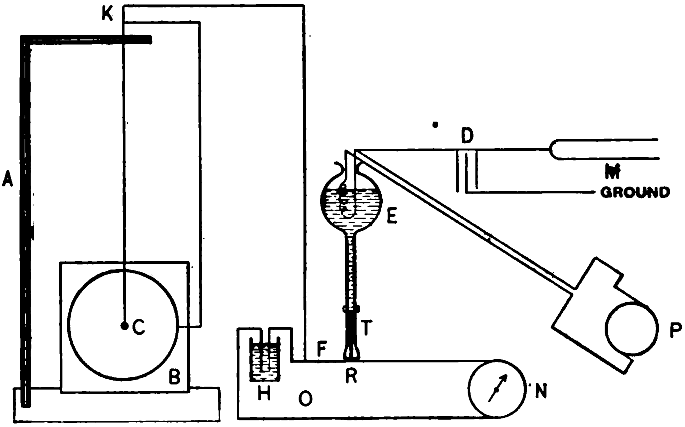

Fig. 4.

Figure 4 is a diagram showing the electric connection between the machine, the battery of Leyden jars, the electrometer, and the water dropper. In that diagram

A is the glass insulating rod;
B is the glass plate;
C is the metallic disk with ball at its center;
D is the condenser;
E is the German silver coil for heating water;
F is the point of connection of the electrometer;
G is the ground circuit;
H is the vessel of constant temperature;
K is the point of support of the electrometer ball;
M is the terminal of the Holtz machine;
N is the galvanometer;
O is the constant junction of the thermo-couple;
P is the dynamo;
R is the dropping orifice with the variable junction of the thermo-couple beneath it;
T is the reservoir and stem.

When charged, the sphere is repelled from the disk and assumes a position of equilibrium in which electric repulsion is balanced by the repellent force due to gravity. In the computation of the potential as measured by the deflection of the ball, the change of surface tension at the edges of the disk was neglected. The method of computation was as follows:

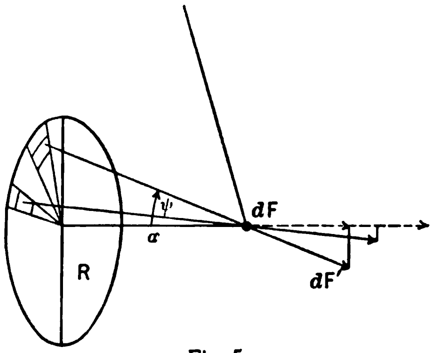

Fig. 5.

In Fig. 5 let

R = radius of the disk;
ρ = radius of the sphere;
a = distance of the sphere from the plate;
σ = surface density of charge;
r = variable distance of an element of area from the center of the disk;
e = variable distance of an element of area from the sphere;
L = length of the suspension wire;
$rd\theta \cdot dr$ = element of area;
M = mass of the sphere;
ϕ = the angle of deflection;
V = the potential;
q = the charge on the sphere;
ψ = angle between a and 1;
dF' = force due to charge on element of area;
dF = horizontal component of force;
F = force required.

We have then:

$$dF' = q\,\frac{d^2}{e^2} = \frac{q \cdot \sigma r d\theta dr}{e^2};$$

and

$$dF = dF' \cos\psi = \frac{q\sigma \cdot rd\theta \cdot dr}{e^2} \cdot \cos\psi. \qquad (1)$$

Now the quantity $rd\theta \cdot dr \cdot \cos\psi$ is the element of area projected on a plane normal to the line drawn from the sphere to the element; and this projected area, when small, divided by the square of the distance between the sphere and the element is, by definition, the solid angle at the center of the sphere subtended by the element of area. Let the solid angle be denoted by ω. Then (1) becomes

$$dF = q\sigma d\omega. \tag{2}$$

$$\therefore\ F = q\sigma\omega, \tag{3}$$

where ω is the solid angle at the center of the sphere subtended by the whole disk.

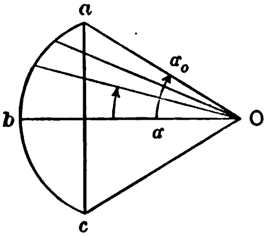

Fig. 6.

To determine ω, let ac (Fig. 6) be a section of the disk and abc a section of the spherical surface with O as a center. Take as element of area an annulus on the spherical surface equal to $2\pi r \sin\sigma \cdot r d\sigma$

$$d\omega = 2 \sin \sigma d\sigma\,;$$

$$\omega = 2\pi \sin d.$$

The capacity of a sphere is equal to its radius:

$$\therefore\ C = \frac{q}{V} = \rho \text{ and } q = \rho V.$$

The capacity of a circular plate is $\dfrac{2R}{\pi}$:

$$\therefore\ C' = \frac{I}{V} = \frac{\pi R^2 \sigma}{V} = \frac{2R}{\pi},$$

and $$\sigma = \frac{2V}{\pi^2 R}.$$

The return force due to gravity is $Mg \sin\phi$, or $Mg\dfrac{a}{L}$. Making this substitution in (3), we have

$$mg\frac{a}{L} = \rho V \cdot \frac{2V}{\pi^2 R} \cdot 2\pi\left[1 - \frac{a}{\sqrt{a^2+R^2}}\right];$$

$$V^2 = mg\frac{a}{L} \cdot \frac{\pi R}{4\rho} \Big/ 1 - \frac{a}{\sqrt{a^2+R^2}} = \frac{\pi mgaR\sqrt{a^2+R^2}}{4\rho L[\sqrt{a^2+R^2} - a]}$$

$$= K\frac{a\sqrt{a^2+R^2}}{[\sqrt{a^2+R^2} - a]}.$$

In this instrument $L = 73.7500$ cm.;
$R = 15.7500$ cm.;
$M = 0.1420$ grams;
$\rho = 0.4360$ cm.;
$g = 980.3800$ dynes;
$V^2 = 53.5540$.

The method of reading the deflection of the electrometer was as follows: A telescope, within the eyepiece of which a ruling on glass consisting of fifty equidistant lines had been mounted, was set up at a distance of about 500 cm. The telescope was adjusted so that the zero line of the scale coincided with the plane of the disk of the electrometer. The field of the telescope was lighted by two large fish-tail burners placed behind the electrometer, so that the fine wire, by means of which the ball was suspended, was seen projected upon a bright background. The definition was such that it was easily possible to estimate tenths of a scale division. A calibration of the eyepiece by means of a standard scale showed each division to be equivalent to 0.2127 cm. The telescope for reading the galvanometer and that for reading the electrometer were so placed with reference to each other as to enable the observer to make alternate readings by merely turning his head.

When the method of experimenting above described was put into operation, a number of interesting phenomena were observed. The first noticeable effect of charging the water was the change in the rate of dropping. Even with a very small charge this increase was easily noticeable. As the charge was increased, the size and shape of the drop could be seen to vary. The accumulated water on the disk of the dropper gradually became cone-shaped; and when the charge was further increased, this cone-shaped body of water broke up into three and sometimes four similar cones, from the apex of each of which issued a fine stream of water. These little cones stood out from each other like so many similarly charged gold leaves. While they generally retained the same relative positions to one another, they did not always remain the same with reference to any particular point on the disk. At times they were all found to be revolving around the central orifice of the disk, while at others they had a harmonic motion of slow period.

The mode of procedure in making measurements was as follows:

(1) The temperature was read.
(2) The voltage was read.
(3) A series of drops was collected.
(4) The voltage was read again.
(5) The temperature was read again.
(6) The water was weighed.

The voltage was then changed, and readings were taken again in the same manner. Readings were taken nearly every day for three months. The data obtained were collated, and the results were used in plotting a curve, with the relation between the weight of a single drop as ordinate, and the deflection of the repelled ball of the electrometer as abscissæ. The data from which this curve was plotted are given in Table III.

TABLE III.

Scale divisions of electrometer readings.	Weight of 1 drop corrected to 20 degrees, in grams.	Scale divisions of electrometer readings.	Weight of 1 drop corrected to 20 degrees, in grams.
0	0.120	35	0.113
20	0.119	36	0.111
21	0.119	37	0.109
22	0.118	38	0.106
23	0.118	39	0.104
24	0.117	40	0.102
25	0.117	41	0.100
26	0.116	42	0.099
27	0.116	43	0.097
28	0.116	44	0.095
29	0.115	45	0.093
30	0.115	46	0.090
31	0.115	47	0.088
32	0.115	48	0.085
33	0.114	49	0.080
34	0.114	50	0.075

Since it was the object of this investigation to determine the *variation* of surface tension with electrification, no attempt to measure the surface tension itself was made. Numerous such measurements exist, and the average of the following values taken from a table published in *The Philosophical Magazine* has been

TABLE IV.

TABLE OF VALUES OF SURFACE TENSION.

Tension at 0°.	Observer.	Tension at 0°.	Observer.
77.4	Artur.	77.1	Jaeger.
75.2	Brunner.	73.7	Magie.
75.2	Desain.	73.3	Quincke.
75.0	Frankenheim.	74.3	Rodenback.
75.4	Frankenheim.	79.7	Sondhauss.
75.6	Frankenheim and Sondhauss.	73.6	Volkmann.
75.4	Gay-Lussac.	75.8	Volkmann.
75.2	Gay-Lussac.	76.5	Wolf.
74.5	Hagen.	77.3	Wolf.

Average, 75.4.

used as a reference value. The values given in the table are for 0°. By applying Brunner's formula[1] for the variation of surface tension with temperature to the mean of the values given in the table, we obtain 72.5 as the value of the surface tension of water at the temperature of 20°. This is the temperature to which our readings were all reduced by means of the curve in Fig. 3. By making use of the readings for voltage as deduced from the deflection of the

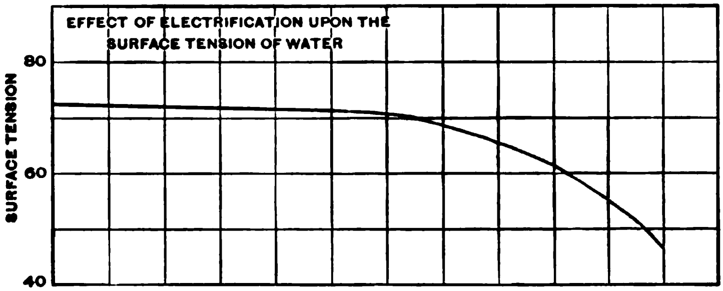

Fig. 7.

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[1] Brunner, Poggendorff's Annalen, 70, p. 481.

electrometer, it is possible to plot a curve which shows the relation between the surface tension and the voltage to which the water was subjected. From this curve, which is given in Fig. 7, and from Table V., it will be seen that the surface tension diminishes slowly, but at an increasing rate with rise of potential, and that at 11,000 volts it has fallen off from 72.5 to about 46.6 in value.

TABLE V.

SURFACE TENSION AND ELECTRIFICATION.

Volts.	Surface tension.	Volts.	Surface tension.
0	72.50	8,000	66.75
2000	72.00	10,000	54.50
4000	71.80	11,000	46.60
6000	71.25		

Having reached this result, it was thought of interest to determine whether the change of surface tension with electrification was dependent upon the sign of the charge. For this purpose, one terminal of the Holtz machine was grounded, and the other terminal, which was found by testing with the proof plane to be positive, was attached to the apparatus. After charging, the machine was stopped, and a series of drops were gathered and weighed. The temperature was noted, and the weighings collated as described above. This process was repeated 11 times. The terminals of the machine were then reversed, and the voltage was brought to the same point as before, and the measurements were repeated with the same range of potential as before. The results obtained with positive and negative charges are given in Table VI.

TABLE VI.

DATA.

Weight of one drop corrected to 20 degrees. Average voltage 7000.

Positive.	Negative.	Positive.	Negative.
0.115	0.114	0.115	0.115
0.115	0.115	0.115	0.115
0.115	0.115	0.115	0.115
0.114	0.115	0.114	0.115
0.115	0.115	0.115	0.115
0.115	0.114		

It will be seen that there is no appreciable difference in the effects of the positive and of the negative charge.

From the foregoing experiments, the following conclusions may be drawn:

(1) The surface tension of water is decreased by giving it a static charge of electricity.

(2) The law of decrease is independent of the nature of the charge. Although no quantitive measurements were made upon other liquids, some preliminary determinations lead the writers to believe that a charge of static electricity exerts a like influence upon the surface tension of other liquids.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
June, 1896.

ON THE MECHANICAL CONCEPTIONS OF ELECTRICITY AND MAGNETISM.[1]

BY W. S. FRANKLIN.

THE following development is based upon the conceptions of Maxwell. It follows, in some particulars, the work of Lodge. The conceptions here developed, of the dependence of electromotive force upon changing magnetic flux, of the dependence of magnetomotive force upon changing electric flux, of the production of electromotive force in a moving wire, of the energy stream, of electromagnetic waves, and of the Hertz vibrator, do not seem to have been pointed out before, except, perhaps, in a general way, as, for example, by Boltzmann, who discusses[2] the mechanical behavior of a medium of which the equations of motion are the electromagnetic equations of Maxwell. The kinematics and dynamics of such a medium can be fairly stated only with the help of differential equations. The conceptions here developed are, of course, approximations. Some inconsistencies arise in the extension of these conceptions to three dimensions, as has been noticed by Poynting.

1. *Fundamental Conception.* — The ether is to be considered as built up of very small cells of two kinds, *positive* and *negative*, in such a way that only unlike cells are in contact. These cells are imagined to be so connected, where they are in contact, that if a cell be turned while the adjacent cells are kept stationary, then a torque, due to elastic reaction of adjoining cells, is brought to bear upon the turned cell, which tends to right it, and which is proportional to the angle turned.

2. *Conception of the Magnetic Field.* — The ether cells at a point in a magnetic field are to be thought of as rotating about axes

[1] A paper read before the Buffalo meeting of the American Association. A portion of this paper which appeared in Vol. II., Nichols and Franklin's Elements of Physics, is given here again for the sake of completeness.

[2] Wied. Ann.

which are parallel to the direction of the field at the point, the angular velocity of the cells being proportional to the intensity of the field at the point. The positive cells rotate in the direction in which a right-handed screw would be turned that it might move in the direction of the field, and the negative cells rotate in the opposite direction. This opposite rotation of positive and negative cells is mechanically possible since only unlike cells are in contact.

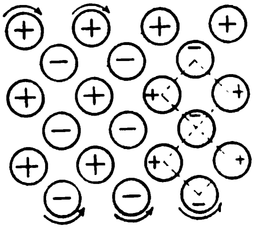
Fig. 1.

This rotatory motion of the ether cells is represented in Fig. 1. The magnetic field is perpendicular to the plane of the paper, and directed away from the reader; all the positive cells rotate clockwise, and the negative cells counter-clockwise. The kinetic energy per unit volume in such a system of rotating cells is proportional to the square of the angular velocity, which is consistent with the fact that the energy (kinetic) per unit volume in a magnetic field is proportional to the square of the intensity of the field.

3. *Conception of the Electric Field.* — The positive ether cells at a point in an electric field are to be thought of as displaced in the direction of the field, while the negative cells are displaced in the opposite direction; this displacement being proportional to the field intensity. Thus Fig. 2 represents the case in which the positive cells have been displaced towards the bottom of the page relatively to the negative cells. Figure 3 represents two meshes. The downward displacement of the positive cells has distorted these meshes which are normally square. Since this cell structure of the ether is elastic, as explained in Art. 1, its distortion, as represented in Figs. 2 and 3, represents potential energy. The amount of potential energy per unit volume is pro-

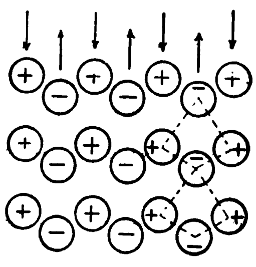
Fig. 2.

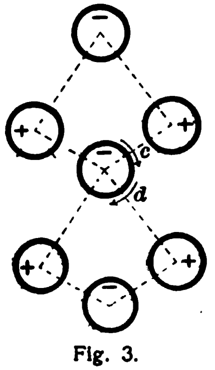

Fig. 3.

portional to the square of the displacement. This is consistent with the fact that the energy (potential) per unit volume in an electric field is proportional to the square of the field intensity.

Remark. — The two positive cells to the right of the middle cell in Fig. 3 being displaced downwards, may be conceived to exert torques upon the middle cell as shown by the arrows c and d; which torques are proportional to the intensity of the electric field, *i.e.* to the displacements of the cells. The cells to the left exert equal but opposite torques upon the middle cell.

4. *The Dependence of Electromotive Force upon Changing Magnetic Flux; Explanation of Induced Electromotive Force.* — The electromotive force around a closed curve is proportional to the rate of change of the magnetic flux through the curve.

Consider a closed boundary $ABCD$ (Fig. 4) in an electric field parallel to DA and CB. Let f be the intensity of the field along CB, and f' the intensity along DA, and let l be the length of DA and of CB. The electromotive force around the boundary $ABCD$ is $lf' - lf$. Now, the total torque acting across BC on the enclosed cells is proportional to l and to f, and the total torque acting across AD is proportional to l and to f', but opposite in direction. Therefore the total torque acting to turn the enclosed cells is proportional to $lf' - lf$. The enclosed cells gain angular velocity under the action of this torque at a rate which is directly proportional to the torque, and inversely proportional to the number of cells which participate, so that the *product* of number of cells (area of $ABCD$) into angular acceleration (rate of change of magnetic field) is proportional to the torque or to the electromotive force around $ABCD$, and this product, area of $ABCD$ into rate of change of magnetic field, is equal to the rate of change of magnetic flux through $ABCD$.

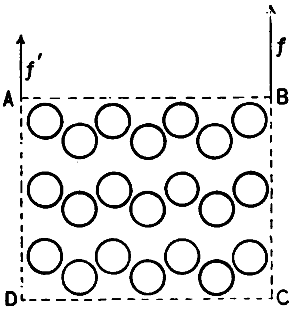

Fig. 4.

5. *The Energy Stream in the Electromagnetic Field; Preliminary Statement.* — Consider three gear wheels A, B, C (Fig. 5). Let A and C exert equal and opposite torque actions upon B.

Then, if the wheels are turning, work will be transmitted from *A* to *C*, or from *C* to *A*, according to direction of turning and to direction of torque action, and the rate of transmission of work will be proportional to the product of torque action into speed.

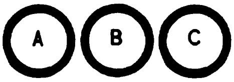

Fig. 5.

Imagine the cells in Fig. 2 to be rotating, positive cells in one direction, negative in the other, about axes perpendicular to the paper. This constitutes a magnetic field perpendicular to the electric field, which is towards the bottom of the page. On account of the torque actions between the cells, as explained in Art. 3, energy will be transferred to the right (or left) by each chain of geared cells at a rate which is proportional to the product of the intensity of the magnetic field into the intensity of the electric field, and the energy per second flowing across an area perpendicular to both electric field and magnetic field is proportional to the product of the respective field intensities into the area; for this area is proportional to the number of rows of cells which are acting as chains of gear wheels. The ***energy stream***, that is, ***energy per unit area per second***, is therefore proportional to the product of magnetic and electric field intensities, and is at right angles to both. In case the electric and magnetic fields are not orthogonal, the energy stream is proportional to the vector part of this product.

6. *The Electric Current.* — Consider a wire *AB* (Fig. 6) along which an electric current is flowing. The magnetic field on opposite sides of *AB* is in opposite directions, so that positive ether cells at *p* and *p'* are rotating in opposite directions, as shown. Since an electric current may be maintained for an indefinite time, this opposite rotation of positive ether cells on the two sides of *AB* cannot be due to an ever-increasing ether distortion (the cells are geared together, as it were), but there must be a *slip* between adjacent cells somewhere between *p* and *p'*. This *slip* between

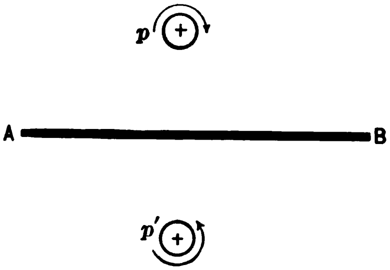

Fig. 6.

adjacent ether cells (positive and negative cells) takes place in the material of the wire, and constitutes the electric current.

7. *Established Electric Currents flow in Closed Circuits.* — Let *AB* (Fig. 7) be a wire carrying an established electric current. If this wire does not form a closed circuit, the opposite rotations of like ether cells on opposite sides of *AB* cannot continue without adjacent cells slipping on each other somewhere along any line passing around the end of *AB*.[1] That is, established *lines of slip* of the ether cells are necessarily closed lines. When a current does flow in a circuit which is not closed, an increasing ether distortion (electric field) is produced around the end portions of the circuit, which distortion produces (constitutes) electric charge there. Compare Article 12 below.

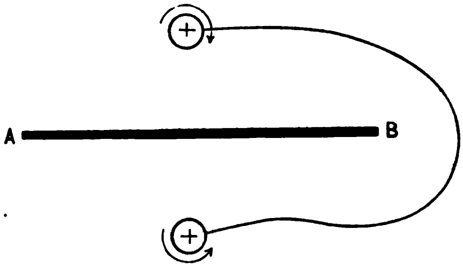

Fig. 7.

8. *Flow of Energy in the Neighborhood of an Electric Current.* — Let Fig. 8 represent the neighborhood of a long wire *AB* through which electric current is flowing. The electric field in the neighborhood is parallel to the wire, and the magnetic field circles round the wire. The product of magnetic field intensity into electric field intensity is the energy stream, and this is directed towards the wire from all sides. This energy streaming in upon the wire changes into the heat which appears in the wire. In case the wire is of high resistance, the electric field (volts per centimeter) is of great intensity, and, for the same current and same intensity of magnetic field,

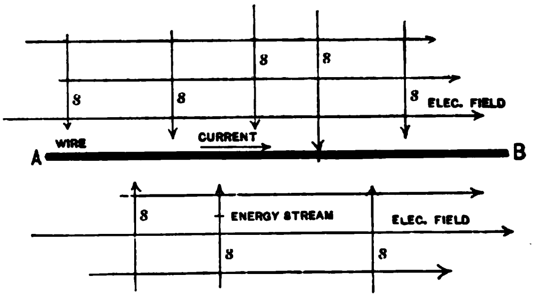

Fig. 8.

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[1] The figure is thought to represent a flow of current in two dimensions.

the energy stream is correspondingly intense, making the wire very hot. At points not very near to the wire the electric field is more nearly perpendicular to the wire, especially near the battery or dynamo which is maintaining the current. The energy therefore streams out from the battery or dynamo through the whole region *surrounding* the wires, and the energy stream turns in upon the wire throughout its length.[1]

9. *The Charge on a Condenser and its Disappearance when the Condenser Plates are connected by a Wire; Preliminary Statement.*

— Consider a row of gear wheels A to B (Fig. 9). If the wheel B is held stationary while A is turned in the direction of the arrow, the wheels will arrange themselves as shown in Fig. 10, alternate wheels being displaced upwards, and the intermediate wheels being displaced downwards. Conversely, a number of geared wheels, which by elastic action of any kind tend to stand in a straight row,[2] will be *relieved* from such a distortion as is represented in Fig. 10, by allowing freedom of rotation, or *permitting slip anywhere across the row.*

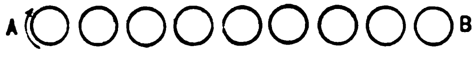

Fig. 9.

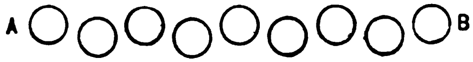

Fig. 10.

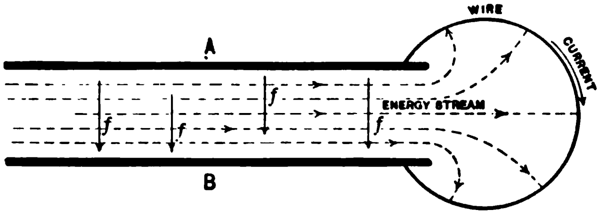

Fig. 11.

Let A and B (Fig. 11) be the plates of a charged condenser. The electric field f between these plates consists of upward displacement of positive ether cells and downward displacement of negative ether cells.

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[1] See J. H. Poynting, Phil. Trans.

[2] The rows of positive and negative ether cells are thought of as standing in a zigzag line, as shown by the horizontal rows in Figs. 1 and 2. In order that the diagrams may be simpler, the rows of positive and negative cells are hereafter to be thought of as straight (or uniformly curved) when free from distortion.

The horizontal rows of cells are distorted as shown in Fig. 10, and if a wire is connected across from *A* to *B* anywhere, this will constitute a line of slip permitting the adjoining ether cells to rotate, and thus entirely relieving the distortion which constitutes the field between the plates. The potential energy of the ether distortion will be transmitted along the rows of geared cells, in the direction of the dotted arrows, to the wire, where it will appear as heat.

An electric spark is a line of slip produced by the breaking down of the mechanism which sustains the electric stress, and the electric energy flows in upon a spark as it does upon a wire carrying current.

The explanation here given of the entire relief of the electric stress between two plates by the establishment of a conducting line (line of slip) between them applies to two adjacent oppositely charged bodies of any shape.

10. *The Dependence of Magnetomotive Force upon Changing Electric Flux.* — The magnetomotive force around any closed curve is proportional to the rate of change of the electric flux through the curve.

Consider a distorted chain of cells *AB* (Fig. 12). The rate at which this distortion is increasing is proportional to the difference in the angular velocities of the cells (intensities *f* and *f'* of the magnetic field) at *A* and *B* divided by the number of cells in the chain (distance *m* from *A* to *B*). Therefore, the product of the rate of increase of this distortion into the area *lm* of *cdeg* (rate of increase of the electric flux through *cdeg*) is proportional to *lf* — *lf'*, which is the magnetomotive force around *cdeg*.

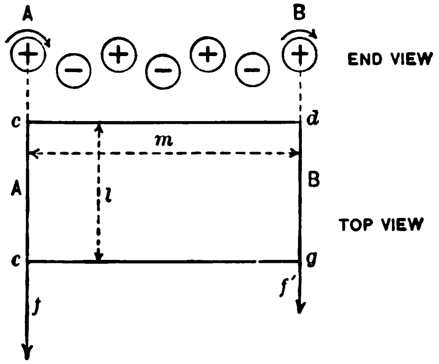

Fig. 12.

11. *Production of Electric Field in the Neighborhood of a Conductor moving across a Magnetic Field; Induced Electromotive Force in a Moving Wire.* — Consider a conductor A (Fig. 13), which is moving at velocity v across a magnetic field of intensity f perpendicular to the paper and towards the reader. This body being a conductor, the ether inside of it cannot sustain electric stress, so that the energy of the magnetic field, which is seated in the region immediately in front of A, cannot be geared across A, but an electric field is called into existence about the ends of A as indicated by the lines of force, and the magnetic energy is geared around A as indicated by the lines marked *energy stream*.

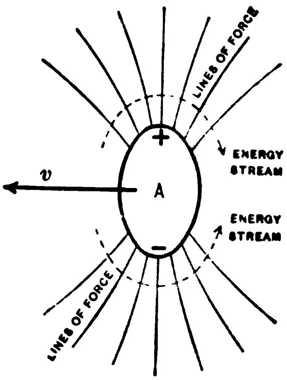

Fig. 13.

In order to explain the equation $E = lfv$, expressing the electromotive force induced in a wire of length l, moving at velocity v across a magnetic field of intensity f, we must consider the following:

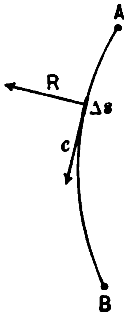

Fig. 14.

Preliminary Statement. — Let the line AB (Fig. 14) be a line of force[1] of an electric field, and imagine the whole region about AB to be a uniform magnetic field perpendicular to the paper and of intensity f. The energy stream, R, at Δs, is normal to Δs and proportional to the product of the intensities of the electric and magnetic fields at Δs; that is, $R = \kappa fe$, where κ is the proportionality factor and e is the intensity of the electric field at Δs.

The energy per second crossing Δs (unit depth) is $R\Delta s = \kappa fe \cdot \Delta s$, and the energy per second crossing the entire line AB is $\Sigma \kappa fe \Delta s$, or $\kappa f \Sigma e \cdot \Delta s$, but $\Sigma e \cdot \Delta s$ is the electromotive force along AB. Therefore, *the energy per second crossing AB is equal to the product κfE, where E is the electromotive force along AB.*

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[1] Chosen as a line of force for the sake of simplicity.

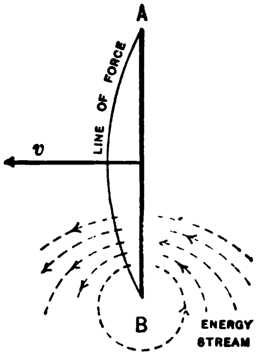

Fig. 15.

Let AB (Fig. 15) be a rod[1] of length l, moving at velocity v across a magnetic field of intensity f perpendicular to the paper. The magnetic energy $\frac{1}{8\pi}f^2lv\Delta t$, which resides in the space, $l \cdot v\Delta t$ (unit depth), which is swept over by the rod during Δt must be equal to the energy $\kappa fE \cdot \Delta t$, which streams across the *line of force* during that interval. That is, $\frac{1}{8\pi}f^2lv\Delta t = \kappa fE\Delta t$, or $E = \frac{1}{8\pi\kappa}lfv$. By properly choosing the unit electromotive force, we may provide that the factor $\frac{1}{8\pi\kappa}$ be equal to unity, in which case we have $E = lfv$. Further, when $\frac{1}{8\pi\kappa} = 1$, then $\kappa = \frac{1}{8\pi}$, so that the energy stream at a point in an electromagnetic field is $R = \frac{1}{8\pi}fe$, in which f and e are the intensities in "electromagnetic" units of the magnetic and electric fields at the point. R, f, and e are of course orthogonal.

When the rod AB (Fig. 15) is a portion of a closed conducting circuit, then electric stress is actually produced throughout the

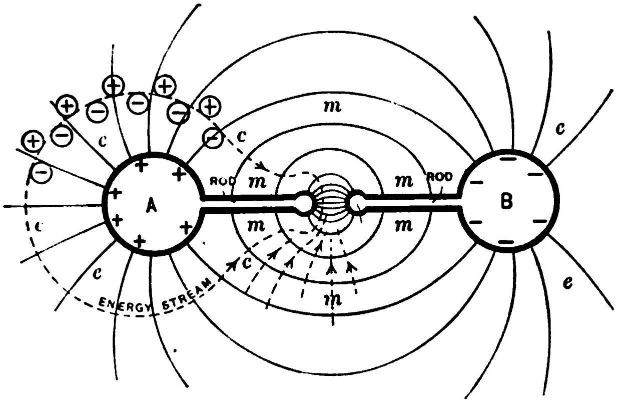

Fig. 16.

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[1] The rod is assumed to be of unit width perpendicularly to plane of paper for simplicity. The formulæ given, however, are true whatever the width, for such energy as is in front of the rod does not leave the plane of the paper as it streams around the ends of AB. Such energy streams as exist in planes above and below the plane of the paper are *re-entrant* streams.

conductor as the magnetic energy streams across it, and it is the continual *slipping of gears*, or breaking down of this maintained electric stress, which constitutes the electric current in the circuit.

12. *The Action of the Hertz Vibrator* is as follows:

Let *AB* (Fig. 16) be the balls of the vibrator upon which electric charge has been collecting. Consider a chain of cells which, when undistorted, lies along the dotted line *c*, everywhere perpendicular to the lines of force. When *A* is positively charged, this chain is distorted as shown (in part), but since it is a closed chain, this distortion is fixed. When a spark occurs at the gap, a line of slip is established across this chain, and the distortion disappears as explained in Art. 9.

If the slip takes place with great friction (high electrical resistance in the gap), the cells at the spark begin turning slowly, and the entire energy of the electric field is geared into the gap and changed to heat. If the slip is almost frictionless (low electrical resistance), the electrical energy is used mostly in overcoming the inertia of the cells as they are set rotating, and after a very short interval of time a very large part of the electrical energy will have been converted into kinetic energy of the rotating cells (magnetic energy). During this conversion the energy, streaming along the dotted line, largely disappears from the regions *ee*, and is distributed mainly in the region *mm*. When the chain of cells has been freed from distortion, the rotatory motion of the cells between *A* and *B* will have reached a maximum, and on account of their momenta the cells will continue turning, and produce a distortion of the chain in a reversed sense. At the same time the energy will, to a large extent, stream back from the region *mm* to the regions *ee*, the ball *A* will be negatively charged, and the ball *B* will be positively charged. This reversed distortion of the chain of cells is then relieved by a reversed slip (a reversed current in the rods and gap), and so on.

These oscillatory changes take place so rapidly that the portions of the distorted ether which are remote from *AB* do not follow the changes promptly. This gives rise to electrical waves, the nature of which at a distance from the vibrator is explained in the following article.

13. *Electromagnetic Waves.* — Imagine the ether cells between the dotted lines *A* and *B* (Fig. 17) to be rotating (magnetic field perpendicular to paper). There will soon be an elastic distortion [electric field towards top (or bottom) of page] between these rotating cells and the stationary cells to right (or left). This elastic reaction will soon stop the motion of the cells between *A* and *B* and set the adjoining cells in motion. These will in turn act upon the cells next beyond, and so forth. Thus the original layer *AB* of the magnetic field will give rise to two waves, to right and left, and these waves will consist of mutually perpendicular electric and magnetic fields. If the cells between *A* and *B* (Fig. 17) are, to begin with, displaced upwards (figure represents positive cells only), their reaction upon adjoining cells will start rotatory motion, and the result will be two waves, to right and left, as before.

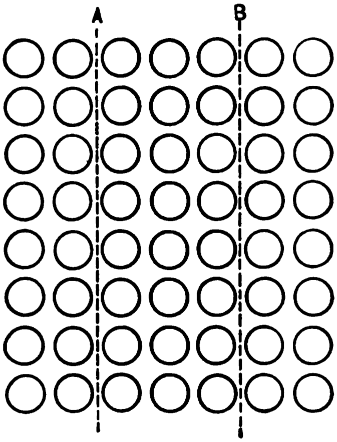

Fig. 17.

14. *Stationary Electromagnetic Wave Trains; Reflection with and without Change of Phase.* — Consider the row of cells *AB* (Fig. 18) along which a train of electromagnetic waves is approaching a conducting wall *W*. Imagine the wall to be a perfect conductor, for simplicity; then the wave train is totally reflected, and the reflected train superposed upon the incident train gives a stationary train. The disturbance at any point is the superposition of the disturbances at that point due to the incident and reflected trains respectively. Now at the face of the wall the electric field must be zero (the ether may be imagined to end here, and the freedom of motion of the end cell prevents elastic distortion between that cell and the next), so that the electric intensities of

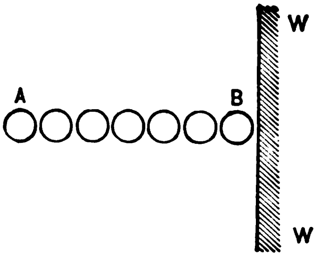

Fig. 18.

the incident and reflected trains must be continually equal and opposite at the wall. This is reflection with change of phase of electric intensity (without change of phase of magnetic intensity).

If the wall *W* be imagined to have infinite magnetic permeability (indefinitely great moments of inertia of ether cells), then the magnetic intensity at the wall would necessarily be zero, so that the magnetic intensities of the incident and reflected trains must be continually equal and opposite at the wall. This is reflection with change of phase of magnetic intensity (without change of phase of electric intensity).

In a stationary electromagnetic wave train the energy streams from regions near the nodes (electric) to regions in the vibrating segments (electric) and back again, in a manner very similar to the back-and-forth flow of the energy in the region surrounding a Hertz oscillator.

MINOR CONTRIBUTIONS.

On a Possible Development of the Idiostatic Electrometer.

By C. Barus.

1. The idiostatic electrometer has not hitherto been developed to an extent comparable with the quadrant electrometer, at least in so far as measurement of small potential differences is concerned. Yet the idiostatic method is apparently the more reasonable, seeing that it introduces no foreign potentials to coöperate with those under investigation. It might seem possible, moreover, to secure an advantage by dispensing with the viscous liquid in which the swinging needle of the quadrant electrometer is damped. I purpose, therefore, in the following paper to describe certain results obtained in an endeavor to perfect the idiostatic electrometer. The experiments proved to be delicate; indeed, one needs to cope with the trying difficulties which I have met with in these attempts to really appreciate the admirable efficiency of the quadrant electrometer.

2. The two forms of apparatus available are the gold-leaf type and the condenser type, but only the latter gave me results which seem worth describing. The pattern adopted is identical with Lord Kelvin's[1] absolute electrometer, and the conditions of sensitiveness are therefore given. To fix the ideas, let the considerations refer to the apparatus annexed, as this is a diagrammatic view of the one used below. Here *A* is the fixed, and *D* the movable, plate of the condenser, *c* the guard ring, Fig. 2 being a

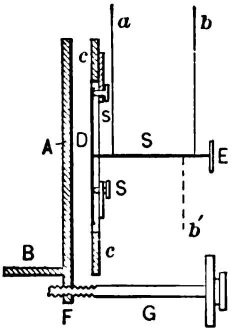

Fig. 1.

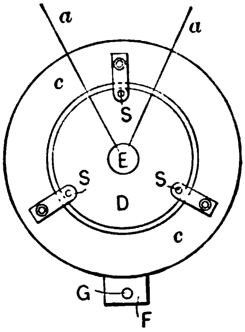

Fig. 2.

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[1] See papers on Electrostatics and Magnetism, p. 281 *et seq.*

front view, Fig. 1 a section at right angles to it. If the disk is to be movable parallel[1] to itself, it may be suspended from parallel silk threads a and b, each of which is broadly bifilar above (see Fig. 1) to prevent lateral oscillation. The disk D is made very light, and the guard ring closed behind in form of a shallow box, with a small central hole at d to admit of free motion of the stem S. Three small screws s serve as stops for the disk D. Finally, the plate A is free to slide on B parallel to itself, and this motion is controlled by the micrometer screw G working in the lug F. E is a small, thin-plate mirror rigidly connected with the movable disk D by the stem S. If the system DSE, with the broad disk D, be made very light, the air damping is excellent.

Suppose, now, that the difference of potential between A and D in this apparatus is 1 volt, and their distance apart d. Then, if A be the area of D in square centimeters, and F the total force on the disk in dynes,

$$F = A/(8\pi \cdot 10^4 \cdot 9d^2).$$

If $A = 8\pi$, or if the area of A be made 25.133 cm^2. (diameter 5.65 cm., a convenient size in practice), we have, simply, $F = (300d)^{-2}$; or

$$\begin{array}{ll} d = 1.00 \text{ cm.} & F = 0.000011 \text{ dynes} \\ = 0.10 & = 0.001100 \\ = 0.01 & = 0.110000 \end{array}$$

Thus the forces to be measured are excessively small even for two disks as near together as $\frac{1}{10}$ mm.

In the above apparatus, forces to equilibrate these values are, for small deflections, $mg \cdot \phi$, where m is the mass of the system DSE, and ϕ the deflection of the pendulum. If l be the length of the pendulum, and x its displacement, $\phi = x/l$; and therefore $x = Fl/mg$.

The value for l being 22.5 cm., the mass of the disk 0.5 g., and its appurtenances 0.71 g., the excursions x for different distances apart d of the condenser plates A and D are:

$$\begin{array}{ll} d = 1.00 \text{ cm.} & x = 0.0000003 \text{ cm.} \\ = 0.10 & = 0.0000352 \\ = 0.01 & = 0.0035200 \end{array}$$

The excursions x to be measured are thus correspondingly small, and, for $d > 0.1$ cm., small even as compared with the wave length of light.

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[1] If rotation of the disk around the horizontal is to accompany the motion, threads a and b' may replace a and b. The mirror E may then be observed with scale and telescope.

3. Having recently had occasion to work with Michelson's refractometer,[1] it seemed worth investigating whether the method could be used for a movable mirror E under the above conditions. The excellent damping secured for the disk D had made this probable. The adjustments are given in the diagram (Fig. 3), where E' is the stationary plane mirror corresponding to the mirror E of the system DSE, P the thick refractometer plate, and T the telescope. The sodium flame and appurtenances are placed beyond L. The optical conditions are very fully discussed in Michelson's great memoir (*l.c.*).

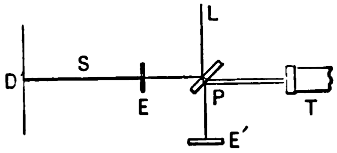

Fig. 3.

I found it convenient to use the large, flat plate of an old Kirchhoff spectrometer, the above electrometer ADE replacing one of the telescopes, and the mirror E' the other. The glass plate P on leveling screws was moved on the plate. E' being revoluble on the axis of the spectrometer, it required fully four complete rotations of the micrometer tangent screw (about 0.4°) to pass the fringes from the initial to the final vanishing point through maximum visibility.

Thus far I have not obtained results with a *free* disk D. The fringes so observed were fleeting; they shifted, rotated, and changed form continually when visible, and reappeared too rarely. But I have not given the matter up, as I think that, with better provision against air currents and earth tremors, the attempt will be feasible. For the present, I added stop screws as shown in the figure at s. These kept the fringes in the field, but did not quite wipe out their slow and regular motion. With a rather heavy disk (3.5 g.) of tin plate, and a distance apart d of about 0.25 cm., I obtained a deflection of one fringe per 20 volts of potential difference between A and D. For a free disk, the excursion x should have corresponded to about fifteen fringes for 20 volts; or we would have obtained fifteen times as much displacement had it not been necessary to lift the disk off the stops.

4. The plan for a feasible apparatus has thus been suggested; viz. to allow the disk to recline very lightly on stops (s, s, s, Fig. 1), and to move the plate A forward by the micrometer screw until the disk is just lifted off by a fraction of a wave length. Then the difference of potential is simply proportional to the distance d between the plates.

Before deciding to make a definite piece of apparatus with a micrometer slide, I wished to test the present arrangement further with the object of ascertaining how far the sensitiveness could be pushed. Accordingly in the following experiments the face F is still reckoned in terms of the number of wave lengths which pass the cross-hair in the telescope when the condenser is charged and discharged alternately.

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[1] Michelson, "Valeur du Mètre en Longueurs d'Ondes Lumineuse," Tome xi., Trav. et Mém. du Bureau internat. des Poids et Mesures, 1894.

To obtain a sensitive system, the disk D was made of thin plate mica, silvered on both sides. The mass of the movable system DSE was thus reduced to 0.71 g. It was charged through the stop screws s. In operating the apparatus I encountered a formidable difficulty, inasmuch as with a very light disk all but free, there was *no contact* between the disk and the stops, and it was now, as a rule, found impossible to charge the disk with small potentials. Liquid contacts, aside from other reasons, are evidently out of the question because of the relatively immense capillary forces involved. After many trials I finally found that a sure contact could be maintained between the guard ring and the disk by connecting the two permanently with a long narrow pendent strip of gold leaf (not shown in Figs. 1 and 2). The ends of this were attached with mucilage to the guard ring C, and the disk D, in such a way as not to hamper the motion. Again, because of the difficulty of obtaining contacts[1] with small voltages it was now possible to move the disk D almost indefinitely near A, and thus obtain relatively great sensitiveness, without the risk of discharging the condenser. Finally, in view of the close proximity in question, the motion of the disk was so slow, that there was no difficulty in counting the fringes on charging or discharging.

A trial of the apparatus in the new form gave me much better results, the fringes passing for a single volt being about 10. Since in an undisturbed apparatus less than $\frac{1}{10}$ fringe may be estimated, the sensitiveness has thus become about $\frac{1}{100}$ volt. Had the disk been about 0.05 cm. from the opposed condenser plate, about 5 fringes would have passed. Hence the disk must have been very near the plate, and the stop screws in good adjustment. Many experiments of this kind were made, but they yielded no additional results. Most of the work had to be done at night.

5. In conclusion it is interesting to inquire into the construction and sensitiveness of divers apparatus of the present kind under favorable conditions. It would be best to make the plate A and guard ring C of plate glass with rounded outer edges and silvered on both sides. The disk D, if made of microscope cover glass, silvered, would not weigh above 0.035 g. per square centimeter, or about 0.88 g. for the area 8π cm^2., which is within one gram for the movable system of disk and mirror. In such a case, parallelism between the silvered plates could be tested in the usual optic way by reflection, and the adjustment of the set screws controlled by

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[1] The usual absence of contact here recalls the case of two soap bubbles which fail to adhere unless pressed together for some time, or otherwise brought in contact. Cf. E. Kaiser, Wied. Ann., LIII., p. 667, 1894.

their own reflected images. If with better protection against air currents, etc., it is possible to work with a free disc *D*, the electrometer would remain absolute in its registry. The sensitiveness of such an apparatus would probably be of the order of 0.001 volt for a minimum of dielectric thickness.

If the absolute character of the instrument be sacrificed in favor of greater sensitiveness, one may profit by the above experience, and replace the movable system *DSE* by a sheet of gold leaf made to cover the whole space on the inner face of the guard ring smoothly. The mirror *E* could then be reduced in size so as to weigh not much above a milligram. In such a system the counter force would be in part flexure and in part gravity. The former may be evaluated by the aid of the equation

$$5x = (r^4/s^3)(p/E),$$

where x is the central displacement due to the pressure of p dynes per square centimeter, r the radius of the disk (2.8 cm.), and s its thickness (say 0.0001 cm.). The pendulum forces are found as above. The results show that in this case $x = F/4$ nearly, where F is the total force on the disk, as given in § 2. Hence such an instrument should in the extreme case register as small a potential as 100 microvolts, supposing $d = 0.01$ cm., and the motion of one-tenth of an interference fringe discernible.

Finally, the best conditions are secured by measuring the counterforces not by gravity, but in the usual way, by torsion. I have made an estimate based on the above experiments, supposing that two identical disks like *D*, in Fig. 1, are symmetrically joined by a light rigid rod, 20 cm. long, and suspended from a fine wire, 0.01 cm. in diameter, attached to the middle of the rod, with the ends fixed 10 cm. above and below it. In such a case, if $d = 0.01$ cm., the motion of one-tenth interference fringe would correspond to less than 20 microvolts. Whether this sensitiveness can be attained or not will depend very largely on quiet surroundings. Beyond this there does not seem to be more serious difficulty than was encountered in the above experiments carried on with very ordinary facilities.

BROWN UNIVERSITY, PROVIDENCE, R.I.

EMPIRICAL FORMULÆ FOR VISCOSITY AS A FUNCTION OF TEMPERATURE.

BY A. WILMER DUFF.

SEVERAL formulæ for the viscosity of liquids at different temperatures have been proposed. The writer has recently required the use of such a formula for a wide range of variation of viscosity. The following classification and discussion of the different formulæ tried, together with the suggestion of a generalized formula, may be of interest. Besides its general importance as a means of interpolation, extrapolation, and tabulation, the viscosity formula at present derives some special interest from the attempt which Stoel, Heydweiller, Thorpe and Rodger, and others have made to use it for studying the relation of chemical constitution to physical properties.

A $\eta = \frac{C}{1 + at + bt^2}$ (Poiseuille,[1] Meyer).

B $\eta = a + bt + ct^2 + dt^3$ (Koch,[2] Wagner[3]).

C $\eta = \frac{C}{1 + at}$ (Meyer, Rosencranz[4]).

D $\eta = C\frac{\alpha - t}{t - \beta}$ (Grätz[5]).

E $\eta = \frac{C}{a + t} - b$ (Slotte[6]).

F $\eta = \frac{C}{(a + t)^n}$ (Slotte, Thorpe and Rodger[7]).

G $\eta = Ce^{-at}$ (Stoel, Heydweiller,[8] Reynolds[9]).

Formula *A* has been chiefly used for water, *B* for mercury; both of them are unsuitable for a wide range of variation of viscosity. *C* is a particular case of *E*; *D* was deduced theoretically by Grätz, and applied to a large number of substances, but it will not represent alcohols, ethyl ether, water, etc. (see also Thorpe and Rodger). The remaining formulæ, *E*, *F*, *G*, are independent of one another, and for each a considerable degree of generality may be claimed; yet the curves they represent diverge considerably from the viscosity curve. Thus *E* and *G* give values that are too small at both high temperatures and low temperatures; while *F* gives values that are too great at high temperatures and too small at low temperatures. Thorpe and Rodger in their very extensive and careful work on about seventy different substances have, after trying other formulæ, chosen Slotte's *F*; but its insufficiency for substances of wide variation of viscosity, such as isobutyl alcohol, amyl alcohol, dimethyl ethyl carbinol, and also for water, is so great that in each case they divide the whole viscosity curve into three parts (for water two) and apply a separate formula of

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[1] Pogg. Ann., LVIII. (1843).

[2] Wied. Ann., XIV. (1881).

[3] Wied. Ann., XVIII. (1883).

[4] Wied. Ann., II. (1877).

[5] Wied. Ann., XXXIV. (1888).

[6] Wied. Ann., XIV. (1881).

[7] Phil. Trans., Vol. 185 (1894).

[8] Wied. Ann., LV. (1895).

[9] Phil. Trans., Vol. 177 (1886).

Slotte's form F to each part. Thus in the case of amyl alcohol the three equations call for the calculation of nine constants from the twelve determinations made, leaving only three values to be represented non-identically. In such cases the value of the formulæ for the purposes above enumerated seems small.

TABLE I.

VISCOSITY OF ISOBUTYL ALCOHOL.

Temperature.	Observed.	Calculated from F (thrice applied).	Per cent error of F.	Calculated from H_2.	Per cent error of H_2.
0.45	0.079111	0.079111	0.0	0.079313	0.2
9.90	0.055735	0.055250	0.9	0.055609	0.2
19.01	0.039779	0.040285	1.2	0.040705	2.3
27.77	0.030658	0.030439	0.7	0.030658	0.0
38.16	0.022398	0.022392	0.0	0.022367	0.1
47.44	0.017217	0.017212	0.0	0.017233	0.3
56.48	0.013571	0.013549	0.1	0.013526	0.3
56.59	0.013502	0.013511	0.1	0.013488	0.1
65.95	0.010697	0.010711	0.1	0.010694	0.0
74.61	0.008748	0.008748	0.0	0.008748	0.0
83.95	0.007173	0.007160	0.2	0.007143	0.4
93.85	0.005864	0.005868	0.1	0.005847	0.3
105.07	0.004753	0.004753	0.0	0.004740	0.3

$$\text{In } H_2 \begin{cases} a = 385.88 \\ \alpha = 0.0074052 \\ \beta = 57.92 \\ C = 0.88930 \end{cases}$$

To compare E, F, and G, let us differentiate the logarithms of each side and invert.

$$\text{From } E \text{ (or } D) \quad -\eta\frac{dt}{d\eta} = C_1(\alpha - t)(t - \beta). \qquad E'$$

$$F \quad -\eta\frac{dt}{d\eta} = C_2(a + t). \qquad F'$$

$$G \quad -\eta\frac{dt}{d\eta} = C_3. \qquad G'$$

Now, since $-\eta\frac{dt}{d\eta}$ is the subtangent of the viscosity curve, let us call the curve got by plotting subtangents against temperatures the "subsidiary curve of subtangents" (or briefly, the "subsidiary curve"). Then we see from E', F', G' that we may interpret E, F, G as meaning, respectively, that the subsidiary curve is a parabola cutting the line of temperatures in real points, a straight line inclined to the line of temperatures, a straight line parallel to the line of temperatures. To test this point I have, for several substances, obtained the subtangents of the viscosity curve by the direct application of a straight-edge, and so plotted the subsidiary curve. This I find to be a parabola, convex to the axis of temperatures, and cutting that axis either in real points (as in the cases of water and most substances of slow variation of viscosity), or in imaginary points (as in the cases of glycerin, amyl and isobutyl alcohols, dimethyl ethyl carbinol, mercury, and most substances of rapid variation of viscosity). The general equation of the subsidiary curve is therefore

Table II.

VISCOSITY OF DIMETHYL ETHYL CARBINOL.

Temperature.	Observed.	Calculated from F (thrice applied).	Per cent error of F.	Calculated from H_2.	Per cent error of H_2.
0.49	0.137969	0.137969	0.0	0.137820	0.1
9.31	0.082034	0.080650	1.7	0.080772	1.5
18.48	0.049978	0.050067	0.2	0.050090	0.2
27.24	0.033643	0.033643	0.0	0.033643	0.0
36.42	0.023322	0.023278	0.2	0.023276	0.2
45.05	0.017135	0.017133	0.0	0.017120	0.1
53.18	0.013199	0.013204	0.0	0.013203	0.0
62.95	0.009943	0.009943	0.0	0.009976	0.3
71.91	0.007931	0.007935	0.0	0.007921	0.1
81.06	0.006400	0.006418	0.3	0.006400	0.0
89.94	0.005301	0.005303	0.0	0.005302	0.0
95.70	0.004718	0.004718	0.0	0.004734	0.3
96.70	0.004643	0.004625	0.4	0.004660	0.3

$$\text{In } H_2 \begin{cases} a = 3.0860 \times 10^8 \\ \alpha = 0.025365 \\ \beta = 101.10 \\ C = 2.14746 \times 10^9 \end{cases}$$

$$y = \left[-\eta \frac{dt}{d\eta} \right] = a + bt + ct^2. \qquad H'$$

For particular values of a, b, c this will, of course, include E', F', G'. Hence, if we integrate H', we must obtain a formula of a sufficiently general type to represent all liquids representable by *E*, *F*, or *G*, and also liquids which give, as a subsidiary curve, a parabola not cutting the temperature axis in real points. This latter class seems to include those substances of rapid variation of viscosity which have hitherto not been reducible to any empirical formula.

TABLE III.

VISCOSITY OF WATER.

Temperature.	Observed.	Calculated from F (twice applied).	Per cent error of F.	Calculated from H_1.	Per cent error of H_1.
1.13	0.01709	0.01712	0.2	0.01714	0.3
3.09	0.01607	0.01609	0.1	0.01605	0.2
4.47	0.01535	0.01535	0.0	0.01535	0.0
5.45	0.01494	0.01494	0.0	0.01488	0.4
13.53	0.01181	0.01179	0.2	0.01177	0.4
22.02	0.00955	0.00951	0.4	0.00951	0.4
30.72	0.00786	0.00784	0.2	0.00785	0.1
39.32	0.00662	0.00662	0.0	0.00662	0.0
47.03	0.00576	0.00577	0.2	0.00577	0.2
55.53	0.00501	0.00502	0.2	0.00502	0.2
64.01	0.004415	0.00442	0.1	0.00442	0.1
72.54	0.003915	0.00393	0.4	0.00393	0.4
80.75	0.003525	0.003535	0.3	0.00353	0.1
89.90	0.003165	0.003165	0.0	0.003165	0.0
98.09	0.00289	0.00289	0.0	0.00289	0.0
100.00	0.00283	0.00283	0.0	0.00283	0.0

$$\text{In } H_1 \begin{cases} \alpha = 6005.52 \\ \beta = \quad 45.28 \\ n = \quad 1.6012 \\ C = \quad 7.1179 \times 10^{-6} \end{cases}$$

To avoid imaginaries, the integrated formula is expressed in two ways. (1) When the subsidiary curve cuts the temperature axis in real points, and when, therefore, in H', $b^2 > 4ac$,

$$\eta = C\left(\frac{t+\alpha}{t+\beta}\right)^n. \qquad H_1$$

(2) When the subsidiary curve does not cut the temperature axis in real points and when, therefore, in H', $b^2 < 4ac$,

$$\eta = Ca^{-\tan^{-1}a(t+\beta)}. \qquad H_2$$

H_1 is of course a generalization of E and F. For let $n = 1$ and we get E. Again write H_1 thus,

$$\eta = \frac{C\alpha^n}{(t+\beta)^n}\left(1 + \frac{t}{\alpha}\right)^n,$$

and let α be supposed infinitely large while n and $C\alpha^n$ remain finite, and we get F. Hence all the substances represented by Rosencranz, Grätz, Slotte, Thorpe and Rodger, and others by means of C, D, E, or F are representable still more perfectly by H_1. Again H_2 includes G as a particular case; namely, that in which β is very small. Hence H_2 will cover the results of Stoel, Heydweiller, Reynolds, and others.

Table IV.

VISCOSITY OF MERCURY.

Temperature.	Observed.	Calculated from B.	Per cent error of B.	Calculated from H_2.	Per cent error of H_2.
21.4	0.01868	0.01847	1.13	0.01865	0.1
0.0	0.01698	0.01697	0.00	0.01698	0.0
9.2	0.01631	0.01640	0.60	0.01634	0.2
10.1	0.01620	0.01631	0.70	0.01628	0.5
12.5	0.01599	0.01618	1.18	0.01612	0.7
18.3	0.01561	0.01582	1.20	0.01568	0.5
98.8	0.01228	0.01224	0.30	0.01218	0.8
99.1	0.01225	0.01222	0.20	0.01217	0.6
154.0	0.01092	0.01090	0.20	0.01089	0.3
176.2	0.01047	0.01045	0.10	0.01052	0.5
272.0	0.009492	0.009477	0.20	0.009477	0.2
314.7	0.009183	0.009172	0.20	0.009185	0.0

$$\text{In } H_2 \begin{cases} a = 1.9165 \\ \alpha = 0.006917 \\ \beta = 34.3 \\ C = 0.01976 \end{cases}$$

Thus to completely establish the generality of H_1 and H_2 it seems only necessary to show that they apply to substances of rapid variation of viscosity, for which F is unsuitable, and also to water and mercury, which A and B, respectively, have been regarded as expressing most satisfactorily. The two substances for which Thorpe and Rodger found F to fail most signally were isobutyl alcohol and dimethyl ethyl carbinol. They will be found represented by H_2 in Tables I. and II., the approximations given by

H_2, once applied, being about as good as those given by F (Thorpe and Rodger's calculations) thrice applied. Tables III. and IV. show H_1 applied to water (Thorpe and Rodger's figures being used) and H_2 applied to mercury (Koch's figures being used).

The only point calling for further mention is the calculation of the constants in H_1 and H_2. Here we have a choice of three methods.

(1) From the viscosity curve subtangents are found directly and employed to obtain the constants of the subsidiary curve; after integration, the remaining constants are found without any difficulty.

(2) The subtangents at three or more points on the viscosity curve are found by applying at such points an approximate formula, such as E or G, and then deducing the values of $\eta\frac{dt}{d\eta}$; the remaining procedure is as before.

(3) Formulæ for the calculation of the constants in H_1 and H_2 are developed. From the viscosity curve, obtain four values of η in geometrical progression and the corresponding values of t. Then in

$$\eta_2^2 = \eta_1\eta_3,$$

$$\eta_3^2 = \eta_2\eta_4,$$

substitute the values of $\eta_1, \eta_2, \eta_3, \eta_4$, in terms of t_1, t_2, t_3, t_4, given by the formula H_1 or H_2; equations, containing only $\alpha, \beta, t_1, t_2, t_3, t_4$, will be found from which α and β may be readily deduced. I have used (1) and (3) for mercury, the better formula (used in calculating Table IV.) being found by (3). For water, isobutyl alcohol and dimethyl ethyl carbinol method (2) was employed.[1] Ways of "improving" the constants and applying the method of least squares will readily suggest themselves. I have not employed such methods; the constants were but once calculated (except in the case of mercury) and the agreement between observed values and calculated values is therefore the more striking.

The above method of constructing empirical formulæ by deriving some recognizable subsidiary curve may be novel; it seems to greatly enlarge the possibilities of finding a suitable formula, and though it may lead to complex expressions it provides its own method of determining their constants.

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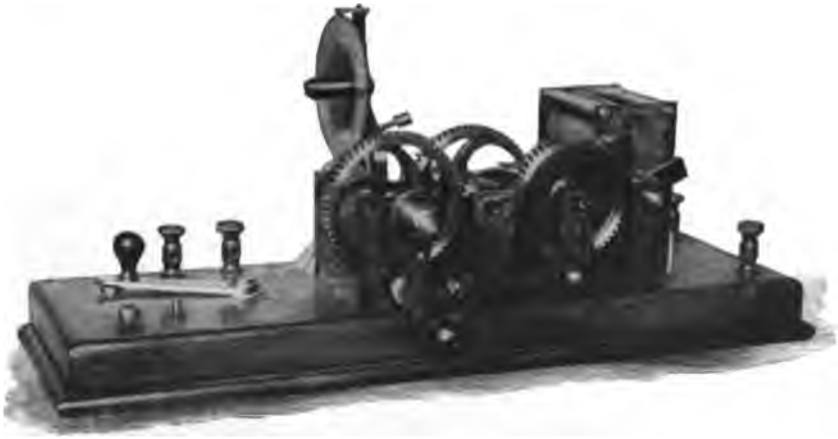
[1] In Table I. the observed value of η at 19°.01 probably contains a large error; and the attempt to include this value in formula F no doubt accounts for the large deviations of the formula at 9°.90 and 27°.77. In Table III. there is probably a large error at 9°.31.

A SYNCHRONOUS MOTOR FOR DETERMINING THE FREQUENCY OF AN ALTERNATING CURRENT.

By George S. Moler.

WHEN an alternating-current dynamo is in operation, there are as many different currents during one revolution as there are poles. One half of these will be in one direction and the remainder in the other, the complete periods during one revolution being equal to the number of pairs of currents, or in other words equal to half the number of poles; therefore the frequency which is the number of pairs per second will be the number of revolutions per minute times one half the number of poles, divided by sixty. If one wishes to determine the frequency of an alternating current when the dynamo is too distant or is inaccessible, some other method than the taking of its speed will have to be employed.

There are several ways of experimentally determining the frequency, but perhaps one of the best of these is by making use of a synchronous motor and a speed counter. A synchronous motor, as its name implies, is one the speed of which depends upon the frequency of the alternating current which drives it. The speed of such a motor is controlled as positively as if the motor were connected by a shaft to the generator itself, the number of poles passed in a second in the motor and in the generator being the same.



The piece of apparatus described in this article consists of a synchronous motor and an electrically operated speed counter built in one piece. The current will only drive it after it has been put up to the proper speed, so a crank handle with gearing is provided for that purpose.

The motor is made by placing two magnet spools on the opposite sides of a small armature shaft and with their axes parallel to it. The spools have cores made of bundles of soft iron wires. Upon the revolving axis are placed two soft iron armatures, each having four arms. They are placed at opposite ends of the spools, and the arms of one are placed parallel to the corresponding arms of the other so that they pass both ends of both spools at the same time. On account of this relation of arms to spools occurring four times in each revolution, it requires four currents, that is, two each way, to cause one revolution of the motor. On account of the above fact, if the number of revolutions made in a minute by this shaft were taken, the frequency would be found by dividing by thirty.

In the apparatus of which a view is given, the speed counter is arranged to give the speed of the crank shaft, which is geared to run only one thirtieth as fast as the armature shaft, so the number of revolutions as read for one minute is the frequency direct without any further calculation. The worm driving the counter is carried by friction and has a ratchet wheel connected with it. There are two electro-magnets operated by a clock closing the circuit once a minute; one of these is for releasing the worm-gear at the beginning of the minute and the other to throw a catch into the ratchet wheel to stop it. Another catch will only operate to stop the worm when the wheel comes around so it reads zero. There is also a graduated circle attached to the worm and this is also stopped at zero at the same time. At the end of a minute's run the electro-magnet throws the catch into some one of the twenty teeth, thus giving the reading to five one-hundredths of an alteration.

The apparatus is small and portable, the base being 14 inches long and the weight being 8.5 pounds. The minimum current which will operate it is 1.36 amperes at about 10 volts, but 2 amperes is a better amount, as it is easier to start the motor. One 32-candle power 50-volt incandescent lamp has a suitable resistance to put in series with the motor when it is to be connected into a 50-volt lighting circuit and the current can be obtained at any 16-candle-power lamp socket usually without changing the fuses. In case a clock and battery are not convenient to attach, the armatures of the electro-magnets may be operated by hand.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
November, 1896.

LECTURE-ROOM DEMONSTRATION OF ORBITS OF BODIES UNDER THE ACTION OF A CENTRAL ATTRACTION.

By R. W. Wood.

NOT remembering to have seen any attempt to show experimentally in the lecture room the motion of bodies acted on by a central attractive force varying inversely as the square of the distance in elliptic, parabolic, and hyperbolic orbits, I have made a few experiments with a view of determining how well these curves could be imitated by the motion of a small steel ball around a magnetic pole. The results were so good that I feel warranted in making them known, and believe that the experiment may be found useful in making more cheerful that portion of the course usually rather destitute of pyrotechnics.

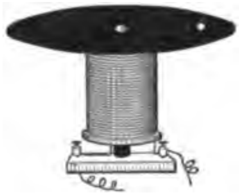


Fig. 1.

The apparatus used was very simple, consisting of a circular glass plate about 40 cm. in diameter, with a small hole in the center through which projected the somewhat conical pole piece of a large electro-magnet (Fig. 1). The surface of the plate was smoked, and it was made level as nearly as possible, the axis of the magnet being of course vertical.

A small, highly polished ball of steel about 5 mm. in diameter (from a bicycle bearing), when projected across the plate, traced its path in the soot and left a permanent record of its motion.

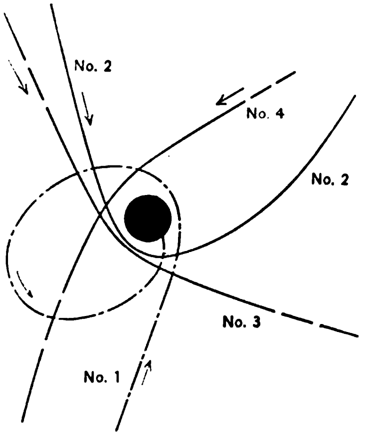


Fig. 2.

Under these conditions gravity exerts no direct influence on the motion, and we have only the initial velocity and the central attractive force to deal with, together with the loss of velocity due to friction. There are several other circumstances which make the conditions unlike those existing in the case of two gravitating bodies in space, and taking everything into consideration, it is quite surprising what good results were obtained.

The ball was blown out of a short piece of glass tubing held in the plane of the plate with varying initial velocities, and curved orbits obtained which were at least good imitations of the ellipse parabola and hyperbola.

Figure 2 is a photograph[1] of a plate showing all three forms, the black spot in the center being the hole occupied by the magnet pole; the arrows indicate the direction of the motion.

Number 1 was produced with low initial velocity, and is a very fair representation of an ellipse, with the attractive force in one focus. The loss of velocity due to friction caused the ball to "fall into the sun" after completing one revolution, a one year's existence of the system.

On another trial an ellipse (*spiral*, strictly speaking) was obtained that was almost reëntering, the miss being not more than a couple of millimeters, while in the one figured it was nearly a centimeter.

The right-hand branch of No. 2 resembles a parabola, and was produced by a somewhat higher initial velocity. It will be noticed that the ball moved to its perihelion position in a path rather like a hyperbola, and on rounding the pole, its velocity having been diminished somewhat, moved off in a parabola. It would be more exact probably if we called this curve an ellipse of great eccentricity, since the conditions governing the formation of a parabolic orbit would be difficult even to approximate.

Numbers 3 and 4 are hyperbolæ, produced by still higher initial velocities.

None of the orbits shown in the figure are as perfect as some that have been obtained by accident on other plates. It is quite difficult to make a plate showing all three forms with only four or five trials, as the velocity has to be nicely adjusted; consequently the curves shown in the figure must not be taken as samples of the best that can be produced by a large number of trials.

The hyperbola is of course the easiest to produce, and the parabola the most difficult. Some device for regulating the initial velocity and aim would be conducive to more uniform results.

Polarization of the steel ball is apt to give trouble, and I have obtained some repulsion orbits where the ball turns back before reaching the center, which are very pretty, but not desirable when one is trying to illustrate central attraction. Soft iron balls would be preferable to steel on this account, but they are not on the market so far as I know, and the others answer the purpose well enough.

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[1] Figure 2 has been reproduced by the engraver from an untouched photograph. — Ed. PHYSICAL REVIEW.

THE REFRACTIVE INDEX OF WATER AND ALCOHOL FOR ELECTRICAL WAVES.

BY A. D. COLE.

IN the September–October number of the PHYSICAL REVIEW, J. F. Mohler criticises my paper on "The Refractive Index, etc., of Water and Alcohol for Electrical Waves," which was given in abstract in the preceding number. I regret that the limits of an abstract did not enable me to be perfectly clear on the points referred to. Reference to the full paper in *Wiedemann's Annalen*, Vol. 57, p. 291, 292, will show that I have *not* ignored the absorption of the medium. Reasons are there given for the belief that the absorption is very small for long waves, although very considerable for short waves. Results published by continental investigators since my work was finished strengthen this conclusion. See Cohn and Zeeman in *Wied. Ann.*, Vol. 57, p. 22, and P. Drude in the same, Vol. 54, p. 352, and Vol. 55, p. 633. Also, Ellinger was easily able to measure the deviation produced on long waves by a thick prism, while my experiments with short waves showed almost complete absorption by a much thinner prism. It was on account of this difference that I used the reflection method in my study of the short waves.

As to my finding but four bridge positions in the trough of water and two in the alcohol, these limits were set by the shortness of the trough, which was but 68 cm. long. There was no room for another bridge in either case.

The bearing of absorption on the reflection method used for the short waves is considered in a footnote on page 300, *Wied. Ann.*, Vol. 57.

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NEW BOOKS.

Higher Mathematics: A Text-book for Classical and Engineering Colleges. Edited by MANSFIELD MERRIMAN and ROBERT S. WOODWARD. 8vo, pp. xi+576. New York, Wiley & Sons, 1896.

The advent of this book marks a new step in the development of teaching facilities in higher mathematics. It is intended to do what no other single mathematical text-book does, viz. gather together in one volume brief presentations of the different extensions of higher mathematics, in order that in each branch the student may become acquainted at least with the nature of the field concerned, with the underlying considerations, with the elementary development, and with its simpler applications.

As the editors well remark in the preface, the master minds of the early part of the century could hope by industry to obtain a comprehensive if not minute knowledge of the various branches of mathematical science; but since that date so great has been the tendency toward specialization, and so extensive and detailed have been the developments in the various branches, that now the mere quantity of information available presents a formidable if not insurmountable obstacle to the attainment of the breadth of knowledge which characterized the mathematician of this earlier period. It is further pointed out, however, that these extensions have been concerned with details rather than with the development of essentially new principles, and hence that the modern student, in entering the higher ranges of mathematics, needs most of all a general guide which shall fix his attention on essentials, and prevent him from wasting time and energy in the pursuit of non-essentials. The editors also point to an existing demand for a text-book in the best classical and engineering colleges for the mathematical work following the usual course in differential and integral calculus, now usually completed not later than the sophomore year. It is with the purpose of supplying such a guide and such a text-book that the editors have associated with themselves various eminent writers, the field to be covered being thus divided among men, each an authority in his particular subject.

The objects thus proposed are most admirable, — the provision of a text which shall aid the student of modern mathematics in laying a good foundation of fundamental principles and elementary development, and which shall broaden his comprehension of the field as a whole, by a fairly rapid survey of salient features, leaving the development of details to later and more specialized work. Taking the book as a whole, it may be said to realize in good degree the ideal thus set before us. The following subjects are treated, each in a separate chapter, and by the authors as noted: —

I.	The Solution of Equations	Mansfield Merriman.
II.	Determinants	L. G. Weld.
III.	Projective Geometry	G. B. Halsted.
IV.	Hyperbolic Functions	James McMahon.
V.	Harmonic Functions	W. E. Byerly.
VI.	Functions of a Complex Variable	T. S. Fiske.
VII.	Differential Equations	W. W. Johnson.
VIII.	Grassman's Space Analysis	E. W. Hyde.
IX.	Vector Analysis and Quaternions	Alexander Macfarlane.
X.	Probability and Theory of Error	R. S. Woodward.
XI.	History of Modern Mathematics	D. E. Smith.

We naturally find in a work of such diverse authorship much difference in style, in mode of treatment, and some unevenness of excellence. Some repetitions are naturally found — hardly avoidable under the circumstances — and as in such cases the subject-matter is approached from different standpoints, they are perhaps advantageous as tending toward breadth of view rather than the reverse. Since the book is intended as a text for students in classical and engineering colleges, it must be considered to some extent, and especially for the latter students, as a compendium of ways and means for the application of advanced higher mathematics to physical and engineering problems. It is in this feature that the work seems most lacking. It was of course no part of the authors' purpose to make a complete mathematical compendium of the branches here treated, and the question is simply as to the amount of recognition this aspect of the book should have received. Some instances may illustrate this point.

Chapter I. on the Solution of Equations takes for discussion designedly only the less well-known methods and points; those not usually treated in elementary texts on the theory of equations. The chapter is thus supplementary to the work usually given in this subject, rather than in any way intended to stand in its place. This is perhaps the more permissible from the fact that the usual course in higher algebra includes the elementary treatment of the theory of equations. For the worker, however, it would be a decided advantage to be able to turn to this chapter and find, in addition to the valuable methods given by the author, a summary or abstract of the more important theorems of the subject as usually presented. From the limitations of space such matter could not be developed in detail, and would not, of course, answer the purpose of a text for students in the theory of equations. It would be, however, a great convenience to the worker who might wish to freshen his memory as to ways and means rather than in the elementary details, and even without such development would better enable the author to broaden and unify his subject than does the fragmentary treatment given.

Again, in the chapter on Probabilities an admirable development of the subject is given leading up to the theory of least squares, and including much not usually given in the elementary treatment of the subject. The theory of least squares is, however, the most important application of the theory of errors, and it is therefore a disappointment to find the development stop short at this point. The space required would perhaps prevent a detailed treatment, but with the admirable foundation previously given, the salient features of the method might have been briefly sketched and its more important formulæ and methods summarized, much to the convenience of the practical worker.

The most important subject lacking as a whole is that of elliptic functions. It is explained that this was included in the original plan of the book, but that the manuscript could not be obtained in time for publication. It may be hoped that in a later edition this defect may be remedied.

At this point mention may also be made of the subject of higher mechanics, which might well have found a place in a book of this character. It may be said, of course, that higher mechanics is not pure mathematics. Such lines of demarcation, however, are quite artificial, and should not be allowed to outweigh considerations affecting the disciplinary or practical value of the work. For advanced students of applied physics and engineering, such a chapter would have been of great value, while the subject-matter itself would have furnished a rich mine of material for the illustration of various methods and operations treated of in the other chapters of the work.

The wisdom of including Chapter XI., on the History of Modern Mathematics, may also be called in question. Students do not usually study this as a text-book subject, and in any event the development possible in 60 pages is not sufficiently great to give more than the merest outline of the subject. The chapter itself gives in condensed form much general information on its subject-matter, and the present criticism refers simply to the wisdom of its inclusion in a book of this character.

The index for the work, as a whole, is fairly comprehensive, though it bears evidence of having been prepared with some haste, several of the references being incorrect.

A few brief points relating to the various chapters may now be noted.

Chapter I. by one of the editors has been referred to above.

Chapter II., on Determinants, by L. G. Weld, is patterned in condensed form after the same author's well-known text-book on the subject. The elementary development is well presented, with the simpler applications and a number of illustrative problems.

Chapter III., on Projective Geometry, by G. B. Halsted, is well written, though in condensed style. Large use is made of the principles and methods of duality and correlation, the propositions relating to which are presented as usual in parallel columns. The subject is treated synthetically, and a selection of problems and exercises is given. In the latter there might have been included with advantage a greater proportion showing the application of modern higher geometry to other fields of mathematical development.

Chapter IV., on Hyperbolic Functions, by James McMahon, gives an exceedingly well-prepared discussion of the subject. The geometrical character of the functions is clearly shown, and their important properties and relations are logically developed. Numerous applications to physical problems are then given, and the chapter closes with short tables of the numerical values of the functions, conveniently arranged for practical use. Among these is found a table of hyperbolic functions of complex variables, $\sin h\,(x + iy)$ and $\cos h\,(x + iy)$, which is believed to be the first of its kind. By the simple relation between hyperbolic and circular functions $(\sin h(i\theta) = i \sin \theta$ and $\cos h\,(i\theta) = \cos \theta)$, this table may also be readily used for the circular functions of complex variables.

Chapter V., on Harmonic Functions, by W. E. Byerly, is in the well-known admirable style of the author. The subject is treated synthetically and concretely. That is, the various functions are first developed and discussed in connection with various special physical problems in the solution of which they naturally arise. Their common origin, however, is shown to lie in certain differential equations, and their particular characteristics are shown to depend on the special form of the equation, and on the nature of the coördinates used. The chapter is well supplied with problems for the illustration of the principles developed, and closes with tables of surface zonal harmonics and of Bessel's functions.

Chapter VI., on the Functions of a Complex Variable, by T. S. Fiske, opens with a well-arranged elementary presentation of the subject, with illustrations in conformal representation, and with various applications to problems in hydrodynamics. The more important modern theorems are also given, and the chapter as a whole is well adapted to broaden and deepen the student's conception of the nature and characteristics of measurable quantity in its generalized sense.

Chapter VII., on Differential Equations, by W. W. Johnson, is in the style of the author's well-known text-book on the subject. The elementary portion is clearly presented, and enough of the higher development, including partial differential equations, is given for all purposes, except those of the specialist, for which, of course, the chapter is not intended.

Chapter VIII., on Grassman's Space Analysis, by E. W. Hyde, is in the style of the same author's text-book on directional calculus. The claims of Grassman's method as a space calculus are well presented, though the problems used for illustration are mostly formal rather than practical. Some further remarks on this general subject will be found under the following chapter.

Chapter IX., on Vector Analysis and Quaternions, by Alexander Macfarlane, follows closely along the lines of the same author's previous papers on this subject. Some departures are made from the fundamental definitions and conventions as laid down by Hamilton, and further developed by Tait. These are intended to bring the development of the system into closer agreement with analogies drawn from other branches of mathematics, and to simplify and broaden its application to such branches as a means of research. The chapter is well written and as a whole produces a result consistent with its fundamental basis. The examples are numerous and for the most part practical rather than formal.

In this connection it may not be amiss to note that the progress which space analysis as a means of research has made in the past twenty-five years has been most disappointing. This is doubtless due to many causes. Among them mention may here be appropriately made of the unsettled condition of the questions relating to fundamental convention and to nomenclature. A space analysis must be the expression of its fundamental conventions or definitions, but just what these should be, modern representative writers such as Tait, Heaviside, Gibbs, Macfarlane, and Hathaway are by no means agreed, to say nothing of Hyde as the representative of Grassman's methods. Until a substantial agreement shall be reached on these points and until an approximately constant nomenclature shall be agreed on, the spread of vector methods of analysis, even admitting its superior advantages, will be much hampered.

Of Chapters X. and XI., mention has already been made.

Notwithstanding the few criticisms made, the book as a whole may be cordially commended as of high mathematical and practical value, and so successful an effort at giving a convenient text-book on such a variety of topics in higher mathematics should meet with generous recognition from those interested in these subjects.

W. F. DURAND.

A Primer of Quaternions. By ARTHUR S. HATHAWAY. pp. x + 113. New York, Macmillan & Co., 1896.

This small volume contains four chapters, of which the first treats of steps; the second of rotations, turns, arc-steps; the third of quaternions; and the fourth of equations of the first degree. It presents a good elementary exposition of the method of quaternions, as taught by Hamilton and Tait. The student who masters it will find that he has acquired some real knowledge, not merely additional dexterity in formal manipulations. At the same time it must be said that he will encounter almost all of the difficulties, anomalies, and imperfections which have long obscured the method, and prevented its general acceptance and development.

In recent years it has been maintained, in opposition to Hamilton and Tait, that a vector is a vector, not a right quaternion. Professor Hathaway recognizes the distinction, and proposes to restrict *vector* to mean right quaternion, while using *step* to denote a directed length. This is a proposal which cannot be commended; for writers agree in using the word *vector* to mean a quantity which has linear direction. The confusion is introduced by the false identification on the part of the quaternionists of the vector with the right quaternion. The former has linear direction, the latter angular direction. The plain principle of procedure is to leave "vector" to its established use; and, if necessary, to devise some other word for "right quaternion."

Hamilton laid great stress on his analysis of a quaternion into the sum of a number and a line. From the preface (p. vi) we observe that the author views the whole quaternion as a number. At p. 53 we have the proposition, "The number r rotated through $2 \operatorname{arc} q$ is the number qrq^{-1}." With Hamilton "number" meant a quantity destitute of direction; with the mathematical physicist it means a quantity destitute of physical dimensions; but what meaning are we to attach to it in order that it may be capable of rotation through an angle? A quaternion is not a number in Hamilton's sense of the word, nor is it always a number in the sense understood by the mathematical physicist. The whole resistance in an alternating circuit is the geometrical sum of an ohmic resistance and an inductive resistance; it is a quaternion, but the dimensions are not zero. To explain the quaternion by means of the complex number is to take a step backwards; for Hamilton strove in vain to extend algebra to space until he gave up the time or number idea, and gave the whole a geometrical basis.

In physical science, both the scalar product and the vector product of two vectors are of frequent occurrence; for instance, work is the scalar product of a force and the displacement of its point of application; electromotive force per unit of length is the vector product of the velocity of the conductor and the density of the magnetic flux. The Hamiltonian theory attempts to explain AB, the vector product of two vectors, by regarding A as an operator on B. But the explanation given does not explain the scalar product; and, from the physical point of view, it is evident that the two vectors enter symmetrically into the product. The operator and operand theory is entirely inadequate to explain the products which occur in mathematical physics. Professor Hathaway gets over the difficulty by omitting the multiplication of vectors ("steps" in his nomenclature), and treating only of the products of right quaternions.

The order in which Hamilton and Tait write the factors in a product of a number of quaternions does not conform to the order of writing of a sum of angles in trigonometry, or to that of the terms in a series, where the natural order of writing, from left to right, is preserved. Hamilton's innovation arose from the same false analysis of a product into operator and operand. At p. 48, the author explains a product of quaternions thus: "If $rOA = OB$, and $qOB = OC$, and $pOC = OD$, then we have $pqrOA = OD$." This is an explanation which does not explain, for here r, q, and p are not general quaternions, but are restricted to being perpendicular to OA, OB, OC, respectively. When p, q, r, are general quaternions, pqr cannot be explained by any such operation. This is admitted by the author; for, in the next article (p. 49), he says: "The product is, however, independent of whether a step OA can be found or not, such that each factor operates upon the product of the preceding factor; *i.e.* we have, by definition,

$$(a) \quad T(\cdots pqr) = \cdots Tp \cdot Tq \cdot Tr.$$

$$(b) \quad \text{arc}\,(\cdots pqr) = \text{arc}\, r + \text{arc}\, q + \text{arc}\, p + \cdots."$$

Here the author inverts the natural order of writing, except in writing out the sum of the angles, where the established convention in mathematical analysis has been too strong. If it is a matter of definition, as is said above, why is the definition not made to conform with the established practice in trigonometrical analysis? As a matter of fact, when a rotation p acts on a vector OA, the expression is $p^{\frac{1}{2}}(OA)p^{-\frac{1}{2}}$, and no argument can be drawn from the supposed fact that the operator always precedes the operand, for here the operator is written both before and behind.

The author bases the principles of the analysis upon a peculiar mixture of formal laws and mechanical truths. At p. 2, we have an arbitrary definition of the equality of vectors; for it is laid down that all vectors of the same length and direction are equal whatever their point of departure. This is arbitrary; for it is not true mechanically that all vectors of equal magnitude and the same direction are equal whatever their points of application. A sum of vectors is said to be both commutative and associative, but no reason is given; on the other hand, a product of quaternions is proved to be associative by means of the highly mechanical consideration of the composition of rotations of a rigid body (p. 39). Surely, all the fundamental rules of the analysis ought to rest on a formal basis, or else they all ought to rest on a geometrical or mechanical basis.

One of the objections to the quaternion method has been its dark symbolic character. In his writings Clifford said that the product of a quaternion and of a vector which is not perpendicular to the axis of the quaternion is an impossible expression without any sense or meaning. In this view he is followed by the author, for at p. 58 it is said that the expression referred to above belongs to that class of products in which the multiplicand does not admit of the operation of the multiplier, as in $\sqrt{2}$ universities, -2 countries, etc. The solution of the mystery is plain; the general product consists of two components, one of which is the resolved part of the vector along the axis of the quaternion, the other being the perpendicular component turned through the proper angle. No doubt, if the projection of the vector on the axis is left out, the product will become mysterious, unless it so happens that there is no component along the axis.

ALEXANDER MACFARLANE.

Alternating Currents and Alternating-Current Machinery. By D. C. JACKSON and J. P. JACKSON. 8vo, pp. 729. The Macmillan Company, 1896.

Some ninety per cent of the ever-growing literature on electrical engineering can be comprised in two classes: theoretical works, written by mathematicians who have never designed, frequently hardly seen, electrical machinery, and thus write books which are as correct and interesting theoretically as useless practically for the engineer, and practical works, whose writers make up for their lack of theoretical knowledge by a generally very limited practical experience. Such latter books may be quite useful as "hints for the dynamo tender," etc., but become disseminators of misinformation when the writer aspires to explain the operation and design of electrical machinery, of which he knows very little himself. Especially vicious this class of books becomes when embodying tables of numerical designing data, either drawn from imagination or by misinterpretation from some antediluvian type of machinery. This state of the literature on electrical engineering is the more deplorable, as all these books are thrust on a public which is not able yet to judge about the worthlessness of the pretended useful information.

It is very gratifying to find in Professors Jackson's work one of the very rare exceptions to the above-mentioned literature, the more as it covers the field of alternating-current apparatus where no large general work of modern nature is in existence, and thus useful information for the student outside of a few of the largest manufacturing concerns is practically inaccessible.

In the first four chapters of the book general matters are discussed, as induction of electromotive forces, resistance, inductance and capacity, wave shapes, armature windings, armature reactions, theoretical methods of investigation, etc. The next four chapters treat of the alternating-current generator, giving amongst others a very good exposition of the problem of parallel operation and of the armature reactions of alternators and their characteristic curves. Chapters IX. to XII. deal in the same thorough and valuable manner with the stationary alternating-current transformer, the next chapter on general polyphase systems. Chapter XIV. treats of alternating-current motors. This chapter, especially the part referring to the induction motor, is decidedly weak compared with the other chapters of the book. The last chapter discusses polyphase transformations. A number of appendices attached to the book give Fourier's series and characteristic features of alternating-current curves, oscillating discharges, and electrical resonance. A very complete index is attached, increasing essentially the usefulness of the book.

While undoubtedly by far the best work on the state of the art of alternating-current engineering and more in touch with modern practice than any other general publication, Professors Jackson's book, nevertheless, cannot quite represent the present state of the art. The development of alternating-current work has practically been monopolized by two or three large manufacturing concerns and an enormous amount of investigation carried on by them. What little of the knowledge thus accumulated is accessible to the general public is due to occasional publications of insiders of these concerns, and to tests made by our universities. Especially in view of the insufficient protection of our patent law, insiders of these manufacturing concerns are necessarily loath to disclose much of a knowledge which has cost enormous sums to accumulate, while tests made by our universities are at the best limited in extent, and frequently made on apparatus which no longer represents the most modern state of the art. This does not distract from the value of Professors Jackson's book, but rather increases it, since, as stated above, the book is in better touch with the most recent development of engineering than any other publication, but it is necessary to mention it so that the student may not be led to believe that he can design modern alternating-current machinery after the study of this work.

Especially noticeable is this feature in the chapter on the induction motor, as the apparatus on which development has progressed most rapidly in the last years. Here the fundamental importance of the internal self induction of the motor is not recognized at all, and in consequence thereof the conclusions regarding the effect of the frequency, and the change from polyphase to single phase are erroneous. For a given size and speed a certain frequency gives the best all-around induction motor. At lower frequencies the efficiency and speed regulation are impaired, at higher frequencies the power factor drops and the current at light load rises. At the same magnetic density a single phase motor gives only somewhat less than half the output of the same motor as a polyphase induction motor, at nearly the same loss, and thus when raising the density so as to get in the single phase induction motor about three-fourths the output as on a polyphase circuit, the efficiency is considerably lower, the exciting current higher, and thus the power factor lower, but the speed regulation better. At the same magnetic density the loss in the iron is shown by tests to be the same in the single phase induction motor as in the polyphase motor. Thus the magnetism in the secondary of the single phase motor is not alternating with the primary frequency but only with the frequency of slip, the same as in the polyphase motor. The representation of the magnetic condition of the single phase motor by two magnetic fields rotating in opposite direction does not represent the actual facts, but would require at synchronism a current equal to the exciting current of the polyphase motor at half voltage plus the primary current of the polyphase motor when driven backwards at synchronous speed and at half voltage, that is, a current more than half the current at standstill. Tests, however, show the current in some single phase induction motors when running light to be less than twenty per cent of the current when standing still. The magnetic field of the single phase induction motor consists of a field of approximately constant intensity — reduced at larger current input only by drop of voltage due to the internal impedance — with a second magnetic field superimposed thereon in quadrature in space and in phase, and approximately proportional to the speed of the motor.

The electrical literature contains a number of erroneous statements made years ago, which have passed from publication to publication without being challenged. Very few of these have entered Professors Jackson's book. One I noticed: the statement of the absence of magnetic leakage in the Mordey type of alternators. If magnetic leakage means anything, it means those lines of force which pass through the magnetic field without contributing to the terminal voltage of the machine. It is thus obvious that in the Mordey type of alternator where the induction is due to the difference of magnetic flux in front of the pole faces and between the pole faces, every line of magnetic force which passes through the space between the pole faces represents two lines of magnetic leakage flux. It is this very type of machine which, other things being equal, has a larger magnetic leakage than the ordinary type of alternator. Thus close regulation in the Mordey type of alternator is not due to the absence of magnetic leakage but to the enormous magnetizing force available by the use of a single magnetizing coil.

Very valuable is the explanation and discussion of the Fourier's series in the first appendix of Professors Jackson's book, since this series has proven of great importance in the theoretical investigation of alternating-current phenomena, and is still very scantily treated in most text-books.

C. P. STEINMETZ.

Transformers for Single and Multiphase Currents. Translated from the German by the author. By GISBERT KAPP. 8vo, pp. 241. London, Whittaker & Co., 1896.

The literature of alternating currents is a product of the last six or eight years, and books dealing solely with this subject now number about a score. A few of these, consisting chiefly of mediocre cuts and meager description, have been written to catch the eye of the so-called "general public," or to delude the dynamo attendant into the belief that after a few weeks of evening study he can undertake the design of systems for long-distance power transmission; but with these few exceptions the books on the subject have been contributions, each independent and filling a field of its own. At the beginning of the year 1896, publishers' announcements called attention to the fact that the year was to bring forth some half dozen works[1] on alternating currents and the alternating-current transformer, the authors — German, French, British, and American — being writers of reputation. The year's addition to the subject has been a most valuable one; with one exception — a volume[2] more in the nature of a general compendium than a methodical treatise — each work has treated the subject along clearly defined lines, the distinctness of treatment being as marked as though each author were aware of the other's work and carefully avoided trespass. The reader thus has the advantage of being able to refer to several books treating the subject from different standpoints to supplement the book selected for chief study.

Although the advantage of individuality of treatment is thus obvious, it is to be noted that the criticism of books of this class is commonly (in substance) that A's book is worthless because not written from the standpoint of B, and (by another critic) that B's book is valueless because not written for the readers reached by A. Both books may be excellent, each for its own class of readers; but they are condemned because written from a different point of view from that of the critic, and because they omit that which it was never intended they should contain.[1] The choice of a book depends upon by whom it is to be read, and for what purpose.

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[1] The belated book of Mr. Steinmetz (1897) is included in this category.

[2] Reviewed in the PHYSICAL REVIEW, November-December, 1896.

For a reader desiring a clearly written, accurate, and readable discussion of the transformer, touching all essential points without dwelling too much upon any particular phase of the subject or upon abstruse theory, the present book by Mr. Kapp is admirably suited. The book contains much substantial material, either for the specialist or the general reader. No book on the subject is written in such a pleasing style; Mr. Kapp keeps the attention of his reader and carries him through much general explanation and accurate detail with ease. If one should seek to learn, however, the theory of alternating currents solely from its pages, he needs must be disappointed; for he would encounter gaps of such magnitude that he could scarcely develop a theory other than incomplete and fragmentary. As stated in the preface, the book is written particularly for those interested in the design of transformers. In the author's words, "I am well aware that through the omission of many theoretical considerations which were not immediately pertinent to the practical object I had in view, my book, considered as a scientific treatise on transformers, is incomplete." He says, further, that his book is written for engineers, whose object must always be to obtain a maximum of practical success with a minimum expenditure of mental labor. In this the author has succeeded.

Although written with this object, the book will prove a valuable source of information to any who may read it. It is not suited for use as a college text-book. Being written by one familiar with the theoretical side of the subject as well as the practical, it is an admirable book to be read as a supplement to treatises devoted more particularly to the principles of the transformer.

Chapter I. deals with fundamental principles and magnetic leakage. The author immediately enters into his subject with commendable directness, general remarks and preamble being omitted. It is not customary and scarcely advisable in treatises of this kind to introduce so early the subject of magnetic leakage. Mr. Kapp introduces it on the very first page, and its discussion constitutes a large portion of the first chapter. Mr. Kapp uses at once the sine assumption (p. 10). Despite the past outcry by so-called practical men, theorists and practical engineers alike have come to recognize the validity of many conclusions based upon an assumed harmonic electromotive force or current. The discussion of the fundamental equations of the transformer is introduced by the following unqualified statements: "*We also assume that the primary current is obtained from an alternator, the electromotive force of which follows a true sine law. This is not always, and indeed very seldom, the case; but it will be shown later on that the equations obtained under these assumptions remain applicable in all cases occurring in practice.*"

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[1] A point well brought out by Professor Trowbridge in a letter in The Electrician (Dec. 25, 1896), and in Nature (Jan. 14, 1897).

The arithmetical proof (p. 15) that the maximum value of a sine function is equal to $\sqrt{2}$ times the square root of the mean square of the instantaneous values is direct and simple. It is convincing to those whose knowledge of calculus does not allow them to follow the proof commonly given.

The losses in transformers are discussed in Chapter II., and the influence of the shape of the electromotive force curve on hysteresis loss. The six pages treating of the latter are very clear, and much more to the point than the wordy discussion of the subject recently published in the technical press. The chapter closes with a discussion of core and shell transformers with parts of different proportions, making obvious the advantages of the generally used modern type.

Chapter III. contains a discussion of the proportion of transformers, and the effect of alterations, of linear dimensions, much of which the author has published before. It is rewritten, and made clearer and more complete. Attention is particularly called to the necessity of providing proper cooling surface in the design of a transformer.

Chapter IV. outlines, in the usual way, familiar principles of alternating currents, the vector addition of currents and electromotive forces, and the determination of the power of an alternating current. The chapter closes with several good pages on the calculation of the no-load current and on the influence of joints in the magnetic circuit.

Chapter V. contains much of value to the designer, discussing at length the best distribution of copper and distribution of losses. Following this, a long chapter is devoted to clock diagrams and the graphic determination of drop. The chapter contains much good material, but a better arrangement of the diagrams would make the work easier to follow.

The next chapter gives, in brief, the usual methods of measuring the power of transformers, dynamometer, wattmeter, three-voltmeter and three-ammeter methods, and methods for testing iron. In a practical book of this kind, more stress might be laid upon the fact that the three-voltmeter and the three-ammeter methods are scarcely suited for common practice.

Chapter VIII. is an eminently practical chapter, discussing safety appliances, substations, house transformers, choking coils, compensating coils, and other subjects relating to distribution. The book closes with a chapter on illustrated descriptions of some transformers. This chapter consists of cuts, chiefly of continental transformers, with brief descriptions.

As we finish the book, we are led to wonder at the title, "Transformers for Single and *Multiphase* Currents." One might conclude that "Multiphase" was introduced into the title merely to give it a fine sound, or to gain the attention, perhaps, of some would-be reader. In vain we turn the pages of the book to find more than a casual mention, and, perhaps, two or three cuts of multiphase transformers. "Multiphase," moreover, is considered by some a hybrid substitute for "polyphase."

As exceptions to the usual lucidity of the book, we may note p. 20, where the discussion of the relation between eddy current and thickness of lamination is not clear upon first reading. On p. 53, the scale employed in plotting the curves showing the rise of temperature in terms of the area allowed per watt loss is not obvious, the omission of the scale being unfortunate as the curves are intended for practical use. In nomenclature, we may take exception to *W* for resistance, and ∼ as a symbol for frequency. The latter is scarcely suited to represent a *quantity* in an equation, no more so than ✱, !, or ∿. Some may object to "effective" for "virtual" (p. 16), to "ampèremeter" for "ammeter" (p. 161), to "an electricity works" (p. 160), to "so-called current heat" (p. 19) as indicating the loss due to ohmic resistance, and to "E.M. forces" for *E.M.F.'s*, or, better still, *electromotive forces*.

Minor typographical errors, for the most part obvious, are all too frequent: thus one of limit of an integral is omitted (p. 16; p. 149); we note *v* for *o* (p. 12); *cos* for *cos*^{2} (p. 14); *sinnsoidal* for *sinusoidal* (p. 17); *less* for *loss* (p. 21); *coil* for *oil* (p. 58); *io* for i_0 (p. 78); *M* for μ (p. 80); 1_1 78 for 1.78 (pp. 80 and 81); *forces* for *fuses* (p. 177).

FREDERICK BEDELL.

Méthode et Principes des Sciences Naturelles. TH. FUNCK BRENTANO. pp. 137. Paris, L. Bataille & C^{ie}, 1896.

The purpose of the writer is evidently twofold; first, to urge upon students of the natural sciences a more logical habit in discussion of scientific principles, and, secondly, to correlate the forces of nature under one pleasing hypothesis of the author's invention. To attain these ends the book is divided into two parts, one, treating of *Method*, having for object the mental development of the reader to a point which will render possible the unreserved acceptance of the second part, the *Principles*. The difficulty and futility of classifications, the relations of the various divisions of science and of thought, experimentation, abstraction, analysis, synthesis, induction and deduction, are all duly discussed. Then follows the hypothesis, with a distinct Cartesian ring, the two realities, form and movement, and the explanation thereby of all chemical and physical phenomena. Affinity, attraction, electricity, light, and heat are all simply and easily explained by peculiarities of form of the atomic and molecular units with the resulting variation in ethereal currents.

An appendix discusses briefly the photography of the invisible, claiming for the *rayons x* a niche in the grand scheme already presented. Light is described as a result of two sets of vibrations, neither alone visible, the primary being those imparted to the ether by the movement of elementary forms in passing from the free state to a molecular combination, and the secondary, those due to the inter-molecular movements of the atoms. The "X ray" is to be considered as an exaggerated development of the latter, the secondary, vibrations.

CHARLES PLATT.

PHILADELPHIA, PA.

Motive Power and Gearing for Electrical Machinery. By E. TREMLETT CARTER. 8vo, pp. xxi+620. New York, D. van Nostrand Company; London, The Electrician Publishing Company; 1896.

The author of this somewhat extensive and very timely work has brought together, in a systematic and logical shape, a large amount of valuable scientific matter and practical knowledge of special value to the engineer engaged in the design, the construction, or the operation of electric light or power "plants." As he states in his preface, he recognizes the facts that electrical engineering is a branch of mechanical engineering of great and growing importance, and that the practitioner must, to succeed to-day, possess a good knowledge both of the science and of the practice of the art. "Just as the marine or locomotive engineers have to make a special study of the steam engine in its relation to marine or railroad practice, so must the electrical engineer study, from his own point of view, the principles and practice of motive power and gearing for electrical machinery." He includes in his field of study gas engines and hydraulic motors, as well as the steam engine. The presentation of the results of trials of various forms of motor, and that of the details of station and "plant" construction constitute extensive and important parts of his work.

In Part I. are given the fundamental principles of mechanical science involved in this branch of engineering, and definitions of scientific and technical terms now accepted by the practitioner and the man of science dealing with such subjects. The problem in hand is stated thus: "What are the best provisions for utilizing the available energy for the performance of useful work and the production of a paying revenue?" The last requirement is one which, though vitally important, is rarely considered in works of this kind; and this may be cited as one of the points of novelty, as well as of excellence, of the book. All such propositions are printed in heavy-faced type, — another good point.

Among the principles thus specially impressed upon the mind of the reader are the following: —

"The most economical design is that in which the annual interest on the cost of any small improvement in the efficiency of the plant is equal to the saving of expenses which would result annually from that improvement being adopted."

"In any arrangement of mechanical or other parts for the transmission of energy, the energy entering the arrangement at an irregular or fluctuating rate, and leaving it at a regular rate, or *vice versa*, the successive parts will be subjected to the least straining action, and will be required to transmit the least power, when the store or reservoir for transforming the rate from the one value to the other is placed at the least possible distance from the actual seat of the fluctuation."

In this portion of the book, the economics of the engineer's problem are studied in various aspects and the principle clearly shown that his work is, in fact, throughout that of securing a stated result at minimum "running cost." The discussion of the bearing of the "factor of idleness" upon this "commercial efficiency," as the writer has called it, is one of the most suggestive parts of this introductory matter.

Part II. is devoted to the steam engine, including its fuels, the thermodynamics of the case, the structure, theory, and action of the engine and of its boiler. Engine testing and the Manchester Steam Users' Association's instructions, the methods of choice of type of engine and boiler and the best system of subdivision of the plant are given considerable space. In the thermodynamic discussion, the author takes $J = 772.55$, and $\gamma = 1.408$. This section is particularly well condensed. It appears to be generally accurate as well; but the author follows Rankine in making, for adiabatic expansion of initially dry steam, $pv^{1.111} = \text{constant}$, instead of adopting the better established index 1.135. The expedients of superheating and steam jacketing are very justly spoken of as thermodynamic errors, but "necessary evils."

Part III. is devoted to gas and oil engines, their fuels, the thermodynamics of this class of motors, and their special adaptation to the development of power in electric stations. The best and the most familiar of recent types are described, and their performance, on trial and in use, discussed. Part IV. treats of water power and hydraulic motors in condensed and well-arranged form. Part V. discusses gearing and Part VI. the arrangement and operation of stations, including, to an exceptional and commendable extent, the questions of costs and profits.

The book is well made, although the illustrations, which are numerous and well chosen, are not fully up to the standard of the best book-making of the time. A good index, a comparatively rare appendix to an English technical publication, completes the book, which will be likely to find many readers, and to be well worth its price to the practitioner in this field.

R. H. Thurston.

The Elements of Electrochemistry. By Max Le Blanc. Translated by W. R. Whitney. 8vo, pp. 282. The Macmillan Company, 1896.

The German edition of this work has been reviewed at length in the Physical Review, Vol. IV., p. 268. The translator has followed the original work as nearly as practicable; the English edition is well written and its mechanical make-up is good.

Mathematical Papers read at the International Mathematical Congress, Chicago, 1893. Edited by the Committee of the Congress, E. Hastings Moore, Oskar Bolza, Heinrich Maschke, Henry S. White. pp. xvi.+411. New York, Macmillan & Co., for the American Mathematical Society, 1896. (*Received.*)

Laboratory Manual of Inorganic Chemistry. By Rufus P. Williams. pp. xxii and 99. Boston, Ginn & Company, 1896. (*Received.*)

Chemistry at a Glance: A Study in Molecular Architecture. By Herbert B. Tuttle. 8vo, pp. 59. New York, 1896. (*Received.*)

Inorganic Chemical Preparations. By Frank Hall Thorp. 8vo, pp. 238. Boston, Ginn & Company, 1896. (*Received.*)

The American Annual of Photography and Photographic Times Almanac for 1897. 8vo, pp. 370. New York, The Scovill & Adams Company. (*Received.*)

Problems and Questions in Physics. By Charles P. Matthews and John Shearer. 8vo, pp. 247. The Macmillan Company, New York, 1897. (*Received.*)

Volume IV. *May–June, 1897.* *Number 6.*

THE

PHYSICAL REVIEW.

ON THE ABSORPTION OF THE EXTRAORDINARY RAY IN UNIAXAL CRYSTALS.

By Oscar M. Stewart.

DICHROISM was observed in the early part of this century by Arago[1] in certain specimens of barite, and by Biot[2] in tourmalin. This variation of color was explained by Biot as being due to the unequal absorption of the ordinary and extraordinary rays. Brewster[3] soon after published a list of some sixty uniaxal and biaxal crystals which presented the same characteristic, although in no case was the dichroism so marked as in tourmalin. More recent researches in the ultra-violet[4] and infra-red[5] have shown that other crystals, Iceland spar for example, apparently entirely transparent to the eye, show this selective absorption.

This selective absorption appeared for a long time anomalous, the extraordinary ray alone showing marked variations for different directions of vibration. On account of its bearing on the laws of double refraction the phenomenon has been given considerable attention.

More recently it has been established beyond all doubt by the experiments of Schwebel[6] and Becquerel,[7] and confirmed by all the later investigations, that the absorption for any given wave length depends upon the direction of vibration. Thus for the ordinary ray, which always vibrates in a direction at right angles to the optic axis, the absorption is the same for all directions of propagation. But the angle made with the optic axis by the direction of vibration of the extraordinary ray varies from 0° to 90° for the different directions of propagation of the ray. Since, when this angle is 90°, the extraordinary vibration coincides with the ordinary, we should expect the absorption of the extraordinary ray to vary from a minimum or maximum, as the case may be, to an identity with the absorption of the ordinary ray, as the direction of propagation of the ray is varied. This is in accord with other properties of uniaxal crystals, as, for example, the variation in the velocity of light in the crystal.

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[1] Journal de Physique, t. 90, 41, 1820. Also Becquerel, Annales de Chimie et de Physique, 14, 170, 1888, and Mascart's Traité d'Optique, t. 2, 205.

[2] Bulletin de la Société Philomathique, pour 1819, 109 et 132.

[3] Edinburgh Philosophical Journal, p. 348, 1817. Also Phil. Trans., p. 11, 1819.

[4] Agafonoff, Archives des Sciences Phys. et Nat. t. 2, p. 349, 1896.

[5] Merritt, Physical Review, Vol. II., p. 424, 1895.

[6] Zeitschrift für Krystallographie, 7, 153, 1883.

[7] Annales de Chimie et de Physique (6), 14, 170, 1888.

The study of this variation in the absorption of the extraordinary ray with the direction of vibration has given rise to a number of theoretical and experimental investigations.

CURRENT THEORIES.

According to the law of thickness, if A_0 be the initial amplitude and A be the resultant amplitude of vibration, we have for any absorbing medium,

$$A = A_0 a^x, \tag{1}$$

where x is the thickness of the medium traversed and a is the factor called the *coefficient of transmission.*

It has been shown that the law of thickness may also be expressed by

$$A = A_0 \epsilon^{-mx}, \tag{2}$$

where m is the factor called the *coefficient of absorption.*

If expressed as a ratio between intensities rather than amplitudes, these formulas become

$$I = I_0 a^{2x},\ I = I_0 \epsilon^{-2mx}. \tag{3}$$

Obviously, $\log_\epsilon a = -m.$

The factors a and m, for any given wave length, are constants for all isotropic media, and for the ordinary ray in uniaxal crystals.

In 1888, Becquerel[1] announced that the law governing the absorption of the extraordinary ray could, for a given wave length, be expressed by the equation

$$A = A_0(a_o \sin^2 \alpha + a_e \cos^2 \alpha),$$

or

$$a = a_o \sin^2 \alpha + a_e \cos^2 \alpha, \tag{4}$$

where α is the angle made by the direction of vibration with the optic axis, a_o the coefficient of transmission of the ordinary ray, a_e the coefficient of the extraordinary ray when its direction of vibration is parallel to the optic axis, and a the coefficient for the direction of vibration making an angle α with the optic axis.[2]

For biaxal crystals, the law was

$$A = A_0(a\lambda^2 + b\mu^2 + c\nu^2),$$

where a, b, and c are the principal coefficients of transmission, and λ, μ, and ν the direction cosines of the vibration referred to the three principal optical directions.

Becquerel believed that he had experimentally verified this law by observations in the visible spectrum taken with a spectrophotometer.

Carvallo[3] has pointed out that Becquerel's law does not conform to the law of thickness. As a result of some experimental work Carvallo came to the conclusion that it was the law of thickness that was at fault, and announced that he had discovered a new phenomenon, "*de signification claire et d'importance capitale.*" A few weeks later Potier[4] showed that it was Becquerel's law, and not the law of thickness, that was at fault. Carvallo, afterwards, came to the same conclusion.

Mallard[5] has developed what has since been called the *ellipsoid of absorption*, or sometimes the *inverse ellipsoid of absorption*. His law may be expressed by the formula

$$m = m_o \sin^2 \alpha + m_e \cos^2 \alpha, \tag{5}$$

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[1] *loc. cit.*

[2] The direction of vibration is the direction as taken in the Fresnel theory.

[3] Comptes Rendus, t. 114, 661, 1892; also, Ann. de Chim. et de Phys. (7), t. 7, 58.

[4] Comptes Rendus, t. 114, 874.

[5] Traité de Cristallographie, 1884, t. 2, 350; Mascart's Traité d'Optique, t. 2, 208.

for uniaxal crystals, and for biaxal crystals by

$$m = m_1\lambda^2 + m_2\mu^2 + m_3\nu^2, \tag{6}$$

the notation being the same as before.

In equation (6) the square root of m is the perpendicular, parallel to the direction of vibration, let fall from the center on the tangent plane to the ellipsoid

$$\frac{x^2}{m_1} + \frac{y^2}{m_2} + \frac{z^2}{m_3} = 1.$$

Mascart[1] has shown that when the absorption is small the formulas of Becquerel and Mallard are identical. The equation

$$\log_e a = -m$$

may be written

$$a = \epsilon^{-m}.$$

Expanding this equation and neglecting m^2 and higher powers, we have

$$a = 1 - m.$$

When this value of a has been substituted in equation (4), an equation identical with equation (5) will be obtained. Thus we see that when m is so small that its square may be neglected the formulas of Becquerel and Mallard are identical.

In 1895, Moreau[2] developed a theory which gives an unsymmetrical relation between the principal coefficients of absorption, and the asymmetry increases as the difference between the principal indices of refraction increases. But with crystals for which the indices are not very different his equation becomes

$$\frac{m}{n} = \frac{m_o}{n_o}\sin^2\alpha + \frac{m_e}{n_e}\cos^2\alpha, \tag{7}$$

where n is the index of refraction for the angle α, and n_o and n_e the maximum and minimum indices of the extraordinary ray. In this form it was verified by data taken by Camichel with tourmalin, in which the indices were practically the same.

Camichel,[3] working with an ingenious spectro-photometer, was able to make more exact determinations than had been previously made. For uniaxal crystals, he worked with different thicknesses and varieties of tourmalin. His results for yellow light confirm Mallard's theory and contradict that of Becquerel.

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[1] Traité d'Optique, t. 2, 208.

[2] Comptes Rendus, t. 119, 327, et t. 120, 258 et 602.

[3] Annales de Chim. et de Phys., t. 4, 433, 1895. Séances de la Soc. Franç. de Phys., 50, 1895. Also Journal de Physique, t. 4, 149.

Carvallo[1] has recently developed a theory based on the equations of Boussinesq,[2] which may be derived[3] from the well-known equations of Helmholtz. He obtains as the equations of the extraordinary ray, for a given wave length,

$$\frac{1}{n^2} = \frac{1}{n_o^2}\sin^2\alpha + \frac{1}{n_e^2}\cos^2\alpha, \tag{8}$$

and

$$\frac{m}{n^3} = \frac{m_o}{n_o^3}\sin^2\alpha + \frac{m_e}{n_e^3}\cos^2\alpha, \tag{9}$$

where the notation is the same as that already used. The first is the equation of the variation of the refractive index of the extraordinary ray, the index being defined as the ratio of sines or of wave velocities, not as the ratio of ray velocities. His second equation reduces to the form of Mallard, equation (5), for substances whose principal indices are practically equal.

Carvallo's experiments were conducted with tourmalin, whose principal indices differed by about one per cent. On account of this small variation of the refractive indices he was unable to experimentally distinguish between his theory and that of Mallard. His data, obtained with a linear thermopile at $\lambda = 1.98\,\mu$, confirm the theory of Mallard and contradict that of Becquerel.

In 1884, Voigt[4] developed a theory, the equations of which reduce to the same form as those of Carvallo. Voigt uses as an absorption coefficient one first used by Cauchy. It is defined by expressing the law of thickness by the equation

$$A = A_0 \epsilon^{\frac{\kappa x}{\tau q}},$$

where x is the thickness, κ a factor called the *extinction coefficient*, $\frac{1}{2\pi\tau}$ the frequency of vibration, and q the velocity of wave propagation in the medium. The product of this extinction coefficient and the refractive index is called the *absorption coefficient.*

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[1] Ann. de Chim. et de Phys. (7), t. 7, 58, 1896.

[2] Poincaré Theorie Math. de la Lumière, 208.

[3] Comptes Rendus, t. 112, 521, 1891.

[4] Wiedemann's Annalen, 23, 577. Also, Kompendium der theoretischen Physik, 2, 708.

Obviously
$$m = \frac{\kappa}{\tau q} = \frac{2\pi\kappa}{l} = \frac{2\pi s}{ln}, \qquad (10)$$

where s is the Cauchy absorption coefficient, and l the wave length in the medium. Voigt gives the following equations for the extraordinary ray, —

$$q^2 = o^2 \sin^2 \alpha + e^2 \cos^2 \alpha, \qquad (11)$$

and
$$2\tau\kappa = \frac{C_o \sin^2 \alpha + C_e \cos^2 \alpha}{o^2 \sin^2 \alpha + e^2 \cos^2 \alpha}, \qquad (12)$$

where q and κ are the velocity of propagation in the medium and the extinction coefficient, respectively, of the vibration making an angle α with the optic axis; o, e, C_o, and C_e are constants for the wave length whose frequency is $\frac{1}{2\pi\tau}$. If equation (11) be divided by the square of the velocity of propagation in air, it reduces to equation (8), the first of the equations of Carvallo.

Equation (12) may be written

$$2\tau\kappa = \frac{1}{q^2}(C_o \sin^2 \alpha + C_e \cos^2 \alpha),$$

and as
$$n = \frac{q_a}{q},$$

where q_a is the velocity in air, we have, from the relations given by equations (10),

$$\frac{\kappa q^2}{\tau q_a^3} = \frac{1}{2\tau^2 q_a^3}(C_o \sin^2 \alpha + C_e \cos^2 \alpha),$$

or
$$\frac{m}{n^3} = \frac{1}{2\tau^2 q_a^3}(C_o \sin^2 \alpha + C_e \cos^2 \alpha).$$

Solving this equation for the constants on the right-hand side by giving α values of 0° and 90° respectively, we have

$$\frac{m}{n^3} = \frac{m_o}{n_o^3} \sin^2 \alpha + \frac{m_e}{n_e^3} \cos^2 \alpha,$$

or equation (9), the second of Carvallo's equations. Thus we see that Carvallo has been anticipated in his theory by a dozen years.

It is thus seen that there is a considerable difference in the theories, and that in some cases the experimental evidence has been conflicting. Becquerel's formula may be regarded as discredited, both by theory and by all recent experiments. The other theories, namely those of Mallard, Voigt or Carvallo, and Moreau, as far as experimental evidence is concerned, have equal claims. Their experimental verification has been carried out with data obtained from crystals whose principal refractive indices differed very little. As we have seen, these formulas are practically identical for such crystals.

The present work was undertaken with the thought of attempting to experimentally distinguish between these different theories. The problem was brought to the attention of the writer by Professor Merritt, and the writer is further indebted to him for his timely suggestions at various stages in the work.

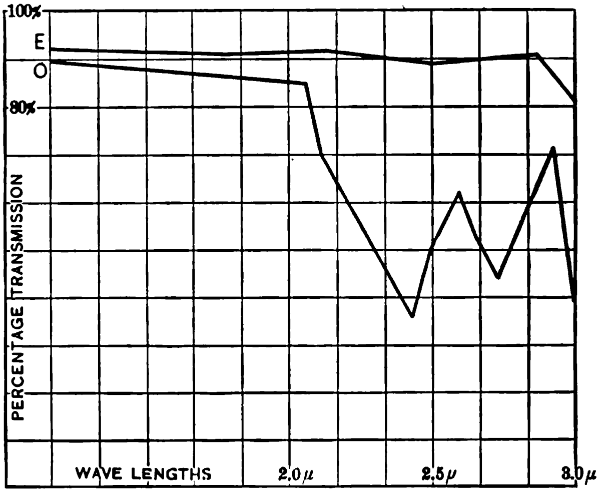

Fig. 1.

As a substance more nearly filling the required conditions than those previously used, Iceland spar was chosen for the work. The difference in the maximum and minimum refractive indices is relatively great, and in the infra-red it presents a suitable absorptive power. As may be seen from the results of Professor Merritt,[1] partially reproduced in Fig. 1, for wave lengths between 2.0 μ and 2.8 μ there is a marked difference in the absorption of the extraordinary ray for different directions of vibration.[1] Yet the absorption is not great enough to modify the reflection or the refractive indices.

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[1] Physical Review, Vol. II., p. 424. Also Wiedemann's Annalen, 55, 49, 1895.

For this region Iceland spar is seen to be an almost ideal substance for the present investigation. The great difference in the principal indices should make the formulas of Mallard, Voigt or Carvallo, and Moreau quite different. And the difference between the absorption of the ordinary ray, and that of the extraordinary ray corresponding to the maximum transmission, presents a variation great enough to make a considerable change in the transmission of the extraordinary ray, when the direction of the vibration is varied.

APPARATUS AND ADJUSTMENTS.

A number of plates of Iceland spar were obtained, whose faces made different angles with the optic axis. These plates were of the purest variety, showing no absorption for the visible portion of spectrum. By means of the spectro-bolometer, measurements were made in the usual manner.

A diagram of the apparatus, as finally used, is shown in Fig. 2.

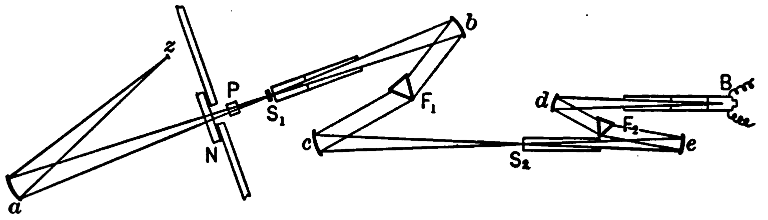

Fig. 2.

Two spectrometers were connected "in series" to obtain the greatest possible purity of the spectrum. It has been shown[2] that unless precautions of this kind be taken, the stray light from the more intense portions of the spectrum will vitiate the results obtained. Energy from the intense zirconium lamp *z* was focused on the slit S_1 by the concave mirror *a*. The rays were rendered parallel by the concave mirror *b*, were dispersed by the fluorite prism F_1, and brought to a focus at S_2. To the mirror *c* was attached a tangent movement, so that the spectrum could be slowly shifted in a direction at right angles to the slit S_2, and any desired portion of the spectrum permitted to fall upon the narrow slit. The rays passing through S_2 were again dispersed by another fluorite prism F_2, and brought to a focus in the plane of the linear bolometer *B*.

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[1] The curves shown in Fig. 1 are from data taken by Professor Merritt with a plate 2.98 mm. thick, the face of which was cut parallel to the optic axis. The upper curve in the figure was obtained when the vibrations were parallel, and the lower when they were perpendicular, to the optic axis. They correspond to the maximum and minimum transmission of the extraordinary ray, the minimum being in this case identical with the ordinary ray. Values, as given by the curves, are not corrected for reflection.

[2] Merritt, *loc. cit.*

Both prisms were kept set for the minimum deviation of the *D* line. The prism F_2 was calibrated, and from values of the refractive indices, as obtained by Paschen,[1] the wave lengths could be computed when the angular deviation from the *D* line had been measured. These angles were read on the circle of the second spectrometer.

At *P* was the polarizer, which will be explained later. Immediately in front of the slit S_1 was a vertical track, sliding along which was a metallic plate. On this metallic plate was mounted the crystal plate to be examined. By this arrangement the crystal plate could be removed from the incident beam, and brought back accurately to the same position. Both this plate carrier and the screen at *N* could be operated by cords from the observer's position at the reading telescope.

Metallic mirrors possess two distinct advantages, viz., that they do not absorb the energy, and that their focal length is the same for all wave lengths — that is, when focused for the visible rays they are focused for all wave lengths. The mirrors used were of glass, silvered on the first surface. During the course of the work it was found necessary to resilver them. This was done by the Brashear method,[2] which gave a coating so tenacious that it could be polished with rouge.

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[1] Wiedemann's Annalen, 53, p. 812, 1894.

[2] Wadsworth, Zeitschrift für Instrumentenkunde, 22, 1895. Also Astrophysical Journal, Vol. I., p. 252, 1895.

The galvanometer used was one built several years ago by the department mechanician, Mr. Fowler. A description of it has already appeared.[1] The coils used when connected in series gave a resistance of 15 ohms. The moving parts, consisting of mirror, glass connecting rod, and eighteen magnets, weighed 0.095 g. This magnetic system was magnetized by the Wadsworth method.[2] It pointed east and west, and gave a period of seven seconds in the earth's field. In practice this field was weakened until the needle had a period of about twelve seconds, which was found to be the most suitable. The irregular vibrations of the needle, due to the mechanical disturbances of the machinery in the neighborhood, were very annoying. An attempt to remedy this was made by placing the galvanometer on a marble slab that lay on a thick bed of sawdust. The slab had a mass of about 30 Kg. This acted fairly well until the sawdust became packed, when it was as bad as ever. Finally a platform was suspended by three similar and parallel cords from three points in the ceiling. On this platform was placed a slab having a mass of about 15 Kg., and upon this the galvanometer. With this arrangement the galvanometer was practically isolated from the mechanical disturbances, and as far as these were concerned its behavior was highly satisfactory.

The Bolometer.

In the bolometer first used the two strips were of iron, each about 1.5 cm. long and of 1 ohm resistance. The width of the exposed strip, as measured on the circle of the spectrometer, was 1.7′. Later a bolometer was used in which the strips were of platinum. They were made by rolling out a platinum drawn-in-silver microscope cross-hair wire. The diameter of this wire, as given by the manufacturers, was 0.006 inch, and of the platinum core, 0.00022 inch. After being mounted, the silver was removed with acid. Each strip was 9 mm. long, and had a resistance of about 4 ohms. After smoking, the width of the exposed strip, expressed in circular measure, was 1.2′. The thickness of the strip was very much less, approaching the dimensions of light waves. The platinum bolometer, notwithstanding that it was narrower and shorter than the iron one, was much more sensitive. This was probably due to its lower heat capacity and higher resistance. Both of these excellent bolometers were made by the mechanician connected with the laboratory.

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[1] E. F. Nichols, PHYSICAL REVIEW, Vol. I., p. 1; also Nichols, The Galvanometer, pp. 75, 76, Figs. 44 and 45.

[2] Phil. Mag., s. 5, 38, p. 482, 1894.

The first bolometer used was connected up in the traditional way; that is, the compensating resistances and the resistance of the galvanometer were made equal to the resistance of the strips. This was afterwards changed, for reasons which are given in an article[1] in this number of the Review. The compensating resistances were made about 26 ohms each, and the electromotive force increased until about the same current as before flowed in the strips. The galvanometer was connected to the junctions of the strips with the compensating resistance coils. The gain in sensitiveness of the bolometer from these changes was about 30 per cent. Later the platinum bolometer was connected up in a similar way. For this bolometer the galvanometer coils were connected in multiple series. This did not quite give the most suitable resistance for the galvanometer, but the gain that would result from rewinding the coils was not sufficient to warrant the change.

The current used was furnished by two storage cells, giving a current of about 0.04 ampere through each of the bolometer strips. In constructing the resistances and the usual balancing adjustments, great care was taken to prevent fluctuations in temperature. As an additional precaution, the spectrometers and all the resistances were inclosed in a constant-temperature room. So successful were these attempts that, when in working order, the disturbances of the needle due to the bolometer connections were not greater than the magnetic disturbances, and only a very slow "drift" of the needle was ever present.

The Polarizer.

At first the light was polarized by reflection from glass, and in the case of the visible rays was tested by a Nicol's prism. As the polarizing angle is a function of the wave length, there is the possibility that when one has complete polarization for the visible spectrum, he may not have it for the longer wave lengths. However, the lack of complete polarization for a glass polarizer has been shown[1] to be very small for wave lengths as great as 3.0 μ. But the greatest objection to this method is the great loss of energy. In the visible spectrum the intensity of the beam reflected from the first surface of glass at the polarizing angle, is only 7 or 8 per cent of the incident intensity, and for the infra-red the percentage, neglecting the effect of absorption, is even less.

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[1] Child and Stewart, p. 502.

Figure 1 shows that the extraordinary ray in Iceland spar, out to $\lambda = 2.8\ \mu$, is transmitted with very little loss, while the ordinary ray suffers rather marked absorption. These results were obtained from a plate cut parallel to the axis 3 mm. thick. Now if the thickness of the plate be increased, according to the law of thickness the difference between the absorption of the two rays should rapidly increase. From these results, correcting for loss due to reflection, the transmission for a plate 21 mm. thick, cut parallel to the optic axis, was computed. The approximate values obtained

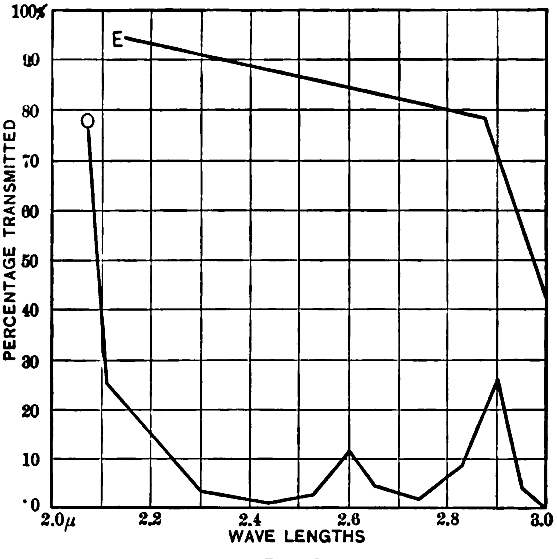

Fig. 3.

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[1] Merritt, *loc. cit.*

are shown graphically in Fig. 3. These curves do not show the loss due to reflection at the two surfaces; this, however, is relatively small.

Figure 3 shows that such a block of Iceland spar would act for some wave lengths almost as a complete polarizer, possessing the great advantage over glass of giving a polarized beam of about *six* times the intensity.

A cube of Iceland spar, 20 mm. on the edge, with faces parallel and perpendicular to the optic axis, was obtained and set with its optic axis perpendicular to the direction of propagation of the light.[1] The optic axis was placed horizontally, so that the emergent beam was polarized in the plane corresponding to the minimum reflection from the prisms; a precaution in the present arrangement of apparatus probably not important. A correction for the slight error due to incomplete polarization was applied to the readings in the manner explained later.

The Crystal Plates.

In all, eight different plates cut from Iceland spar were used. Each of these crystal plates, which were 1 cm. square on the face, and about 0.4 cm. thick, was mounted on one of the carriers, previously referred to, which moved along in front of the slit S_1. On each of these carriers was a small divided circle. And as the crystal plate was placed over a small opening at the center of this circle, it could be rotated through a known angle about an axis perpendicular to its face. As the crystal and the small opening at the center of the carrier were both larger than the slit S_1, and close to it, no error was introduced by any screening effect on the slit.

With the aid of Professor Gill of the Department of Mineralogy, the angle made by the optic axis with the face of each crystal plate was measured in the following manner. A small fragment having been chipped from one corner of the plate, an almost microscopic portion of one of the natural cleavage planes was revealed. At another corner, a second cleavage plane was similarly obtained.

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[1] See P in Fig. 2.

With a Fuess No. 2 reflection goniometer the angles between these cleavage planes and the face of the plate were accurately measured. The angle between the optic axis and the face of the plate was computed from these angles and the known constants of Iceland spar.

To set the crystal plates perpendicular to the beam of light an inclined piece of plate glass was temporarily placed in the path of the rays between the mirror *a* and the slit S_1. By observing in this glass the beam of light reflected from the surface of the crystal plate, the latter was adjusted on its carrier until it was perpendicular.

For the adjustment of the principal planes, two crossed Nicols were temporarily mounted, one in front of the polarizer *P* and the other between the slit S_1 and the mirror *b*. By the usual optical method their principal planes were set parallel and perpendicular, respectively, to the principal plane of the polarizer. One at a time the crystal plates were introduced before the slit S_1 and rotated about an axis perpendicular to their faces until their principal planes were parallel or perpendicular to the principal plane of the polarizer, and the readings of the divided circles of the carriers noted.

Table I. gives the thickness of each plate and the angle α made by the optic axis with the face of the crystal.

TABLE I.

Plate.	Thickness in mm.	α	Plate.	Thickness in mm.	α
A	4.03	0° 9′	*E*	3.83	44° 36′
B	3.98	22 4	*F*	3.98	59 41
C	3.76	39 30	*G*	3.64	77 4
D	3.83	44 11	*H*	3.93	82 1

The angle for plate *E* was not measured. This plate was cut parallel to one of the cleavage planes of the spar, and its angle could therefore not be readily determined by the method used; the angle given in the table is that of the cleavage plane. Plates *D* and *E* were obtained from Steg and Reuter, and the remainder from Brashear.

OBSERVATIONS AND RESULTS.

The bolometer was set by the circle of the second spectrometer to the angular deviation from the *D* line corresponding to the desired wave length. The screen *N* was raised, and by the slow tangent movement the mirror *c* was turned until the maximum deflection was indicated by the galvanometer. This maximum was well defined, and when attained the slit S_2 admitted the wave length desired. The first throw of the galvanometer when the screen was raised was observed, first when the crystal plate was not in the path of the rays, and next when in place. The ratio of the two throws gave the ratio of the two intensities, or the percentage of transmission.

The plane of polarization was perpendicular to the principal plane of the polarizer. Therefore when the principal plane of the crystal plate was parallel to that of the polarizer, the extraordinary ray was transmitted through the plate. When the plate was set with its principal plane perpendicular to the principal plane of the polarizer, the ordinary ray would be transmitted by the plate. Hence, by a rotation of the plate about an axis perpendicular to its face, either the ordinary or the extraordinary transmission could be obtained. This rotation could easily be effected by the use of the divided circle previously referred to. By the method of adjustment employed the principal plane of the crystal plate was either perpendicular or parallel to that of the polarizer; the method did not permit a distinction to be made. But as the extraordinary ray was known to be always more freely transmitted, the two were readily distinguished by observing the transmission.

In general, ten or more readings were taken alternately, first with the crystal in position before the slit, and when removed. As the galvanometer gave deflections of 150 small scale divisions for the wave lengths used, and as successive readings usually agreed to within one or two per cent, the mean of such a series gave the galvanometer deflections to a rather high degree of accuracy. The error in the reading of the galvanometer throw is probably of much less magnitude than several other errors involved.

The coefficients of transmission are often, for convenience, defined as the ratio of the intensities, and not of amplitudes as given in equations (1) and (2). It is a matter of indifference, since, as one may see from equations (3), the same result is obtained by merely changing the arbitrary unit of thickness by the factor 2. For convenience 3.98 mm. has been used as an arbitrary unit of thickness; and the ratios of the intensities corresponding to this thickness are taken as the coefficients of transmission. If twice the unit given be taken as the unit of thickness, then the coefficients are the ratios of amplitudes.

Owing to the wave lengths used, and the great variation of the refractive index of the extraordinary ray, the usual device for getting rid of the reflection at the face of the crystals by immersing them in some liquid of the same index was not used. The transmission as observed was corrected for reflection by the formula,

$$B_1 = B_2 (1 - r)^2,$$

where B_1 is the percentage of transmission observed, and B_2 the corrected percentage. The factor r is the percentage reflected from the first surface at normal incidence, and may be obtained from the Fresnel formula, as the absorption here is not great enough to modify the reflection. For normal incidence this reduces to the well-known form due to Young,

$$r = \left(\frac{n-1}{n+1}\right)^2, \tag{13}$$

where n is the refractive index. The refractive indices of the ordinary ray and the minimum indices for the extraordinary ray were calculated from the dispersion formula, using the constants for Iceland spar obtained by Carvallo.[1]

In Table II. are given the indices as computed from Carvallo's constants.[2]

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[1] Journal de Physique, 9, 265, 1890.

[2] The value of the constant a in the dispersion formula for the ordinary ray, as given in the place cited, is incorrect. As computed by the writer it should be $+ 0.371069$.

TABLE II.

Ordinary.		Minimum extraordinary.	
λ	n	λ	n
1.009 μ	1.644	1.007 μ	1.481
2.193	1.624	2.066	1.476
2.351	1.621	2.301	1.475
2.430	1.620	2.447	1.474
2.586	1.616	2.593	1.474
2.664	1.615	2.667	1.473
2.742	1.613	2.886	1.472
2.974	1.608	3.032	1.472

From the values given in this table, curves were accurately plotted, by means of which the indices for any intermediate wave length could be obtained.

For a given wave length the refractive index of the extraordinary ray transmitted by the plate whose optic axis made an angle α with the face, was obtained from the formula

$$\frac{1}{n^2} = \frac{1}{n_o^2}\sin^2\alpha + \frac{1}{n_e^2}\cos^2\alpha, \tag{14}$$

where n_o and n_e are respectively the maximum and minimum indices obtained from the dispersion formulas, and n the desired index. Carvallo's constants for the dispersion formula give indices which agree remarkably well with observed results, and were verified by him in the infra-red as far as $\lambda = 1.98\,\mu$ for the ordinary or maximum index, and to $\lambda = 2.15\,\mu$ for the minimum index.

The method of correction for reflection used supposes a perfect surface of the crystal plate. On account of its softness and marked cleavage, perfect surfaces of Iceland spar are hard to maintain. The tendency for the surfaces to deteriorate is more marked for plates cut in some directions than for others. Before the work was completed, the surfaces of two of the plates used became slightly imperfect as seen by the eye. Although this would not affect the transmission of the longer waves as much as it did the visible rays, yet it would have a tendency to make the observed transmission too small or the absorption too great. The crystal plate *C* shows this effect, and to a less extent it is also shown by plate *G*, as will appear from the results given.

As the plates differed slightly in thickness, a correction for this was applied to the values observed, after they had been corrected for reflection. This correction was computed in accordance with the law of thickness, and was simply equivalent to finding the percentage of transmission for the unit of thickness.

The last correction applied was for the error due to the incomplete polarization. The vibrations transmitted by the polarizer may be resolved into two components, viz., a vibration in the plane of polarization, and one at right angles to it. If the percentage of the intensity of the component vibrating in the plane of polarization be known, the correction may be applied by the following formulas :[1]

$$B_o = a_o(1 - x) + ax,$$

or

$$B_o = a_o + (a - a_o)x, \tag{15}$$

and

$$B = a(1 - x) + a_o x,$$

or

$$B = a - (a - a_o)x, \tag{16}$$

where x is the percentage of the component of the vibrations in the plane of polarization. B_o and B are the observed percentages, after other corrections were made, transmitted by the plate for the ordinary and extraordinary rays respectively. a_o and a are the coefficients of transmission, or the percentage not absorbed, of the ordinary and extraordinary ray.

If P_o and P_e are respectively the actual percentage of the initial intensity of the ordinary and extraordinary rays of the polarizer transmitted,

$$x = \frac{P_o}{P_o + P_e}. \tag{17}$$

But

$$P_o = (1 - r_o)^2 a_o^d,$$

and

$$P_e = (1 - r_e)^2 a_e^d,$$

where d is the thickness of the polarizer in terms of our arbitrary unit of thickness, 3.98 mm., and a_e is the maximum coefficient of transmission of the extraordinary ray.

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[1] The Fresnel hypothesis is here followed, viz., that in a beam of plane polarized light the direction of vibration is perpendicular to the plane of polarization.

Neglecting the difference between r_o and r_e we have

$$x = \frac{a_o^d}{a_o^d + a_e^d}. \tag{18}$$

With a plate cut perpendicularly to the optic axis, no matter whether the light is polarized or not, a_o is the coefficient of transmission which will be obtained. This may be seen from equation (15); since for this particular plate the extraordinary and ordinary vibrations become identical, or $a = a_o$. But if x is not zero, the a_e of equation (18) cannot be directly observed. However, it was obtained in the following way. The observed transmission, for the extraordinary ray, of a plate cut with its optic axis parallel to the face was corrected for reflection and thickness. This value gives B in equation (16), which equation, as this is the plate corresponding to maximum transmission, may be written

$$B_e = a_e - (a_e - a_o)x. \tag{19}$$

This value of B_e is a little less than a_e, usually only 1 or 2 per cent. This numerical difference was estimated, and the value obtained for a_e substituted in equation (18), and a value of x obtained. This value was used in (19) to obtain a_e more accurately, and this new value of a_e used in equation (18), and a corrected value of x found, and so on. As x was always a small quantity, and the correction therefore small, it was not necessary to obtain x very accurately. However, the method admitted of a higher degree of accuracy than was used.

Knowing x and a_o, a was obtained for the different plates used, from equations (15) and (16). From these values of the coefficients of transmission the coefficients of absorption were obtained by the relation

$$-m = \log_e a.$$

As has been previously stated, the transmission for the ordinary ray should be the same for all the different directions of propagation. In Table III. are shown values obtained for the transmission of the ordinary ray.

TABLE III.

ORDINARY RAY. $\lambda = 2.43 \mu$.

Plate.	Observed percentage transmission.	Corrected for reflection.	Corrected for thickness.	Coefficients of transmission.
A	0.410	0.460	0.464	0.452
B	0.416	0.467	0.467	0.458
C	0.419	0.470	0.450	0.445
D	0.407	0.457	0.443	0.439
E	0.399	0.448	0.434	0.430
F	0.393	0.441	0.441	0.440
G	0.404	0.454	0.422	0.422
H	0.405	0.455	0.450	0.450
Mean	0.406	—	—	0.442

As shown in the table, data were taken on all the plates. In the second column are given the percentages actually observed. In the third, the observed values are given after correction has been made for reflection. In the next column these values have been reduced to the values corresponding to the unit thickness. And in the last column are given the values of the coefficients of transmission, which are obtained from the fourth column by applying the correction for polarization.

TABLE IV.

EXTRAORDINARY RAY. $\lambda = 2.28 \mu$.

Plate.	Observed percentage transmitted.	Corrected for reflection.	Corrected for thickness.	Cöefficients of transmission.		Coefficients of absorption.		
				Observed.	Becquerel computed.	Observed.	Mallard computed.	Voigt-Carvallo computed.
A	0.891	0.959	0.960	0.983	—	0.017	—	—
B	0.823	0.892	0.892	0.911	0.921	0.093	0.099	0.081
D	0.664	0.729	0.720	0.729	0.771	0.316	0.301	0.260
E	0.671	0.737	0.728	0.737	0.768	0.305	0.305	0.265
F	0.575	0.638	0.638	0.643	0.659	0.442	0.452	0.419
G	0.518	0.580	0.551	0.551	0.570	0.596	0.572	0.563
H	0.505	0.567	0.563	0.563	0.556	0.574	0.590	0.587
Ordinary ray	0.505	0.567	0.571	0.548	—	0.602	—	—

TABLE V.

EXTRAORDINARY RAY. $\lambda = 2.38\mu$.

Plate	Observed percentage transmitted.	Corrected for reflection.	Corrected for thickness.	Coefficients of transmission.		Coefficients of absorption.		
				Observed.	Becquerel computed	Observed	Mallard computed.	Voigt-Carvallo computed.
A	0.891	0.959	0.960	0.979	—	0.021	—	—
B	0.823	0.892	0.892	0.907	0.913	0.098	0.112	0.092
C	0.671	0.733	0.720	0.728	0.791	0.318	0.282	0.241
D	0.662	0.726	0.717	0.725	0.752	0.322	0.335	0.291
E	0.649	0.712	0.702	0.710	0.749	0.342	0.340	0.296
F	0.560	0.622	0.622	0.626	0.632	0.468	0.503	0.467
G	0.492	0.550	0.520	0.520	0.536	0.654	0.636	0.626
H	0.470	0.527	0.523	0.523	0.522	0.648	0.656	0.651
Ordinary ray	—	—	—	[1]0.513	—	0.668	—	—

[1] Obtained from the curve.

TABLE VI.

EXTRAORDINARY RAY. $\lambda = 2.49\mu$.

Plate.	Observed percentage transmitted.	Corrected for reflection.	Corrected for thickness.	Coefficients of transmission.		Coefficients of absorption.		
				Observed	Becquerel computed.	Observed	Mallard computed.	Voigt-Carvallo computed.
A	0.882	0.949	0.950	0.962	—	0.039	—	—
B	0.796	0.862	0.862	0.871	0.890	0.138	0.145	0.122
C	0.642	0.702	0.688	0.693	0.756	0.367	0.344	0.295
D	0.626	0.687	0.677	0.682	0.715	0.383	0.405	0.354
E	0.613	0.673	0.662	0.667	0.711	0.405	0.410	0.359
F	0.501	0.556	0.556	0.558	0.583	0.583	0.600	0.559
G	0.450	0.503	0.472	0.472	0.478	0.751	0.754	0.744
H	0.421	0.472	0.467	0.467	0.463	0.761	0.777	0.774
Ordinary ray	0.408	0.458	0.453	0.453	—	0.792	—	—

In Tables IV., V., and VI. are given the values for the variation in the transmission of the extraordinary ray. These are tabulated similarly to the values given in Table III., but with the addition of the coefficients of absorption, and the theoretical values according to the theories of Becquerel, Mallard, and Voigt-Carvallo. These theoretical values were computed from the observed coefficients corresponding to the maximum and minimum transmission, viz., the coefficients obtained from plate *A* and from the ordinary ray.

In Plates I., II., and III. the observed and theoretical values are shown graphically. The attention of the reader is especially directed to this graphical representation, where conclusions may be more quickly formed than in an inspection of the tabulated data.

In the upper set of curves in Plates I., II., and III. are plotted the observed percentages transmitted and the coefficients of transmission, while the broken curve represents the theoretical values of Becquerel's theory. It is at once evident that the results are not in harmony with the theory of Becquerel. The same conclusion is reached by an inspection of the numerical values given in the tables.

In the lower set of curves in the plates are given the coefficients of absorption. The continuous line is the curve obtained from Mallard's formula, while the broken line is that obtained from the Voigt-Carvallo formula. It will be noticed that the Mallard curve agrees closely with the curve that one would draw for the observation curve, while the Voigt-Carvallo curve lies so far below the Mallard curve as to make the distinction obvious.

The point plotted for the abscissa 44°.4 is in each case the mean of the values obtained from the crystal plates *D* and *E*. On this account, and from the fact that the surfaces of these two plates were almost perfect, greater weight should be given to these values. Although they occur at about the maximum divergence of the Mallard and the Voigt-Carvallo curves, in every case they agree closely with the Mallard value. The observed coefficients plotted for the angle 39°.5 are the values obtained from the crystal plate *C*. This plate is the one that had the poorest surface, and for this reason it always shows a greater absorption. But the variation is not great. The crystal plate *G*, the plate which was cut at an angle of 77° to the optic axis, also showed slight irregularities of its surface.

The Moreau formula has not been compared directly with the observed results. The variation in the refractive indices of the plates used was so great that his formula as given in equation (7) does not apply. When there is a marked difference in the refractive indices, this formula is unsymmetrical, while the data given here show a symmetrical relation.

It is also apparent from these results that the error due to incomplete polarization was very small. In Table III., if any error of this kind were present, the observed coefficients of transmission of the crystal plates *A* and *B* would be larger than the coefficients obtained from the plates *F*, *G*, and *H*. For in the case of the plate *A*, if the rays were not entirely plane polarized, the plate would transmit both an ordinary and an extraordinary ray. As the transmission by this plate of the extraordinary ray is almost perfect, the result would be an increase in the transmission observed. On the other hand, the transmission of the extraordinary ray by the plates *G* and *H* is nearly the same as that of the ordinary ray, and the presence of the former would not change to a noticeable extent the transmission observed.

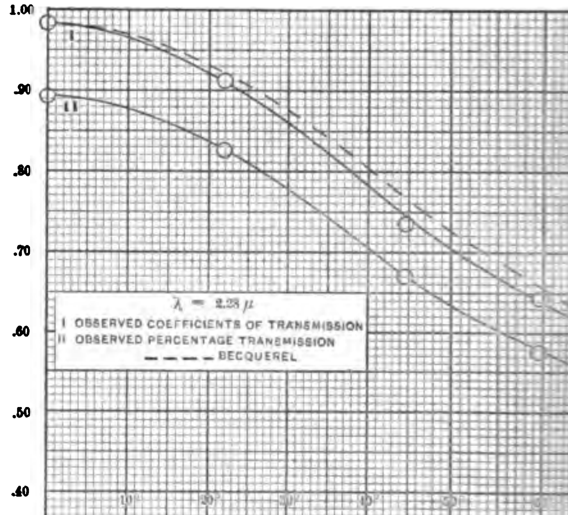
Again, it is seen that the maximum coefficients of transmission obtained for the extraordinary ray are only slightly less than that corresponding to a perfect transmission, while the minimum, or that of the ordinary ray, is relatively small. If any considerable error, due to incomplete polarization, was present, the observed values for the maximum transmission would be smaller than those actually obtained. That there is a slight absorption of the extraordinary ray corresponding to the maximum transmission, follows from the observations of Professor Merritt. He found that a slight absorption of the extraordinary ray that corresponds to the maximum transmission was present in this region, and that the absorption increases slightly with the wave length. This is confirmed by the data obtained by the writer.

The crystal plates *D* and *E* were cut with faces respectively parallel to the negative and positive unit rhombohedron. Although the angles made with the optic axis by the faces of two such plates are equal, other crystalline properties are different, the face of one plate being parallel to one of the cleavage planes, while the face of the other was equally inclined to two of the cleavage planes.

These plates were obtained for the purpose of testing whether the direction of the cleavage plane affected the absorption. The plate *E* shows, in Tables V. and VI., a slightly greater absorption than plate *D*. This is explained by the fact that the face of plate *D* was not quite parallel to the negative rhombohedron, and the angle made by the face with the optic axis was a little smaller than the angle made by the plate *E*. But in Table IV. the errors of observation have concealed even this effect. The conclusion is therefore drawn that if the direction of cleavage has any effect on the absorption, the effect is less than the errors of observation.

This work was begun before the papers of Carvallo and Camichel had been published. After the publication of the work of Carvallo, the writer was more inclined to favor the Voigt or Carvallo theory than that of Mallard. But from the preceding results the conclusion must be drawn that the theory of Mallard agrees with experiment more nearly than any of the others.

PHYSICAL LABORATORY, CORNELL UNIVERSITY.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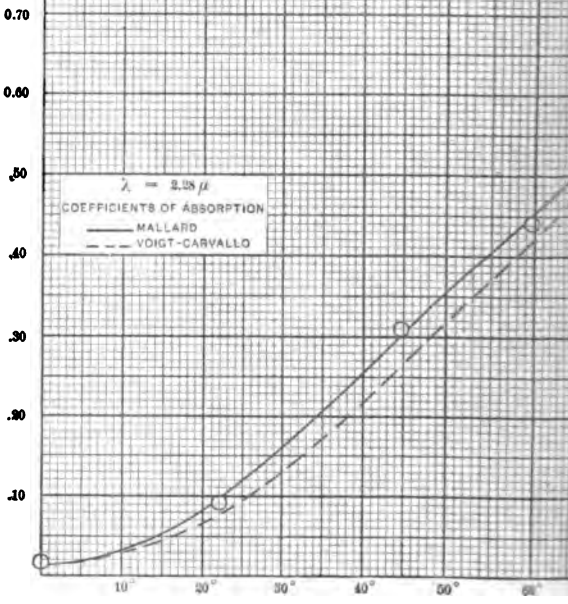
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COEFFICIENTS OF ABSORPTION
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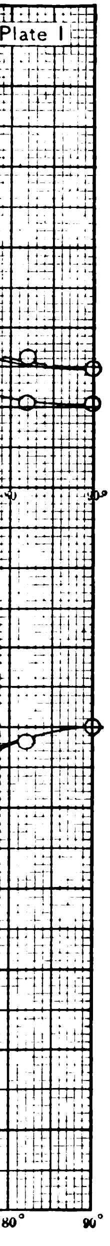
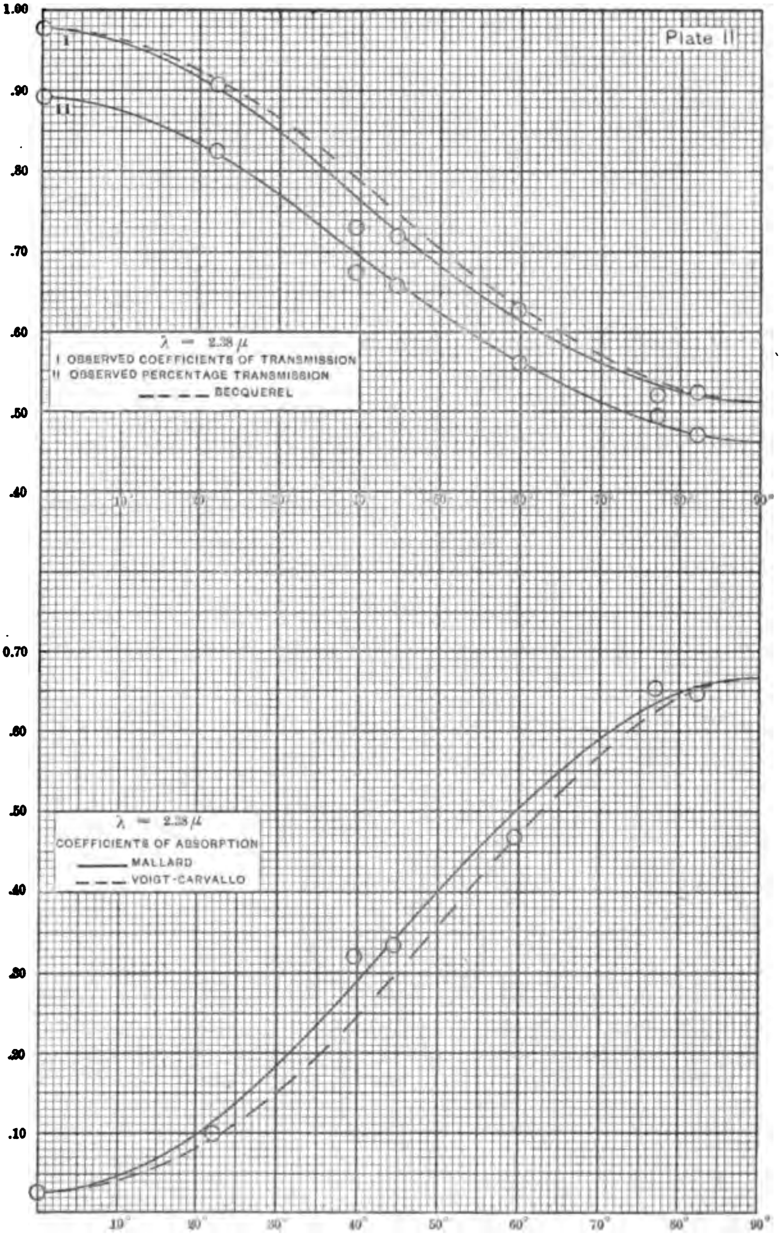
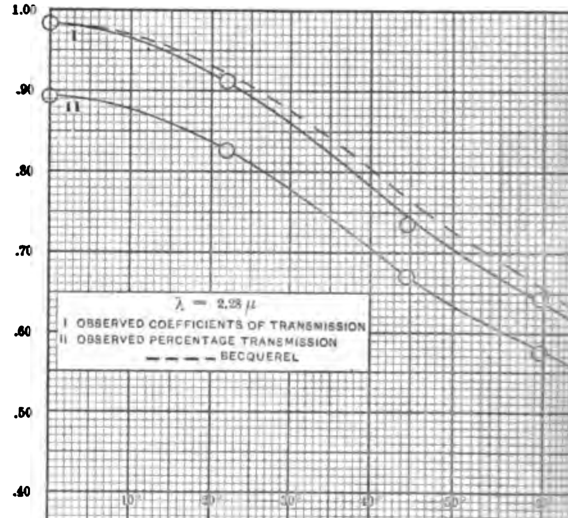
VOIGT-CARVALLO
10°
20°
30°
40°
50°
60°





STEWART: ABSORPTION IN UNIAXAL CRYSTALS.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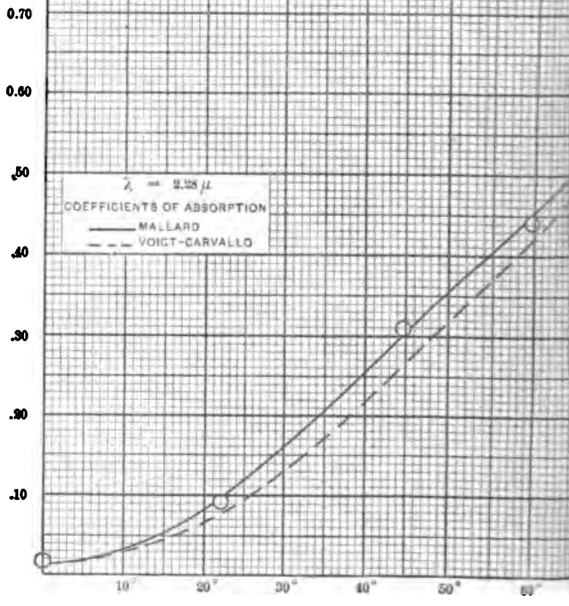
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λ = 2.28 μ
COEFFICIENTS OF ABSORPTION
MALLARD
VOIGT-CARVALLO
10°
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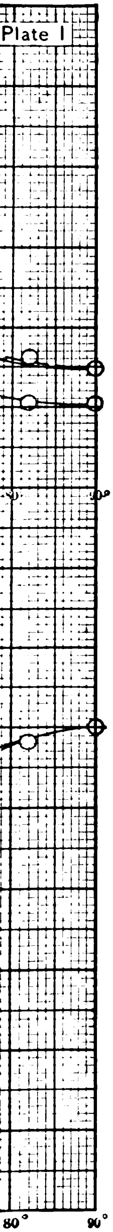
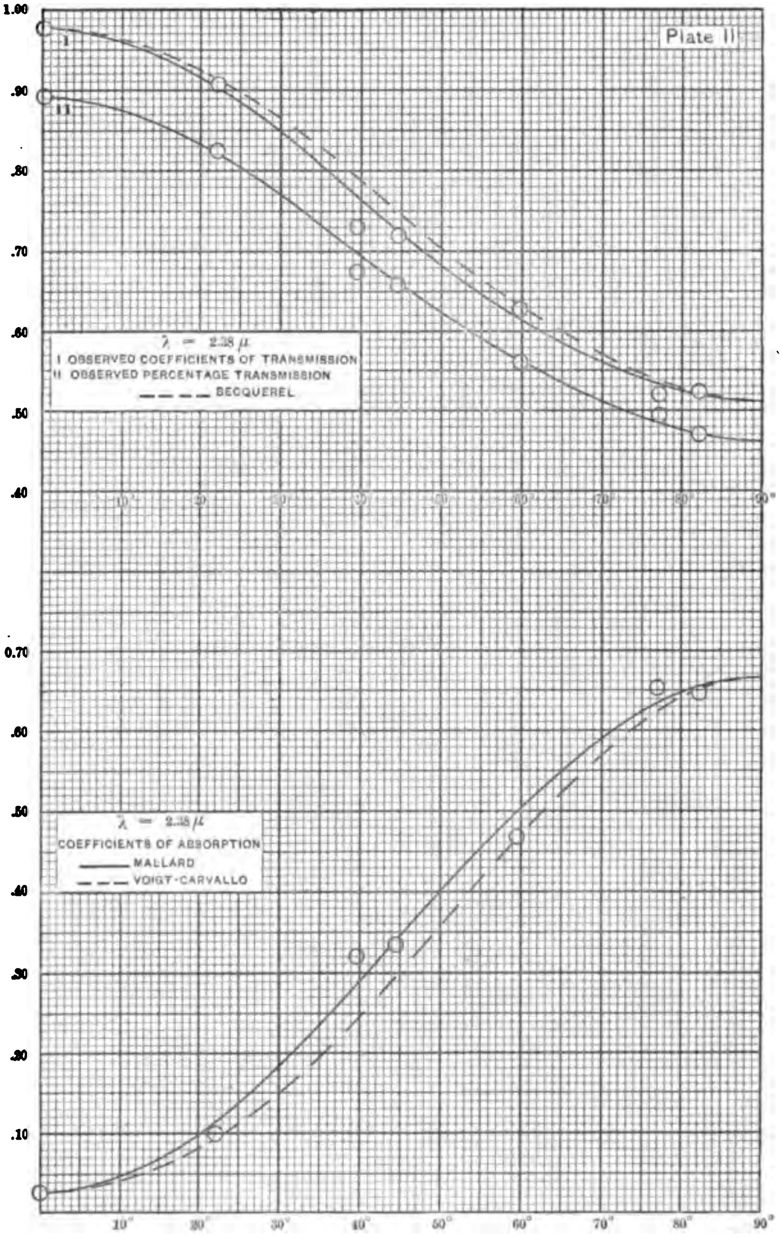
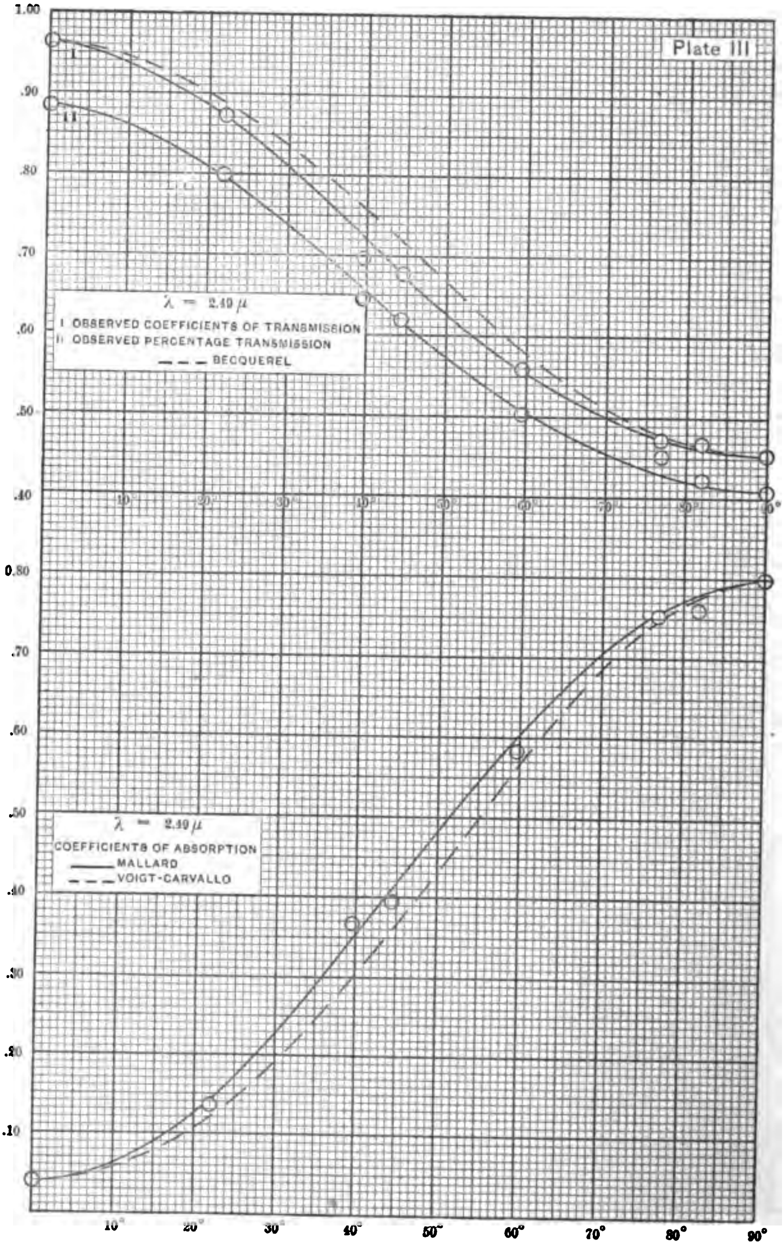




STEWART: ABSORPTION IN UNIAXAL CRYSTALS.

Plate III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II OBSERVED PERCENTAGE TRANSMISSION
– – – BECQUEREL
10°
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λ = 2.49 μ
COEFFICIENTS OF ABSORPTION
——— MALLARD
– – – VOIGT-CARVALLO
10°
20°
30°
40°
50°
60°
70°
80°
90°

ON THE CONVERSION OF ELECTRIC ENERGY IN DIELECTRICS. I.

By Richard Threlfall.

IN the *Journal de Physique* for 1892, p. 5, is a very interesting paper by M. Bouty on the properties of mica. It appears that under certain circumstances the residual charge of M. Bouty's mica condensers may fall to about $\frac{1}{80}$ or less of the original charge.

In September, 1892, I was experimenting with films of exceptionally well purified sulphur, and found, conformably with other observers, that the residual charge of crystalline sulphur, regarded as a dielectric, is remarkably small. This led to a repetition of some of M. Bouty's work on mica, for I desired to compare the behavior of mica and sulphur under exactly corresponding conditions. In the course of this comparison it appeared that *prolonged* drying over phosphorus pentoxide materially reduced the apparent residual charge both with mica and sulphur. As this observation seemed to throw some doubt on the sufficiency of previous work in this direction, the literature of the subject was closely examined, with the result that, with the exception of an investigation by Rowland and Nichols (*Phil. Mag.*, 1881, Vol. 12, p. 418), the distinction between "residual charge" and "creeping surface charge" does not appear to have sufficiently engaged the attention of observers. This led to a desire to examine the general properties of dielectrics in a comprehensive manner; but it was not until September, 1895, that I was able to make any serious attack upon the subject. Meanwhile, I became acquainted (through the technical journals) with the fact that when ordinary mica or paraffin paper condensers are used with alternating currents, great heating generally occurs. This heating was usually ascribed to "dielectric hysteresis," although no definition of the term had been given and no experiments of an unexceptionable kind on the subject had been published.

Lord Kelvin expressed an opinion, in the course of a discussion at a meeting of the Society of Electrical Engineers some years ago, that the cause of condenser heating was most likely to be found in the occurrence of internal discharges rendered possible by the presence of air and moisture accompanying the dielectric employed.

Mr. Swinburn, Mr. Tesla, and perhaps others have given attention to the production of condensers on the commercial scale, though with only a certain measure of success, so far as the tests at my disposal have enabled me to ascertain. From the experiments described above, however, I knew that success must depend not on the partial removal of air and water, but on the absolute removal of these substances, and by experimenting in this direction I have been able to make condensers in which the heating by alternating currents is practically negligible, although the dielectric used was merely paper saturated with a mixture of vaseline and hard paraffin. As I have described the mode of preparation of these condensers at full length in a book on Laboratory Arts, I will not refer to it here, but will merely offer the following data for comparison.

A certain condenser of a capacity of 0.123 microfarads was tested at an alternating potential difference of 3000 volts and with a frequency of 60 periods per second. This condenser when packed in a box and thermally insulated was found to rise in temperature by an amount certainly less than one fifth of a degree centigrade per hour. Compare this with some tests by Messrs. Bedell, Ballantyne, and Williamson (PHYSICAL REVIEW, I., p. 95, Sept.–Oct., 1893) on a waxed paper condenser. This condenser was tested at a potential difference of 500 volts and with a frequency of 160. The loss was ascertained to be from 2 to 3 per cent of the power transmitted; and the rise of temperature, under circumstances similar to my own tests, was at the rate of about 1° C. per hour. In order to see what this comparison means, it is more convenient to consider that energy moves first in and then out of the condenser, than to begin by writing down the activity equation, which, in the case of no condenser loss, is equivalent to saying that no energy is supplied. Looking at it in the way I propose, it appears that the energy supply in the case I tested was, per unit capacity, about thirteen times greater than in the case treated by Messrs. Bedell, Ballantyne, and Williamson. In making a comparison, the actual capacity is, roughly, immaterial, since the volume of condenser matter to be heated and the capacity of the condenser increase together. Assuming that the specific heat of the condenser materials and the heat insulation was the same in both cases, we find that the percentage losses are in the ratio of about 1 to 60 — or the condenser I tested was only losing energy to an extent of clearly less than $\frac{3}{60}$ per cent = 0.05 per cent of the energy supplied. I mention this chiefly as an illustration of the immense effect which small quantities of air or moisture have on the apparent behavior of dielectrics. Of course it is evident that no information as to the purely dielectric action of dielectrics as opposed to their conducting action can be drawn from such experiments, unless it be known with certainty that the conduction losses may be neglected — and this, in general, it is next door to impossible to ascertain. I need only refer to the experiments of Koller (*Wien. Bericht,* 98, II., 201, 1889) on the specific resistance of oils, for a justification of this statement, for this investigation shows how the specific resistance is a matter of previous history, voltage, etc., a conclusion entirely in accordance with my own experience where the resistance of insulators is in question.

The question which I desired to approach experimentally in 1893 was this: Is it true that when surface action is properly guarded against and an elementary volume of a dielectric is carried round a cycle of electric polarization, that some of the energy of electrification is converted into heat, or, at all events, disappears? Assuming that this question is answered in the affirmative (it is), the further question arises as to whether this loss can be explained on the lines suggested by Maxwell (*Electricity and Magnetism*, § 328) and more recently elaborated by Hess (*Journal de Physique*, 3d series, Vol. 2, 145, April, 1893). Although I cannot claim to have finally answered this question, I think the experiments which will be described lead to the conclusion that Maxwell's explanation, though a possible one, is not sufficient in all cases. A third question arose during the investigation, viz., as to whether dielectric loss under prescribed circumstances is a fixed physical property of the dielectric, or whether it is of the nature of an accidental circumstance depending on such minute peculiarities that we are unable in any particular case to predict what the loss will be. Does it in fact resemble the problem of the prediction of hysteresis loss in an untested specimen of iron? This question is finally resolved in the sense that the dielectric loss, though perfectly determinate in any given specimen, is exceedingly variable in passing from one specimen to another, even though every care be taken to secure uniformity. Thus, the loss in the outer layers of a rod of ebonite is different to the loss at the center (per unit volume, of course), and two spheres of resin cast at the same time from the same melting of resin may behave quite differently.

The investigation has been extended to very rapidly varying electric forces, produced by the use of oscillating discharges, in the hope that some light may be thrown on the curious opacity of certain dielectrics — black waxes for instance — but with the unexpected result that dielectric losses become either zero, or, at least, get too small for detection at frequencies of the order of 10^7 per second.

A large number of subsidiary matters, such as the absence of losses in liquid dielectrics, were also investigated.

Before giving an account of my own experiments, it is necessary to deal briefly with the work of other observers in the same field. A detailed consideration would be out of place here, for in the *Electrician* for November 1, 1895, there is an elaborate discussion of the whole position by P. Gasnier. I may as well say here, however, that I consider that in this matter there has been too great a tendency to take things for granted. M. Gasnier assumes the fact of a dielectric hysteresis analogous to magnetic hysteresis, and discusses the question as to whether this hysteresis is "static" or "viscous." The following brief criticism of the then available evidence will, I think, show that this discussion was premature, but my own experiments have, I think, settled the question in favor of the latter hypothesis, at least in certain cases, in the event of Maxwell's suggestion being regarded as insufficient, but not otherwise.

Steinmetz, *Electrotech. Zeitschr.*, 1892, April 29. This paper has been often quoted as proving the existence of dielectric hysteresis. It does no such thing. Mr. Steinmetz measured the phase angle between current and potential difference in a paraffin paper condenser circuit by a dynamometric method, and deduced the condenser losses. The insulation resistance of the condenser was determined by means of a steady electromotive force of the order of the alternating potential difference. The results thus obtained are, of course, quite fallacious on the grounds of uncertainty of resistance; independently of this, however, the greater portion of the loss was probably due to creeping and air discharges.

A very persistent worker in this field has been Riccardo Arno.[1] This investigator made use of an original method of great ingenuity and convenience. An electrostatic field is caused to rotate by the device of combining two mutually perpendicular, harmonically varying electric fields. The harmonically varying potential differences with the necessary phase difference are obtained by placing a small condenser in series with a water resistance and the secondary of a transformer. The general arrangement will be clear from a reference to the diagram which accompanies the latter portion of this paper. The direction of rotation of the field can be reversed by reversing one pair of connections. The armatures bounding the rotating field are formed of portions of a cylindrical surface fashioned in metal and supported on an insulating base.

A small hollow cylinder of ebonite was found to rotate when placed in the space between the armature surfaces.

Arno takes the supposed fact of dielectric hysteresis for granted, and refers to Steinmetz as having established it.

In the second paper on this subject Arno suspends a small hollow ebonite cylinder by a bifilar suspension in the rotating field, and thus measures the moment of the forces brought to bear upon it. This immediately gives the loss of energy per revolution — or per cycle of electric polarization. It is important to note that M. Arno dried his cylinder by a cake of calcium chloride which was inclosed in the apparatus. The potential difference at the terminals of the transformer (an induction coil) varied from .95 to 3.33 electrostatic units. The results of the measurements were represented by a formula of the form

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[1] Riccardo Arno, Accad. dei Linc., Oct., 1892; Elec. Review, Jan. 20, 1893; Accad. dei Linc., April, 1893; Electrician, 31. 201. Also Electrician, 32. 222; Electrician, 33. 210; Electrician, Jan., 1895; Electrician, May 15, 1896.

$$D = CV^{1.6},$$

where D is the angular deflection of the cylinder, C a constant, and V the potential difference employed.

In the third paper the range of electric force is considerably extended, and a long series of numerical results are given as deduced from experiments on paraffined cardboard cylinders: the frequency of field rotation was in all cases 40 per second. Arno thinks that the similarity of his results to those obtained by Ewing and Miss Klaassen in the case of iron lead to the conclusion that he is dealing with "static" hysteresis.

In the fourth paper the frequency of rotation as well as the potential difference is varied, the material still being paraffined cardboard, with the result that the energy waste is found to decrease as the speed of rotation increases — both at low and high potential differences. This causes M. Arno to reconsider the conclusion come to in his former papers.

Arno's conclusions, as distinguished from his experiments, have been sharply criticized by M. Hess in *L'Éclairage Électrique*, Vol. 7, p. 450 (June, 1896). M. Hess very properly takes exception to the use of such heterogeneous material as paraffined cardboard, and shows that Maxwell's suggestion is quite sufficient to explain the results obtained without any reference to the doctrine of hysteresis at all.

I have made large use of M. Arno's method, though in a modified form, for it has many advantages; and I therefore reserve for the present my own remarks on the subject. I regret to say that the *Rendiconti dei Lincei* is not available in Sydney, and I therefore had to trust to translations and abstracts for this brief summary: I can only hope that I have not been misled by them, and have done M. Arno no injustice.

Perhaps the most interesting work which has yet appeared on the subject of dielectric polarization is contained in various notes sent to the *Comptes Rendus* by M. Janet.

In *Comptes Rendus*, 115, 875 (1892), Janet explains his method of experimenting, which consists in breaking the circuit of a condenser, induction coil, and dead resistance, and the instantaneous measurement of certain potential differences by means of "M. Mouton's Disjoncteur." In this paper the theory of the apparatus is given and the oscillations of the condenser discharge are traced out.

In *Comptes Rendus*, 115, p. 1286, the apparatus is applied to determine the inductance of the circuit, and the result is checked and found to be correct by a measurement of the same quantity by Lord Rayleigh's method.

The third paper, to which the others must be regarded as preliminary, is in *Comptes Rendus*, 116, 373 (1893), and is devoted to the study of a certain mica condenser. The ratio of the charge on the condenser to the potential difference of its terminals, forms the subject of inquiry, and it is found that for given values of rapidly changing potential difference the charge is *less* with rising than with falling values. If charge and potential difference are plotted round a cycle, a closed curve or hysteresis diagram is obtained. It is obvious, however, that this is just the effect to be expected if we allow the possibility of the creeping of the charge of the condenser armatures over the imperfectly dried mica surface. But this possibility, I think, has been established by the repetition of M. Bouty's experiment, described at the commencement of this paper. In addition to this, I consider that some criticism of the results obtained by M. Janet may be applied on other grounds; but as this does not materially affect the position, it may be omitted here.

Some rather interesting experiments are described by Benischke (*Wien. Sitzber.*, 102, Part II., p. 1345) on the heating of condensers. A bolometric method was employed, with the result that with glass, mica, and ebonite, some heating was observed; with resin and paraffin, however, no heating was noticed. The author's conclusion is that the heating is due to Joulean wastes and mechanical effects. Benischke refers to several other workers at the subject, but on reference I found that their researches have no bearing on the present inquiry.

In the *Comptes Rendus* for May, 1893, M. Borel described a new phenomenon under the title of "Phénomènes dynamiques dus à l'électrisation residuelle des diélectriques." A disk of some dielectric material is suspended between two conducting plates so that the plane of the disk is horizontal, and in the plane of the lines of electrostatic force. The plates are supplied with an alternating potential difference by commutating the current from a Toepler machine. It is found that when a glass rod is held near to one edge of the suspended disk a sort of repulsion is observed which leads to a rotation of the disk. The drift of M. Borel's explanation will be gathered from the title of his paper. This explanation is, however, merely speculative, and though no doubt M. Borel's experiments deserve careful repetition and variation, I am unable to see that they necessarily prove the fact of "hysteresis."

A large number of experimenters have drawn attention to the very variable values of the dielectric constant obtained when very rapid methods are compared with slower ones. M. Blondlot, Professor J. J. Thomson, Boltzmann, and Northrup may be mentioned in this connection.[1] The general result appears to be that when the dielectric constant is determined by very rapid methods (oscillatory discharges) it tends towards a smaller value than that arrived at by charges of longer duration.

Perhaps Mr. Northrup's paper on this subject places the matter in the clearest light (*Phil. Mag.*, Vol. 39, Jan., 1895).

In the foregoing summary I have confined myself to work which I have seen quoted as leading to the probability of dielectric hysteresis. The list of investigators of the charge of condensers might be immensely extended, and hardly any observer in this field has failed to note some matter requiring further explanation. With regard to dielectric losses, however, there are no experiments, so far as I know, which shut out the possibility of explanation by means either of conduction or of creeping, or of both.

During the progress of an investigation of the properties of sulphur made some years ago, I was much struck by the huge differences assigned to the specific inductive capacity of that substance by different observers; differences which I satisfied myself were really quite outside the range of experimental error, and which I, at the time, attributed to want of definition in the specification of the chemical state of the sulphur employed. In the course of the investigation referred to, I made a very large number of determinations of the dielectric constant of sulphur under varying conditions, and encountered variations in the behavior of even well-specified samples, which, if not entirely outside the range of experimental error, still left in my mind the impression that the specific inductive capacity of apparently similar samples was rarely exactly the same.

Perhaps the most startling inconsistency occurs in the case of muscovite (mica), for which M. Bouty found a well-defined specific inductive capacity of about 8 (*Journal de Physique*, I., 1892, p. 5), while M. Klemenčič by equally careful experiments obtained the value 6 for the same constant. With the help of Wiedemann's *Sachregister*, 1824–1877, and the Beiblätter of Wiedemann's *Annalen* since that date, I have found a large number of observations on specific inductive capacity, many of which, of course, are largely affected by imperfect methods of measurement, but which, nevertheless, give an impression that the constant depends very much on the sample of material selected for observation.

Experimental Work. — In September, 1895, I set up, with the assistance of Mr. Pepper, a copy of Arno's apparatus, with the object of finding out whether it was suitable for qualitative work on the subject of dielectric losses. There is a considerable difficulty in making a suitable high non-inductive resistance, and like Arno, I was driven back upon the use of a column of water. This has the disadvantage of being appreciably heated by the currents employed; for the main condenser must have considerable capacity to swamp the effect of the armatures of the rotating field apparatus. No doubt the variability in resistance which gave rise to the trouble might have been got over, but I soon found that the arrangement was unsuitable in another particular.

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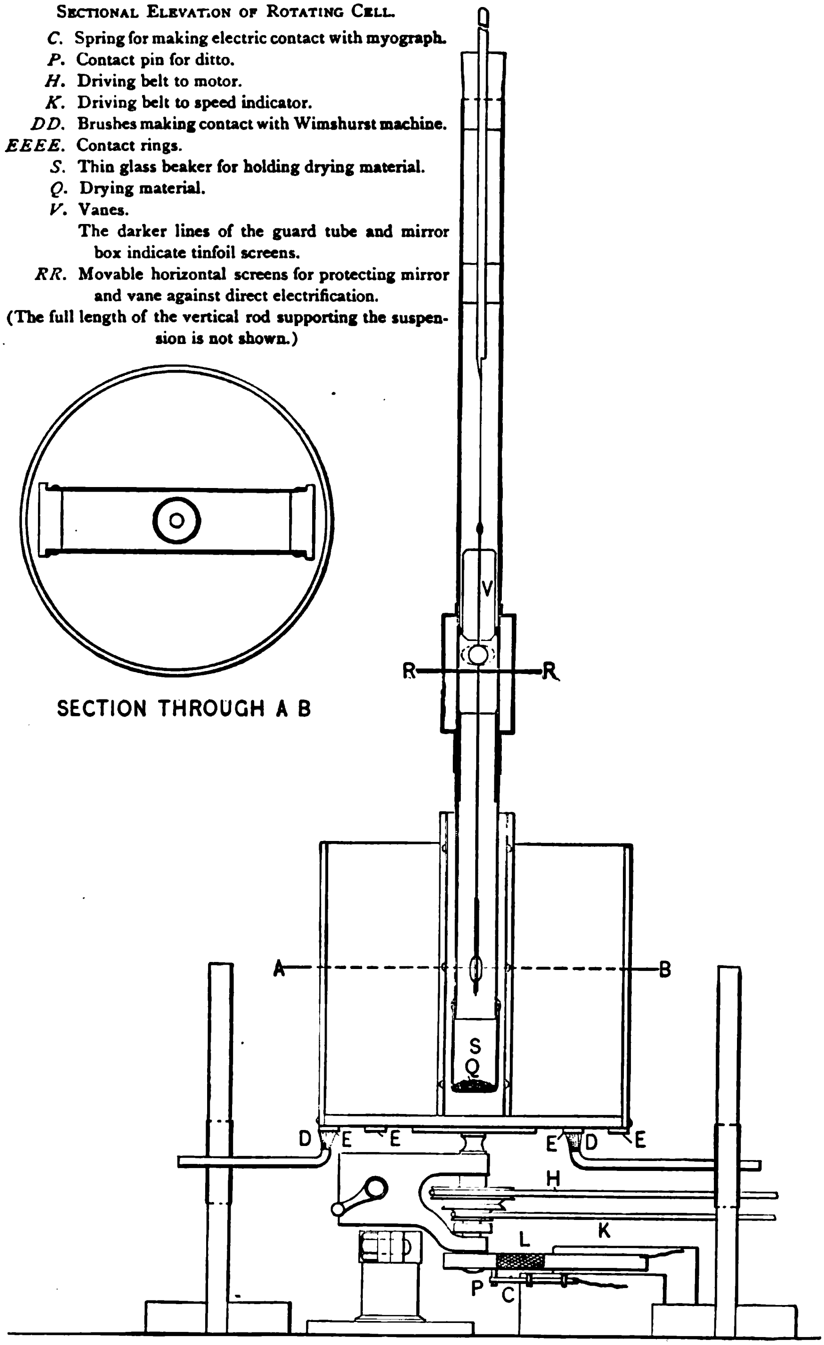
[1] J. J. Thomson, Phil. Proc. R. S., 46, 292 (1889); Blondlot, Comptes Rendus, 112, 1058 (1891); Boltzmann, Gordon's Electricity and Magnetism.

Consideration of the diagram will render it obvious that the field produced within the armatures can only be very roughly "circularly polarized." The leakage of induction between the quadrants placed nearest to one another varies considerably during each cycle, with the result that the harmonically compounding fields, though doubtless of the same mean intensity, are not so at any prescribed moment. By this, of course, I mean that there is a variation of intensity quite large in comparison with the sinusoidal variation postulated by the theory. The polarization must, therefore, be elliptic, and the ellipticity must be variable during each cycle. By roughly drawing the field, under certain arbitrary conditions, I was able to see that in some cases and at points in the field not far from the center, the polarization must be almost *plane*, and this I verified by suspending a mica needle, and observing its behavior at the points in question. As had been anticipated, the needle refused to rotate when placed in certain positions, and vibrated instead about a well-marked direction. It was clear, therefore, that the intensity of the field could not be exactly determined in absolute measure, even at a specified epoch, while if the whole cycle were considered, the variations of strength were so great as to render any exact quantitative conclusion entirely ridiculous.

On the other hand, the method has such conspicuous merits that I considered it advisable to adhere to it as far as possible. In the first place, the quantity sought, viz., the loss of energy, forms the direct subject of the measurement, and that measurement is, moreover, of a remarkably precise and simple kind, viz., the torsion of a quartz fiber. The material employed can also be perfectly and continuously dried; and is used in a form very favorable for the observation of its physical condition. Most important of all, the slope of the potential gradient is very gradual all over the surface of the sample; a condition of affairs exceedingly favorable to the reduction of creeping charges, the uncertain and deceptive action of which I was particularly anxious to avoid. A further check on this source of error can also be easily attained by coating the samples of dielectric superficially with paraffin, varnish, etc.

The modification of Arno's method which I was led to adopt was (1) the substitution of a mechanically rotated uniform field for a field produced by the composition of harmonic motions, and (2) the presentation of the



Sectional Elevation of Rotating Cell.

C. Spring for making electric contact with myograph.
P. Contact pin for ditto.
H. Driving belt to motor.
K. Driving belt to speed indicator.
DD. Brushes making contact with Wimshurst machine.
EEEE. Contact rings.
S. Thin glass beaker for holding drying material.
Q. Drying material.
V. Vanes.
The darker lines of the guard tube and mirror box indicate tinfoil screens.
RR. Movable horizontal screens for protecting mirror and vane against direct electrification.
(The full length of the vertical rod supporting the suspension is not shown.)

SECTION THROUGH A B

Fig. 1.

substance to be examined in the form of an ellipsoid, which permits us to calculate the actual field to which it is exposed, and hence to reduce the results of observation to absolute electrostatic measure.

The alterations proposed were first thoroughly tested by means of an apparatus chiefly constructed of wood; and the experience obtained was then applied to the construction of the permanent apparatus, which is shown partly in elevation and partly in section in Fig. 1. The base of the rotating part is formed of the head stock of a watchmaker's lathe, suitably supported. This carries the revolving cell, consisting of two zinc plates with suitable stiffening and wire edges, mounted on a disk of ebonite and surrounded by a case of varnished paper which rotates with the cell and prevents it acting too much as a fan when it is driven round at a high rate of speed. The drawing is to scale, but the following dimensions may be quoted. The zinc plates were 3.8 cm. apart. The width of each plate was 15.8 cm., and the depth, *i.e.* the dimension parallel to the axis of rotation, was 17.8 cm. The two zinc plates were of rolled zinc and were about 1 mm. thick; they were held in position by milled ebonite end pieces; and these were firmly screwed to the face plate of the lathe. The diameter of the varnished paper cylinder surrounding the whole was 19.6 cm., and its thickness was 15 cm. Paper was chosen for this purpose, as when wrapped upon a cylindrical former it makes a light but strong protection and can readily be fashioned so as to have the exact size required. The whole of the rotating parts were most carefully balanced — a precaution absolutely necessary. The driving cone of the apparatus was belted to a shunt electric motor, driven from a constant potential circuit, and was governed by the Foucault current resistance which a large brass disk, mounted on the motor shaft, experienced when exposed to the action of a large horseshoe electromagnet with poles at equal distances from the axis of rotation. The electromagnet was provided with current through a variable resistance, and could also be moved with respect to the rotating disk. This method of governing is the precise electrical analogue of the method used by Lord Rayleigh to govern a water motor in his determination of the absolute value of a resistance by the British Association and Lorenz methods. The fanlike action of the cell itself also assists the governing.

The second belt from the driving cone of the cell passed to a speed indicator of novel but simple form. My object was to employ a speed indicator of such a kind that very little power would be required to work it, and which would work over a large range of speed. The advantage of using a speed indicator which required but little power is that it can be driven by a belt without fear of the indications being affected by the slipping of the belt. The speed indicator really consisted of two parts, one arranged as an inspectional instrument to afford information as to the constancy of the speed, and the latter designed to afford an exact measurement of the speed at any specified instant. The inspectional instrument consisted of a minute fan revolving in a brass case and measuring about 5 cm. in diameter over all. The air pressure produced by this fan was observed in the following way — it was designedly too small for a hydrostatic gauge to be of any use. A wide piece of brass tubing, 7 cm. diameter and 8 cm. long, was closed by means of a cork at one end and was provided at the other with a diaphragm of fine rubber tissue very evenly stretched. The tube was placed with the plane of the tissue vertical, and at one end of the horizontal diameter of the face a galvanometer mirror was fastened. The position of the mirror was observed by a lamp and scale. The air in the brass tube was put in communication with the fan by means of rubber tubing, so that when the fan rotated the air pressure produced was communicated to the diaphragm and caused it to bulge outward, thus slightly rotating the mirror.

The apparatus for registering the speed of rotation — or calibrating the speed indication — consisted merely of a contact arrangement on the vertical shaft, below the driving cone of the lathe head stock. The contacts were made once every revolution and worked the electromagnetic arrangements of a myograph in an adjoining room. The general practice was to calibrate the speed indicator by the myograph so that a rough idea of the frequency of rotation could be obtained, and then this speed being kept constant it could be measured exactly, if necessary, at any convenient moment.

The arrangement worked perfectly from the start, almost any desired degree of sensitiveness being attained by adjusting a leak from the brass tube; and by a pinch clamp working on the rubber tube connecting the indicator with the fan. It takes almost as long to describe the apparatus as to make it. Mine was made during one morning.

The Suspended Apparatus. — A brass tube about 2.5 cm. diameter was soldered to a strong brass bar, so that when the bar was horizontal the tube was vertical. The heavy slide rest of a lathe was placed on a wall bracket immediately above the rotating apparatus (which was clamped to a slate table), and the brass bar was carried by the slide rest. Everything was exceedingly strong, heavy, and rigid, and this is necessary. The brass tube carried the suspended ellipsoid of dielectric matter, and was furnished in the usual way for a quartz suspension. The tube terminated in a cylindrical wooden box in which a window was cut. Below the box and coaxial with the brass tube was a glass tube of thin, flint glass, carefully dried and varnished. This tube ran into a shallow, thin glass beaker containing phosphorus pentoxide, and was cemented to it by soft wax.

The slide rest served as a means of adjusting the suspension tube concentric with the axis of rotation of the rotating plate system, and being itself supported on three screws, perfect leveling could also be obtained.

The thick lines in the diagram which bound the upper part of the glass tube both outside and in, indicate the tin-foil screening which it was found necessary to solder on to the brass tube — itself carefully earthed. Two brass screens were also introduced just below the suspended mirror, for, of course, perfect screening of the mirror is essential. These side screens could be taken in and out as required for the purpose of raising or lowering the suspended dielectric.

The Dielectric and its Mounting. — Samples of dielectric were prepared in various ways. Ellipsoids of various shape and size were employed (including spheres), the largest of which, actually employed for observation, had a horizontal diameter (when mounted) of about 1 cm. The inner diameter of the glass tube was 2.504 cm., and the outer diameter 2.750 cm. It will be shown further on that the field inside such a tube, arranged in an initially uniform field, is itself uniform. When the dielectric is introduced, the field becomes non-uniform in the space outside the dielectric, and the ellipsoid is in unstable equilibrium when suspended coaxially with the tube. For this reason it is desirable, from the point of view of convenience of observation, to keep the diameter of the ellipsoid down to about 1 cm. This conclusion was the result of many trials.

It was a question as to whether the suspension of the ellipsoid should be conducting and earthed, or as perfectly non-conducting as possible. Owing to the convenience of using quartz threads and the inconvenience of silvering them, the latter alternative was adopted, but it led to much trouble. It was not until the ellipsoid was carried on a quartz needle that electrification of the mirror and vane was got rid of. Solid dielectrics, such as ebonite, were prepared by turning on a lathe, and were correctly drilled to receive the quartz needle. Other dielectrics were cast in a sort of zinc bullet mold around a short length of filamentary glass tube thrust through the mold by means of properly bored holes. These narrow tubes were prepared just to admit of the passage of the quartz suspension needle, and formed an easy and accurate mode of suspending the ellipsoids in position. When an ellipsoid had been mounted the quartz needle was heated by a minute Bunsen flame so as to make sure of its insulation, and the ellipsoid was diselectrified by the flame, after which the suspended apparatus was carefully lowered into the suspension tube, which was leveled to center it, and the whole was then adjusted to the rotating cell by means of the screws of the slide rest. A little soft wax made the affair quite air tight, and gave the phosphorus pentoxide an opportunity to exert its proper influence.

As the weight of the suspended parts was not of importance, a substantial glass mirror, carefully figured to a radius of about 1 meter, was employed to indicate deflections, and this was surmounted by a mica vane nearly filling the tube. The semi-transparent scale was of celluloid bent to a radius of 96 cm. — the invariable distance of mirror from scale. Some of the earlier observations were taken on a straight scale and reduced to angles by a graphic method.

Electrification of the Rotating Cell. — As the result of a number of trials, one plate of the cell system was kept to earth, the other being charged by an electrical machine. The plates were kept in contact with earth and the source, respectively, by means of brushes bearing on rings beneath the ebonite disk which formed the base of the cell.

An eight-plate Wimshurst machine, driven by a constant potential motor, was used as a source of electrification. A battery of Leydens was employed to steady its action. The potential difference of the plates was measured by a Kelvin electrostatic voltmeter, whose readings were found to be self consistent, but the absolute value of which was not tested.

The potential difference was kept constant by interposing a short circuit consisting of a pile of vulcanized fiber plates and oiled paper. These were mounted under a screw on an iron stand, and an assistant kept the voltmeter steady by working the screw. A good many plans were tried, but this was found to be the most convenient one.

Sources of Error. — The most important and persistent source of error was found to be a shifting of the zero on the dynamometer scale. At first this was large, but careful screening of the mirror, etc., by tin foil, and the introduction of the horizontal screens under the mirror, immensely reduced it. The ebonite used in the construction of the cell itself also tended to become electrified, and produced shifting. This was prevented from becoming serious by wiping the ebonite ends of the cell now and then with a rag moistened with kerosene. The ebonite base between the conducting rings also became electrified, but all such effects were removed by a thick coating of tin foil well earthed, which lined the whole of the base of the cell and the inside of the paper guard cylinder. This involved raising the zinc plates about 1.5 cm. from the ebonite base, the space being filled in with mica to reduce the fan action. The electrification of the glass guard tube at first gave trouble, but this was got over by never applying the potential difference until the cell was in rapid rotation. The remaining cause of error, viz., the electrification of the dielectric ellipsoid itself, was never entirely got over. It depended on the material of the ellipsoid, and some materials, *e.g.* resin, never gave really satisfactory results from this cause. It was not till near the close of the observations that it was certainly discovered that the shifting zero was dependent on the nature of the ellipsoid. With glass, sulphur, and paraffin I have had samples giving a zero absolutely steady, so far as I could discover, *i.e.* the light spot did not move by more than 0.1 mm. on the scale. At an early stage it was noted that this shift of zero was a phenomenon requiring time, and it was also happily independent of the direction of rotation of the cell: it depended merely on the potential difference and the length of time this had been applied.

An exhaustive series of experiments was made in order to discover the best mode of observing the deflections, a certain ebonite sphere which exhibited considerable "creeping of the zero" being selected as the subject of examination. A series of measurements was made by electrifying the plates to varying potential differences, observing the rotation of the sphere, and then connecting the plates and observing the zero. This series of observations was compared with a set in which the deflections and potential differences were simply read off with a cell rotating alternately in one and the other direction. The differences only amounted to one or two per cent, and the curves expressing the relation of the logarithms of the deflections of the sphere and the logarithms of the potential difference were indistinguishable. This being established, the latter mode of observation was invariably adopted, subject to the following precautions: The zero reading at the commencement of each set of observations was made identical by diselectrifying the ebonite end pieces, and waiting, if necessary, for the dielectrics under examination to lose their charges. Observations were then made after equal lapses of times at the potential differences agreed upon, first with the cell rotating in one direction, and then with the direction of motion reversed. The observations thus obtained were found to be perfectly consistent on repetition, and even after the material had been dismounted and put back again.

Effect of the Position of the Sphere between the Plates. — The guard tube was always set so as to be coaxial with the rotating system, but experiments were made to test the effect of a sensible want of centering. It was found that when the centering was imperfect, the zero changes became more troublesome, but that only a slight effect was produced on the mean deflections.

Raising and lowering the dielectric in the guard tube had an inappreciable effect even when the range of motion approached a centimeter in either direction. As the cell had been designed to give a uniform field over a range of some centimeters, within the limits of about 1 per cent, this was only to be expected. I was unable to make a rigorous investigation owing to the finite width of the plates, but considerations of symmetry show that in the axis the distribution of potential is relatively the same as between plates extending to infinity on either side. This case, therefore, was considered, and the design based upon it. My present position, however, is that the design was justified by experiment.

Effect of Want of Centering of the Sphere or Ellipsoid within the Glass Tube. — The effect of a want of centering here is to aggravate the normal instability, and the effect on the observations was to immensely increase the uncertainty of the zero position owing to the increased electrification of the suspended dielectric. Care was always taken, therefore, to center the ellipsoid with respect to the tube as perfectly as possible, and this could be done by leveling in perpendicular planes with the requisite accuracy.

I may say that had I known the extreme variability of the quantity I set out to measure from the commencement, most of this work would have been unnecessary; but, at first, whenever I got two pieces of ebonite or sulphur to exhibit different effects, I assumed that the apparatus or mode of observing was at fault, and it was only after satisfying myself that this was not the case that I adopted the hypothesis of variability in sample, which afterwards turned out to be the most strongly marked feature of the phenomenon.

Influence of the Guard Tube. — The influence of the cylindrical guard tube in an originally uniform field can be calculated by means of Maxwell, Vol. II., § 432, together with the consideration that in a uniform field the strength of field within such a tube is the same as that within a spherical shell inscribed in the cylinder. Taking the origin of coördinates at the center of the shell, Fig. 2, and supposing that the direction of the field is in the plane xz and parallel to the x axis, it is clear that the influence of the sphere and shell are identical along that axis. But Maxwell shows that the field inside the sphere is uniform, and if the extension in the z direction is infinite, the field inside a cylinder must be uniform in virtue of symmetry: hence the field at any point inside the cylinder is the same as that inside the shell. This, of course, follows at once if we put the z axis of the ellipsoid infinite in Maxwell's investigation.

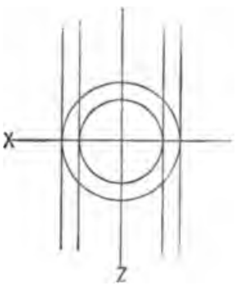


Fig. 2.

Making use of Maxwell's equation, § 433, we find that the force inside the cylindrical tube of radii of curvature a_1 and a_2 for the two surfaces is equal to the force outside multiplied by

$$\frac{9(1+4\pi K)}{9(1+4\pi K)+2(4\pi K)^2\left[1-\left(\frac{a_1}{a_2}\right)^3\right]}$$

where $K=\frac{\mu-1}{4\pi}$, where μ is the specific inductive capacity.

The above expression is, therefore, equivalent to

$$\frac{9\mu}{9\mu + 2(\mu - 1)^2\left(1 - \frac{a_1}{a_2}\right)^3}.$$

Now with the glass tube employed I had no means of determining μ, for it so happened that I had only about a foot in length of tubing of a suitable diameter and thickness of wall. This tubing was of flint glass, made by Powell, and had a density of 3.22. The specific inductive capacity of a flint glass of density 3.21 was examined by Hopkinson, or rather he examined the specific inductive capacity of Chance's light flint of density 3.21, and I have assumed that this would be practically identical with the glass I used. The value for μ found by Hopkinson was 6.85. Using the value 6.8, the value of the above expression becomes, in the case quoted, 0.786, which is hereafter referred to as the "screening factor." Now my object was primarily to investigate the laws of dielectric loss under similar circumstances in different dielectrics, and as the exact value of the screening factor only affects the very variable *absolute* value of the losses, it is not of any great importance. A little consideration, however, will show that the screening by the guard tube is a matter requiring close attention. In the first place, the field outside the glass tube is not rigorously uniform, consequently the method of calculating the screening does not rigorously apply. In the second place, the whole process is doubtless vitiated to some extent by the fact that the polarization of the material of the guard tube is not exactly in phase with the electric intensity any more than it is in the suspended dielectrics. The result of this action is to give, so far as effects of the second order are concerned, an exceedingly complex field inside the guard tube, the nature of which it is not easy to forecast exactly. However, the chief effect will be to cause the rotating field within the guard tube to lag slightly behind the field corresponding to air alone, and, at the same time, to slightly reduce the screening effect. Now the experiments to be described, as well as those of Arno already quoted, show that the magnitude of the dielectric losses depends on the speed of field rotation; so that, strictly speaking, a special screening factor should be employed for each separate speed of rotation. It appeared that nothing but an extensive series of experiments could afford the required evidence as to the complex effects of screening. Accordingly, the months of November and December, 1895, were taken up in investigating this matter in the following way. Three guard tubes were used: the one already mentioned, one of thin ebonite, and one of mica — bare, varnished, and paraffined. Complete series of observations were made on ellipsoids of glass, ebonite, and sulphur at potential differences of the plates from 250 to 6000 volts, and even to 12,000 in some cases. The speed of rotation varied from 5 to 6 per second, up to 28, the greatest frequency attained. The results of observation were first plotted simply as curves of angular deflection and potential difference, and afterwards by plotting the logarithms of these variables. The reproduction of the laboriously acquired results, however, would serve no useful purpose, and I will, therefore, merely state that the logarithmic curves were in all cases straight lines, and that their regularity was such that on submitting them to my friend, Mr. Knibbs, an expert in the reduction of observations, he was able to assert that, so far as the numbers went, the value of the index n in the equation $D = aV^n$ (D, angular deflection of dielectric; V, potential difference of plates) could be relied upon to about unity in the third significant figure. The curves referring to the same dielectric in the three guard tubes were, as nearly as could be seen, exactly parallel; and though the intervals were not quite the same at high as at low speeds, the differences were not worthy of attention from the point of view of the present inquiry. It also appeared, as was to be expected, that the hygrometric state of the air had a very large influence on the result; and, as this could not be very well controlled, one result of the investigation was to show that it was useless to make measurements except in dry weather, and that it was very necessary to have plenty of drying material inside the guard tube.

The calculated screening factors were:

Glass tube	0.786
Ebonite tube	0.966
Mica tube	1.000

With regard to the deflections in glass and ebonite: In a certain experiment which may be selected as having involved high voltages, the deflection of a sulphur ellipsoid in the glass tube was compared with its deflection in the ebonite tube. The calculated ratio of deflections was 1.50 to 1, and the observed ratio was 1.54 to 1.

In the case of the mica tube the results were less satisfactory, but as this was traced to the poor insulating power of the only mica of suitable size I could procure, nothing need be said about them.

The glass tube was finally selected as the more suitable on account of its transparency, and the whole of the subsequent observations were made in it. Up to this point I had enjoyed the advantage of the assistance of my friend, Mr. Pepper, to whom I hereby desire to express my indebtedness.

Effects of Moisture. — The invariable effect of a damp day was to decrease the deflections, owing to the extra screening of the guard tube. By drying the tube externally by hot dry air the deflections could be increased for a short time. The action of the rotating cell is to produce an enormous down draught all round the guard tube, so that unless the air is dry, *i.e.* contains not more than about 60 per cent of the maximum moisture, no reliable absolute results can be obtained. The only remedy is to abstain from working in damp weather; and after this fact was discovered I made no measurements when blowing in hot air was found to have an appreciable effect. It was afterwards found, however, that different samples of the same substance, tested under exactly similar conditions, gave results differing much more from one another than any difference produced by a rise of humidity to, say, 75 per cent, and consequently a large number of observations which I at first threw aside as likely to be affected by moisture have a certain value.

Effect of Want of Centering of the Suspended Dielectric. — Owing to the mode of preparation adopted, this never occurred to an appreciable extent.

Effect of Want of Perfection of Form in the Suspended Dielectric. — When the period of rotation of the field is small compared with the period of vibration of the ellipsoid, want of precision of form has no direct effect on the rotation produced. This was roughly checked by comparing the behavior of two cylinders of ebonite, one of which was perfect in form and the other intentionally mutilated by filing. Of course, there is an indirect effect in so far as the actual electric intensity inside the ellipsoid depends upon its shape.

Effect of Want of Perfect Drying of the Dielectric. — I had hoped that the absence of points of sudden variation of electric intensity over the surface of the dielectric would have removed the necessity for drying. This hope, however, proved to be vain. The effect of the slightest moisture was generally to increase the deflections for a given potential difference, so that in most of the experiments the material was left in presence of phosphorus pentoxide for at least three days, and the readings were not passed till they became invariable. On the other hand, the effects of paraffin and shellac varnish on the deflections of glass, sulphur, and ebonite ellipsoids were studied, with the result that differences of the order of a few per cent only were ascertained; that is, of course, when the drying was thorough. Great pains were taken over this part of the work, as the whole object of the investigation was to separate effects due to creeping from the other phenomena. The sort of agreement observed may be illustrated by an example of the behavior of an ebonite sphere of 1 cm. diameter at a frequency of 16.

The reason for the fall observed in the observations 2 and 3 is the slight but inevitable damping of the inside of the guard tube by the admission of damp air (the humidity was 72 per cent). In the second and third observations the ebonite was dried by flame gases before it was put back in the tube, and this was done while it was warm, *i.e.* at about 40° C. in both cases. Many cases both of better and of worse agreement could be cited, but this is a fairly typical one.

TESTS OF AN EBONITE SPHERE UNDER VARIOUS CONDITIONS OF ITS SURFACE.

State of surface.	How dried.	P. D. of cell.	Rotation observed.
1. Paraffin.	Exposed to the action of P_2O_5 for 48 hours.	3500	253′
2. Paraffin washed away by benzine.	Dried by hot air, and placed while hot in the guard tube.	3500	207′
3. Reparaffined.	Dried by hot air, and placed while hot in the guard tube.	3500	204′

METHODS OF OBSERVING.

The method of making observations was as follows: The conditions as to dryness having been found to be satisfactory, the cell was started and the deflection of the tachometer noted. This was reduced to absolute value by means of a curve, and was, when necessary, further checked by an observation at the myograph.

The zero of the dynamometer was noted, the earth connection broken, and the cell raised to a high potential. A shift of zero indicated insufficient screening of the mirror and vane, and when this occurred the apparatus required attention. If no shift of zero appeared the earth connection was restored and observations taken at every 5 divisions of the voltmeter, *i.e.* at intervals of 500 volts. It was the business of the assistant at the regulator to keep this constant. If the needle moved off the mark to the extent of half a division on either side, he gave a signal and the observer at the dynamometer (myself in all cases) suspended operations. The practice was to take ten elongations of the light spot at each potential difference — five in each direction — and some time was allowed both for reasons already stated, and to eliminate any want of attention to the adjustment of the potential difference. As a rule, the observations were carried from 0 to 60,000 volts, after which the cell plates were connected first by a high and then by a low resistance, and the dynamometer zero was reobserved. In many cases no shift whatever was noted, and no series of observations for standard purposes was passed until this condition became satisfactorily satisfied — it was, with some exceptions, chiefly a question of drying. The cell was then stopped, its motion reversed, and the same series of observations gone through. The deflections were then reduced to "single" deflections, *i.e.* the actual rotation of the spheroid, and expressed in minutes of arc by reference to a curve; and the logarithms of these rotations and of the potential differences were plotted out on squared paper.

TREATMENT AND REDUCTION OF OBSERVATIONS.

Let θ be the rotation in minutes of the ellipsoid under the influence of a potential difference V (measured in units of a certain number of volts, generally 100). Then by plotting logarithms it was found that, within the limits of errors of observation, the relation could be represented by a formula of the kind $\theta = \alpha V^n$, where α and n are constants.

n was determined from the plot, and then a pair of simultaneous values selected from the curve gave α.

It is required to reduce the relation thus obtained to electrostatic measure, and express the waste of energy per cycle in ergs as a function of the electric intensity inside the dielectric. The perpendicular distance of the plates of the cell was 3.8 cm., and the screening factor for the glass tube was 0.7865.

Let S be the numerical factor required to reduce the external electric intensity to the intensity inside the ellipsoid.

Let F be the internal electric intensity.

Then, with the above data and the value of the ratio of the electrostatic to the electromagnetic units,

$$F = 0.069\, SV = qV. \tag{2}$$

The value of S is calculated from the shape of the material used and from its specific inductive capacity — exactly as in the magnetic case — for which Ewing's *Magnetism in Iron and Other Metals* may be consulted.

There is no need to reproduce the argument here, but it may be mentioned that the *correction* is important.

For instance, for a certain spheroid of ebonite with diameters 2.646 and 0.984 (suspension being about the longer diameter, the value of S was 0.396, and for a sulphur ellipsoid of similar form $S = 0.43$.

The energy lost per revolution is evidently proportional to the angular deflection of the ellipsoid. Let W be the energy loss per unit volume per revolution, *i.e.* per cycle of electrification. We may write

$$W = k\theta, \tag{3}$$

where k is a constant.

Let ω be the volume of the ellipsoid.

Let the thread be vibrated by a body of moment of inertia m, and have a period of vibration t, and let p be the value of one minute of arc in circular measure.

Then
$$k = \frac{4\pi^3 mp}{\omega t^2}. \tag{4}$$

Then, from (1), (2), (3),

$$W = k\alpha\left(\frac{F}{q}\right)^n = CF^n.$$

Let K be the specific inductive capacity of the dielectric employed. Then the energy per unit volume due to the electrification is

$$\frac{KF^2}{8\pi}.$$

There is no meaning to be attached to the phrase "percentage of energy lost per cycle," because n is not in general equal to 2. Since, however, the experiments to be described give values for C and n, the percentage loss can be calculated for any assigned value of F (of course within the limits covered by the experiments).

One illustration of the methods of calculation will suffice. I select at random the following:

Sulphur Ellipsoid K. — The sulphur was composed of what I have already described (Royal Soc. paper unpublished) as "aged monoclinic sulphur" combined with 0.708 per cent of insoluble sulphur. I say "combined" advisedly as opposed to "mixed."

Major diameter of spheroid, 2.55; minor diameter, 0.97; mass, 2.5772 g. Specific inductive capacity 3.2 (*loc. cit.*); volume, 1.29 cc.; frequency of field rotation, 18 per second.

The observations gave $n = 1.857$, and the relation

$$\theta \text{ (in minutes)} = 0.143\, V^{1.857},$$

V being in 100 volt units.

The value of S in this case is 0.43. The moment of inertia of a cylindrical vibrator was 0.079; and the period of vibration 30″.

Using these data we find

$$W = 0.00048\, F^{1.857}.$$

In the case where $F = 1$, this gives a percentage loss of about 0.0038 per cent per cycle, at 16 cycles per second.

As it happens, the observations leading to this relation were about as irregular as any ever admitted, and I will, therefore, tabulate them. It is no use to draw the logarithmic curve, for when reduced to the size of these pages, all the observations would probably appear to fall upon the line. In many cases, using other substances, even when plotted on $\frac{1}{10}$-inch paper, on a scale of 10 inches to unity in the logarithms, the errors were of the same order as the error of ruling of the paper, and could not be detected without close scrutiny.

READINGS OF VOLTAGE AND DEFLECTION.

Sulphur Ellipsoid K. May 12, 1896.

Voltmeter readings at 100 volts per division.	Reading of dynamometer.	Reading of dynamometer.	Double deflection in scale divisions; i.e. centimeters at 96 cm. from mirror.	Rotation of ellipsoid in minutes of arc.	Log: minutes.	Log: (100) volts.
	Rotation counter-clockwise. Number of rotations 18 per second.	Rotation clockwise. Number of rotations 18 per second.				
10	18.13	17.09	1.04	10.0	1.000	1.000
15	18.37	16.76	1.61	15.0	1.176	1.176
20	20.07	16.16	3.91	35.0	1.544	1.301
25	21.41	15.37	6.04	54.0	1.732	1.398
30	23.17	14.35	8.82	78.5	1.895	1.477
35	25.21	12.80	12.41	111.5	2.047	1.544
40	27.21	11.61	15.60	139.0	2.143	1.602
45	29.00	10.08	18.92	168.0	2.225	1.653
50	31.37	8.70	22.67	202.0	2.305	1.699
55	33.76	7.40	26.36	234.5	2.370	1.740
60	37.40	5.64	31.76	282.0	2.450	1.778

The observations at 4500 volts were selected from the curve for calculation.

EBONITE.

A large number of samples of this substance were examined, and it was found by their aid:

(1) That the index n of the curve in the equation

$$W = CF^n$$

is independent of the nature of the guard tube, or the state of dryness of its surface, but that, on the other hand, it depends almost entirely on the dryness of the ebonite surface.

(2) When the speed is varied the index remains constant, though the value of C varies a little with a guard tube which does not insulate perfectly. Though I never entirely succeeded in eliminating this effect, it became, under suitable circumstances, so small that I think there is no doubt that with the samples examined there is no change of n as the

speed of rotation increases from about 10 to 30 per second. Even in those cases in which C appeared to vary there was no variation of n.

(3) The constants are independent of the nature of the surface, *i.e.* it does not matter whether the ebonite is polished dry; is smeared with hard paraffin or vaseline; or is rough, or varnished with shellac varnish, so long as the surface is in good insulating condition.

(4) No two samples of ebonite have given the same result, though the same sample may be dismounted, remounted, and examined under circumstances as varied as possible without giving variable results.

The following data may be regarded as typical:

Ebonite spheroid	"Long A."
Longer diameter	2.646 cm.
Shorter diameter	0.984 cm.

The spheroid was suspended about its longer axis.

Weight of spheroid	1.610 g.
Volume of spheroid	1.382 c.c.
Specific inductive capacity, assumed	3.5.
Internal screening factor	0.396.

After being dried by phosphorus pentoxide for 14 days, and on a day when the air contained 45 per cent of the possible moisture, I found

$$W = 0.029\,F^{1.765}.$$

Percentage loss, 21.2 when $F = 1$.

Percentage loss per cycle, $21.2\,F^{-0.235}$.

This was at a field rotation of 15 revolutions per second. On varying the speed and keeping the potential difference (5000 volts) constant, the following deflections were obtained:

Field revolutions per second.	Double deflections in centimeters.
11	31.97
15	31.82
30	32.21

The very slight increase at the higher speed is, possibly, to be attributed to the effect of a slight residual conductivity of the surface of the guard tube.

UNIVERSITY OF SYDNEY, NEW SOUTH WALES,
November 23, 1896.

[*The conclusion will appear in the following number of the* PHYSICAL REVIEW. *The article is divided arbitrarily at this point.*]

MINOR CONTRIBUTIONS.

THE APPLICATION OF THE INTERFEROMETER TO THE MEASUREMENT OF SMALL ANGULAR DEFLECTIONS OF A SUSPENDED SYSTEM.

BY F. L. O. WADSWORTH.

IN an interesting paper in the last number of the PHYSICAL REVIEW, "On the development of the Idiostatic Electrometer," Professor Barus has described an application of the Michelson "interferometer" to the measurement of the small *linear* movements of the electrometer disk, and has suggested the use of the same instrument in the measurement of the *angular deflections* of an ordinary suspended system. If such an application is practicable, it is at once evident what a field of usefulness is open to the interferometer in the measurement of minute deflections, which our present means of reading do not enable us to measure with any degree of accuracy. It may therefore not be uninteresting in this connection to state that this application has already been made, and its practicability under proper experimental conditions demonstrated. The difficulties encountered, however, particularly by those who should attempt to use the instrument for the first time, are even greater than those indicated by Professor Barus. Indeed I have found that many find difficulty at first in finding and adjusting the fringes at all, even when working with the most stable form of instrument with all the mirrors fixed. These difficulties are rendered greater than they need be by the fact that, although the theory of the instrument has been fully discussed and explained by both Professor Michelson and Professor Morley,[1] and the general types and special forms for both linear, angular, and spectroscopic measurements described by Michelson in a number of papers,[2] there has not been published, so far as I am aware, any description of those details of construction and manipulation which seem comparatively unimportant to those rendered thoroughly familiar with the instrument by long use, but which nevertheless are likely to occasion difficulties to those taking up the construction or the use of one of these instruments for the first time, particularly when, as in this case, they are not commercially obtainable in standard forms. For this reason I take this occasion to go over the general construction and details of adjustment common to all forms of interferometer before describing the special forms of instrument with which the experiments on the measurement of angular deflections of a suspended system were made.

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[1] See paper, Interference Phenomena in a new form of Refractometer, A. A. Michelson, Phil. Mag., Vol. 46, p. 395, 1882; Théorie de L'Appareil D'Interférence, A. A. Michelson, Trav. et Mem. du Bureau internat. des Poids et Mesures, Annexe II., Tome XI., p. 115; On the Measurement of the Expansion of Metals by the Interferential Method, Part I., Optical Principles on which the Method depends, E. W. Morley, PHYSICAL REVIEW, Vol. IV., p. 3. Also joint papers by Michelson and Morley, On the Feasibility of making the Light Wave the Ultimate Standard of Length, Amer. Jour. Science, Vol. 38, Sept., 1889, and Journal of Association of Eng. Soc., May, 1888.

[2] Measurements by Light Waves, Amer. Jour. Science, Vol. 39, p. 115; Application of Interference Measurements to Astronomical Measurements, Phil. Mag., Vol. 30,

In his paper "Measurements by Light Waves" in the *American Journal*, Michelson divides interferometers into five general groups, the distinction between which in some cases does not seem very clear. It has always seemed to the writer that a division into two general classes, according to the character of the source of light with which the instrument is to be used, would be better and more comprehensive. On this basis the first class would include all those forms of interference apparatus in which the source is necessarily of very small angular magnitude; types of these being Fresnel's and Loyd's mirror systems, Billet's bi-lenses, and Michelson's "telescope refractometers."

The second, and by far the more important class, comprises those instruments which may be used with a source of any angular magnitude, *i.e.* an extended luminous surface. In this class, of which the earliest example was Jamin's refractometer,[1] there is necessarily a plane of separation, generally an optical surface of some transparent material, at which the incident light is divided into two beams, one reflected, the other transmitted; which pass away from the surface in different directions, and are afterwards brought together again by any desired combination of reflecting surfaces or lenses. If the transmitted ray is allowed to emerge from a second surface parallel to the first (surface of separation), it is evident that we may, by simply changing the inclination of the latter to the incident light, obtain any angular divergence of the two separated beams from 0° to 180°. Figs. 1, 2, 3, and 4 show four forms in which the separation β of the two rays is respectively 45°, 90°, 135°, and 180°.

In these figures the single heavy arrow indicates the beam of incident light, the two thin arrows the reunited rays which produce interference. The shaded plates are mirrors (silvered on front surface), the unshaded

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p. 1; Application of Interference Measurements to Spectroscopic Measurements, Part II., Phil. Mag., Vol. 34, p. 280; and Smithsonian Contributions to Knowledge, Application of Light Waves to Metrology, Nature, Nov. 16, 1893.

[1] Ann. de Chem. et de Phys., 3d Ser., Vol. 52, p. 163, 1858.

plates are plane parallel glasses, the heavy shaded side of the one marked *A* being the side at which the separation of the two rays takes place. In Figs. 1 and 2, the two rays are reunited at this same surface; in Fig. 3, at a second surface, *A'*.[1] In the former case a compensator *C* is required, in the latter none is needed.[2]

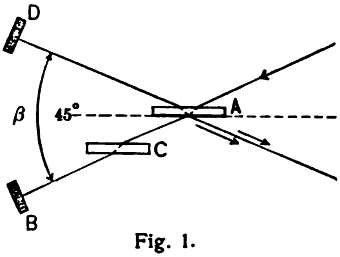

Fig. 1.

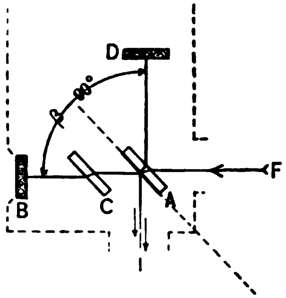

Fig. 2.

It is evident that an infinite number of individual forms may be produced by changing the angle β and the subsequent arrangement of the mirrors which bring about reunion. None of these, however, differs in any essential particular either as regards the method of mounting and adjusting the mirrors, or the character of the interference phenomena produced. As

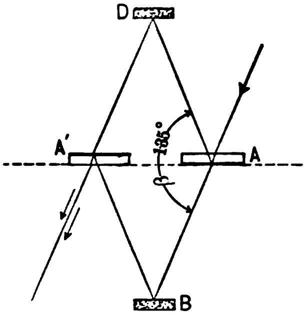

Fig. 3.

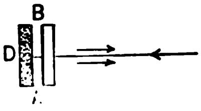

Fig. 4.

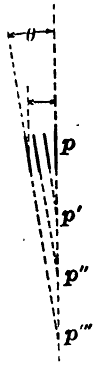

Fig. 5.

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[1] In this case the function of the surfaces *A* and *A'* may of course be interchanged; *i.e.* *A'* may be the surface of separation, *A* that of reunion.

[2] The exact correspondence between this case and any of the others shown may be easily seen by projecting the path of the reflected ray on the path of the transmitted one by cutting the paper along the dotted lines (the prolongation of the intersection of the separating surface *A* with the plane in which the angle β is measured) and folding the parts to the right over upon the parts to the left. This process also makes clear the part played by the compensator *C* in making the two paths optically the same.

will be most readily seen from Figs. 4 and 5, this is simply that produced by reflection when the light falls upon a thin air film whose thickness at the center is a, b, and whose sides are inclined to each other at an angle θ. When the angle θ is 0, as in Fig. 4, the interference fringes are circles whose diameters are determined by the distance a, b (the thickness of the film); when θ is not zero, as in Fig. 5, the fringes are hyperbolas or parabolas which are symmetrically arranged on each side of a single straight fringe lying at the line of intersection p of the two sides of the film.[1]

In all the instruments which have so far been used for either angular or linear measurements, the general arrangement shown in Fig. 2, for which $\beta = 90°$, has been adopted either in the exact form there shown or with some simple modification for securing (as in the case of the Paris interferometer) greater economy of construction and greater convenience of manipulation; or (as in the case of the form adopted for the angular measurements presently to be described) a more symmetrical arrangement of parts about the axis of rotation. This form may therefore be taken as the general type of construction for all instruments of this class.

In the descriptions of the details of setting up and adjusting the interferometer which follow, I shall assume that the reader has made himself familiar with the general theory and method of operation of the instrument from the papers already referred to. For convenience of description I will indicate the four principal optical parts by the following letters (see Fig. 2):

A indicates the separating surface.
C indicates the compensating glass.
B indicates the fixed mirror of the interferometer. Sometimes called the reference plane.[2]
D indicates the movable mirror of the interferometer.

The difference in path between the two rays is in every case twice the thickness of the equivalent air film ab. This will be indicated by the symbol Δ.

In regard to the general design of the instrument the following points must be considered:

1. The bed of the whole instrument should be very heavy and rigid in order to afford an unusually steady and stiff support for the whole mirror system. It should be H or U shaped in section, of a width of at least twice the aperture of the mirrors B and D, of a height at least one and one

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[1] For fuller discussion of these phenomena see papers already referred to, particularly that of Michelson's in the Phil. Mag. for 1882, and that of Morley's in this journal: Vol. IV. p. 3.

[2] In Fig. 4, A performs also the function of B.

half times the width, and of a length depending on the range of movement allowed for the movable mirror. The extreme difference of path Δ at which the interference fringes have yet been observed is about 45 cm. (with the light from the green mercury line), and since, as just stated, the difference of path introduced by the movement of *D* is twice the movement itself, it is not generally necessary to provide for a greater range of motion than 20 cm. The carriage upon which *D* is carried should be long (three to five times its width), in order to minimize the effect of error in the ways. The best form for the latter is that shown in Fig. 6,[1] which is a cross section through the bed, movable carriage, and mirror. These ways should be as true and smooth as possible, a good way of securing the desired result being to grind them by the method described in a recent paper in the *Journal of the Franklin Institute*.[2]

The screw used to move the carriage does not usually need to be an especially accurate one, but it should work very smoothly in the nut.

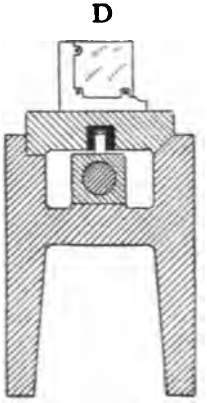

Fig. 6.

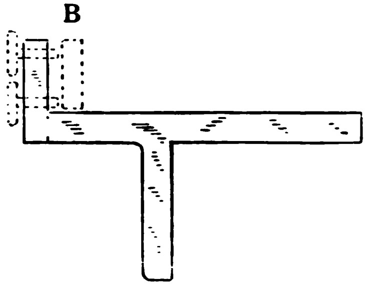

Fig. 7.

The nut may for the same reason be a simple square solid nut, as shown, with its lower side sliding against the surface of the cross bar of the bed, and with a pin projecting from its upper surface and entering a slot in the carriage. This slot should be considerably wider than the diameter of the pin, so that when the screw is turned backward a half turn no part of the nut will touch the carriage. At the forward end the screw should be provided with a graduated head, and also some simple form of index for indicating the whole number of turns. It is convenient to have the screw itself of a comparatively coarse pitch (not finer than 1 mm., and better 2 mm.), and have at the front end a slow motion device, such as a tangent screw or worm gear that may be readily thrown in when desired to give a very slow motion to the carriage.

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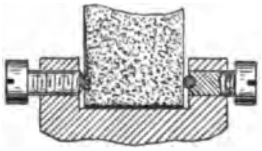
[1] This is the form adopted for the Paris interferometer. For a discussion of its advantages see paper On the Manufacture of Very Accurate Straight Edges, F. L. O. Wadsworth, Jour. Franklin Inst., July, 1894, and American Machinist, Aug. 23, 1894.

[2] *Ibid.*

The head plate at the front of the bed which carries the glasses *A*, *B*, and *C* (see Fig. 7) should be thick and heavy (not less than one and one half to two times the thickness of the glass plates themselves), and, to prevent springing, should be screwed to the bed on the side only.

2. *Mountings of the Mirrors.*—The proper mounting of the mirrors is of the utmost importance, as upon this more than upon anything else depends the successful and convenient manipulation of the instrument. They must be held firmly and at the same time in such a way as to introduce no strain, and so as to allow each one to be independently adjusted in both a vertical and horizontal plane. The most convenient and efficient way which I have found of fulfilling these conditions is to support each mirror (if rectangular, as is generally the case) on an L-shaped piece at whose vertex and ends are three inwardly projecting forks (see Figs. 8 and 9), the opening in which is slightly greater than the thickness of the mirror. The whole piece is milled out of the solid metal, and its thickness is about one and one half times the thickness of the mirror, so as to be very stiff and rigid in the direction at right angles to the plane of the mirror. Through each arm of the forks are threaded fine screws, *a*, *b*, *c*, between the points of which the glass is lightly clamped.[1] On one side the glass rests directly against the points of the screws; on the other the points are hollow, as shown in the figure, and little rounded plugs of cork are inserted to allow for any expansion or contraction, and to allow also of a slight adjustment of the glass without moving the screws on both sides.



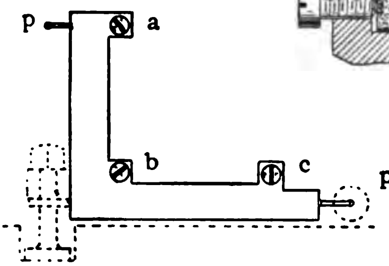


Fig. 8.

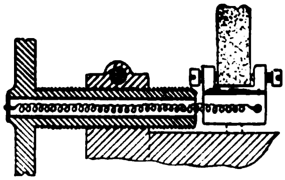


Fig. 9.

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[1] One great advantage of this form of mounting is that mirrors of different sizes may be held in the same mounting.

In the case of glasses *A* and *D* the supporting piece is screwed down to the head plate and carriage respectively, by means of screws through the lower arm of the L; but in the case of *C* and *B* it is mounted so as to permit of a more delicate motion of adjustment than can be obtained by means of the screws *a* and *c* alone. This may be and has been done in a number of ways in different instruments, but the plan I have found most reliable and convenient is one similar to that used in the Paris interferometer for the back support of the "intermediate standards," and for the support of the compensating mirror. This is to mount the L piece on a short, stiff steel spring, a convenient form of which is a steel column, one end of which is screwed into the bed plate and the other into a lug on the side of the L near the vertex (see Fig. 8, dotted lines). The required small horizontal adjustment is effected by slightly *twisting* this column about its axis, best by the varying pressure of a very light spring acting on the end of the horizontal arm of the L; the vertical adjustment by *bending* the column by a similar arrangement acting on the end of the vertical arm.

The most recent arrangement which I have used for this purpose, and which has been found more compact and satisfactory than any heretofore tried, is that shown in Fig. 9. It consists of a light spiral spring, one end attached to a pin in the end of the arm of the L, the other to the outer end of a hollow screw through which the spring passes freely. The thread of the screw should not be finer than 1 mm. pitch,[1] and should work very smoothly in the nut, which should be at least two diameters in length. The strength of the spring should be so adjusted that one revolution of the screw shifts a fringe by not more than twice its own width. The screws should have a run of about 2 cm., corresponding to a total movement of about 40 fringes, or for a mirror 2 cm. wide, to a linear movement of about 0.01 mm. It will be at once seen by those familiar with the inherent defects of even the very best micrometer screws, how very difficult it would be to move the mirror by an amount corresponding to a tenth of a wave (*i.e.* 0.000025 mm.), as is necessary in the final adjustment, by any screw bearing directly upon the mirror or any arm of practicable length attached thereto. Not only do the errors of the screw far exceed this amount, but what is more, the unavoidable longitudinal pressure exerted by the fingers in turning the head, moves the screw in the nut and springs the support of the latter, no matter how rigid we may attempt to make it, by an amount far greater than the desired movement. With this spring arrangement the effect of these sources of irregular motion is reduced in the same proportion that the movement of the screw for a given effect is increased, *i.e.* about 1000 times. To reduce the effect in the same proportion by simply lengthening the arm of the L would require that this be made 20 m. long.

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[1] Heretofore it has been customary to use screws of much finer pitch bearing against much stiffer flat steel springs

In the case of the compensating glass there is no necessity for a delicate adjustment in the vertical plane, and only one spring movement, *i.e.* that acting on the end of the horizontal arm of the support, is needed. In this case we need a movement corresponding to only two or three fringes, but since this movement is used to measure fractions of a wave length, the inequalities in the motion must not exceed a quantity corresponding to $\frac{1}{100}$ wave. The pitch of the screw should therefore be much coarser than before (a double or triple thread or spiral of about 5 mm. pitch is not too coarse), and the spring still finer, so that one complete turn corresponds to a shift of one fringe. This screw should have a large head, graduated to hundredths of a turn (so that one division corresponds approximately to $\frac{1}{100}$ wave length).

Setting up and Adjusting the Instrument.

a. Silvering of Mirrors. — Mirrors *B* and *D* are full silvered on their front surface; *A* is "half silvered" on the surface nearest the compensating glass. Of the different processes of full silvering, the writer prefers above all others that of Brashear, the details concerning which were given in the *Astrophysical Journal* for March, 1895.[1] For "half silvering," *i.e.* depositing a semitransparent coat of such density as to reflect about half the incident light, I have, however, obtained the best results with the ordinary rochelle salts solution prepared as directed by Boettger.[2] For "half" silvering the solution should be comparatively new, and the glass cleaned with the greatest care in order to obtain a uniform film. The plate should be supported in the liquid by one of the glass "forks" described in the article above referred to, so that it may readily be lifted out of the solution from time to time and the density of the deposit examined. When of about the right thickness it has a peculiar blue color by transmitted light. A few trials will be necessary at first to enable the operator to determine by this means just when the right thickness of film has been obtained. It should then be immediately lifted from the silvering solution, plunged into a large vessel of distilled water and thoroughly rinsed, and then turned on edge on a sheet of blotting paper to dry. Great care must be taken not to touch the silvered surface with the fingers either before or after drying, as it is so thin that it is very readily rubbed off.

b. Putting Mirrors in Place. — First put *D* and *B* into their supports, and by means of a small steel square adjust them so that their surfaces are at right angles to the side and end of the bed, and the faces of the carriage and head plate, respectively. Then if the mechanical work on the machine has been properly done these two surfaces ought to be very nearly at right angles to each other. Next put in *A* and *C* and adjust *A* until the image of *D* seen by reflection is exactly superposed on *B*; then finally adjust *C* until it is parallel to *A*, either mechanically by the use of a pair of outside calipers (outside calipers are used instead of inside ones to avoid injuring the silver film which is on the side nearest the compensator), or optically by making the two images of a bright distant object seen by reflection from the surfaces of *A* and *C* coincide. It is, perhaps, necessary here to call attention to the fact that the thickness of the two plates *A* and *C* should be the same within a few thousands of a millimeter, and that both ought to be as nearly as possible plane parallel, *i.e.* both surfaces should be optically flat, and as nearly parallel to each other as possible. The best way is to have them both cut from one large piece.[1] If it is necessary, as sometimes happens on account of the loss or breaking of one of the "pair," to use two pieces of slightly different thickness, we may up to a certain point still obtain practically perfect compensation by inclining *C* slightly to *A*, so as to make the length of the path traversed by the rays in the glass equal in the two cases. If *C* is thicker, it is of course turned so as to make the angle of incidence smaller than on *A*; if thinner, so as to make the angle of incidence larger. It is further to be mentioned that these adjustments are only necessary in order to obtain the fringes in white light; with monochromatic light we might dispense with *C* altogether, as is sometimes done.

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[1] "Silvering Solutions and Silvering"; F. L. O. Wadsworth, Astrophysical Journal, Vol. I., p. 252.

[2] Kohlrausch, Physical Measurements, p. 146.

c. To Find the Fringes. — First measure, by means of a thread or scale, the distances from any point on the edge of the silvered surface of *A* to the surfaces of *B* and *D*, and move the latter forward or backward until these distances are the same to within about 1–4 mm. Then cover the whole instrument with a wooden or pasteboard box (as indicated by dotted lines in Fig. 2), having two holes about 3 cm. square cut in front of the mirror *A*, as shown. It should also have an opening in front through which projects the handle and the slow-motion device for turning the screw, and another opening in the side opposite the mirror *B*, through which the screws which adjust the latter can be reached. This box is not essential if the room is very quiet and free from air currents, but it is always desirable to use it when possible, as it will frequently greatly facilitate the finding of the fringes. Next place a sodium flame at *F*, about 30 or 40 cm. from *A*, and between the two an ordinary condensing lens which will render the rays falling on *A* approximately parallel. Place the eye at *I*, in front of *A*, looking in the direction of the double arrow toward *D*, and move the flame and condenser until the whole field of the mirrors is uniformly filled with light. Place a needle point just in front of the flame near the focus of the condenser, and with the fine motion screws of *B* at the middle of their run, adjust this mirror with the screws *a* and *c*, until the two images of the point seen by reflection in *A* (one formed by reflection from *D*, the other by reflection from *B*) are in exact coincidence. (Besides the pair of images formed by reflection from the silvered surface of *A*, there will be another pair formed by reflection from the front unsilvered surface. If the glass is exactly plane parallel this second pair will also be in coincidence when the first pair is, but not otherwise. Care, therefore, should always be taken to select the right pair of images. They may be easily distinguished by their greater brilliancy.) This adjustment to coincidence needs to be performed with great care, for unless it is very exact the fringes will not appear. It is, therefore, best to use at first a telescope at *I*, by means of which the images of the point will be considerably magnified, and the coincidence more easily determined. It is also a help to change the position of the needle, holding it with its axis horizontal when adjusting in a vertical plane, by means of the screw *a* (Figs. 8 and 9), and vertical when adjusting in the other plane by means of screw *c*. When coincidence is judged to be exact, remove the telescope (if one has been used) and needle point, and place the eye about 25 or 30 cm. from the mirror *A*. If the fringes do not at once appear (generally they will appear as fine, more or less curved lines), first move the eye back and forth (there being a position of maximum visibility for each particular value of Δ and θ). If this does not bring them into view, move *D* slightly (about 0.2 mm. each time) by means of the screw, first in one direction and then in the other, and examine again after each movement by moving the eye backward and forward as before. After each movement of *D*, reëxamine the images of the needle point to see that they are still in coincidence. The fringes will be certain to appear in one of these positions if the preceding directions have been carefully followed.

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[1] Mr. Brashear has attained such skill in the manufacture of these plane parallel pieces that he can furnish at merely nominal cost a "pair" that will always give satisfaction.

d. Adjusting the Fringes. — After the fringes have once been seen, it will be found an easy matter, after a little practice, to adjust them to any desired width or position by means of the fine-motion screws on *B*. A little manipulation of these screws and a study of the effect produced is worth more than any number of directions. A word, however, may be said regarding the adjustment for circular fringes, which appear when each of the mirrors is exactly perpendicular to the incident beam of light falling on it; and the finding of the central straight fringe in white light. To produce circular fringes, the mirror D should be moved back until there is a considerable difference of path between the two rays (1000 to 2000 wave lengths), and the screws moved so as to *increase* the curvature of the fringes. The center of the system will then soon appear, and may, by a further movement of the screws, be brought to the center of the field. Then move the eye to the right and left, and if the diameters of the circles change, move the screw acting on the horizontal arm of the mirror mounting B in such direction that this change grows less and less, and finally nearly disappears. Then move the eye up and down, and if a corresponding change takes place, adjust the other screw until this also disappears. When the fringes remain the same in all parts of the field, the adjustment is perfect.

To find the fringes in white light, the fringes are first made straight and preferably vertical. The screw is then turned so as to move the mirror D towards the zero position. It is easy to determine when this position has been nearly reached by the increasing straightness of the fringes, and also by the fact that there is very little tilting or change of curvature as the eye is moved up and down. The slow-motion screw is then thrown in, the sodium light replaced by a gas flame, and the carriage moved very slowly (at such speed that about 10 fringes cross the field per second) until the colored fringes appear. This position, as determined by the graduated head of the screw, should be noted for convenience in finding these fringes again when desired. Since only some thirty fringes are distinctly visible in white light, the position of the mirror must be right to within less than 0.01 mm. in order to make them appear at all.

e. Observing and Measuring with the Fringes.—The straight fringes obtained when the two mirrors are slightly inclined to each other are in general most conveniently observed by the eye alone, without the aid of the telescope. If a telescope is used it must be focused for the central white fringe on the surface D. For the fringes to the right or left of this, corresponding to a definite difference of path Δ between the two rays, the focus of the telescope changes. This fact, together with the continual change in the curvature, and consequent apparent shifting of the fringes with a change in the position of the eye, makes their use unsuitable in measuring any large differences of path. The circular fringes are free from all these difficulties, for being formed at infinity (see theory of instrument) they are always visible for all values of Δ in a telescope focused for parallel rays; and their position and appearance are likewise independent of the position of the eye. They are therefore always to be used when large differences of path are to be observed. For measuring small movements, however, the straight fringes are more convenient, it being only necessary to count the number which pass a given point in the field (most conveniently a line engraved on the surface of one of the mirrors *B* or *D*) when the movement takes place. When the displacement is so rapid that the counting is impracticable, the central fringe of the colored system seen in white light must be used to mark the initial and final position, this being the only fringe of the system that can be identified.

It may appear from what has preceded that the interferometer is an instrument of such extreme delicacy that its use in ordinary laboratory measurements is impracticable. Such, however, is not the case if the instrument is properly constructed. Its stability and steadiness is then perfectly surprising, considering the minute movements which it will detect. It is far less difficult to work with, for example, than a delicate galvanometer or electrometer, and in applying the interferometer to the measurement of the deflections of these instruments, the difficulties we meet with are not those due to the interferometer proper, but those due to the unsteadiness of the suspended system of the galvanometer or electrometer. It was for the purpose of determining whether these difficulties could be overcome that the experiments already alluded to were, at Professor Michelson's suggestion, undertaken, in 1891, at Clark University, by Dr. Austin, now of the University of Wisconsin, and the writer.[1]

The type of optical system adopted was that first described by Professor Michelson in his paper "Application of Interference Methods to Astronomical Measurements" (*Phil. Mag.*, July, 1890), and shown in Fig. 11 of that paper. Figures 10, 11, and 12 show how this is derived from the type of instrument for linear measurement. It is evident that this latter may be used as well for angular measurements, by mounting the mirror *D* on the end of the arm whose deflections are to be measured (see Fig. 10). This arrangement, however, has the great disadvantage of altering the inclination of the mirror, and thus changing the width of the fringes at the same time that they are shifted in the field by the linear change in path.[2] This objection can be at once overcome by the modification shown in Fig. 11,

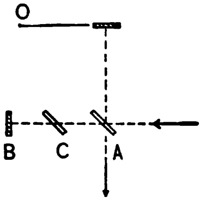

Fig. 10.

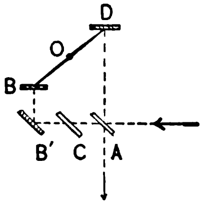

Fig. 11.

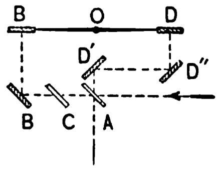

Fig. 12.

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[1] Unfortunately Dr. Austin left the University before the experiments were completed.

[2] This same objection applies to the type of instrument first proposed for angular measurement by Michelson in his paper, Measurement by Light Waves, Amer. Journal, Feb., 1890, Plate III., Fig. 1; also Group III., Figs. 4 and 5.

in which the mirror B is mounted parallel to D, and on the same swinging arm which carries the latter. This is accomplished by introducing an additional mirror B', placed parallel to A and C. The effect of the angular motion of one mirror is compensated by the equal angular motion of the other, while if the point of suspension is between the two mirrors, as at O, the sensitiveness is doubled by the fact that a movement of the suspended system shortens one path at the same time that it lengthens the other. The only practical objection to the arrangement is that the mirrors B and D are not in the same plane, and are not therefore as easily mounted and managed as they would be if this were the case. The form in Fig. 12 accomplishes this last object by the use of the two additional mirrors, D' and D''.

The complete instrument as designed on these lines is shown in Fig. 12 *a*. It was built partly by Mr. Claflin, of the Claflin Dental Mfg. Co., of Worcester, and partly by Mr. Francis and myself in the machine shop of the university. Some features of construction differ from those since adopted as more desirable (see earlier part of this paper). For example, the fine adjustment of B (in this case applied to B') is accomplished by means of very fine watch screws, bearing directly against the mirror frame, and moved the required minute amount by long steel levers (one of which can be seen in the photograph) inserted in the heads of the pins.[1] In order to adapt the optical part of the instrument to suspended systems of different magnitude, the separating mirror A and mirror D' are mounted on one carriage, D'' on another, and B' on the third. These may be slid along to any part of the long V-shaped bed and clamped in any desired position. The mirror B' is mounted on a short auxiliary slide, and moved parallel to itself a small amount by a screw which takes the place of the screw which moves D in the linear type of instrument. To impart a slow motion to this screw, the nut in which the main screw takes its bearings is itself threaded, the pitch of this thread being very slightly greater than that of the screw. When this nut, therefore, is turned, as it may be when desired by the large milled head seen in the photograph, the screw is moved in or out at each revolution of the latter by an amount equal to the difference in the pitch of the thread on the outside of the nut and that on the inside. The screw itself is kept from revolving when the nut is turned by a friction collar which is not, however, tight enough to prevent its being turned directly by the small head when a rapid motion of B' is desired.

The whole bed may also be rotated about a heavy steel pivot at the center (by means of the very fine screw with large milled head seen at the back of the instrument), in order to bring these parts of the interferometer system parallel to the suspended system. The compensator C is mounted,

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[1] The objections to this method have already been stated; see page 486.

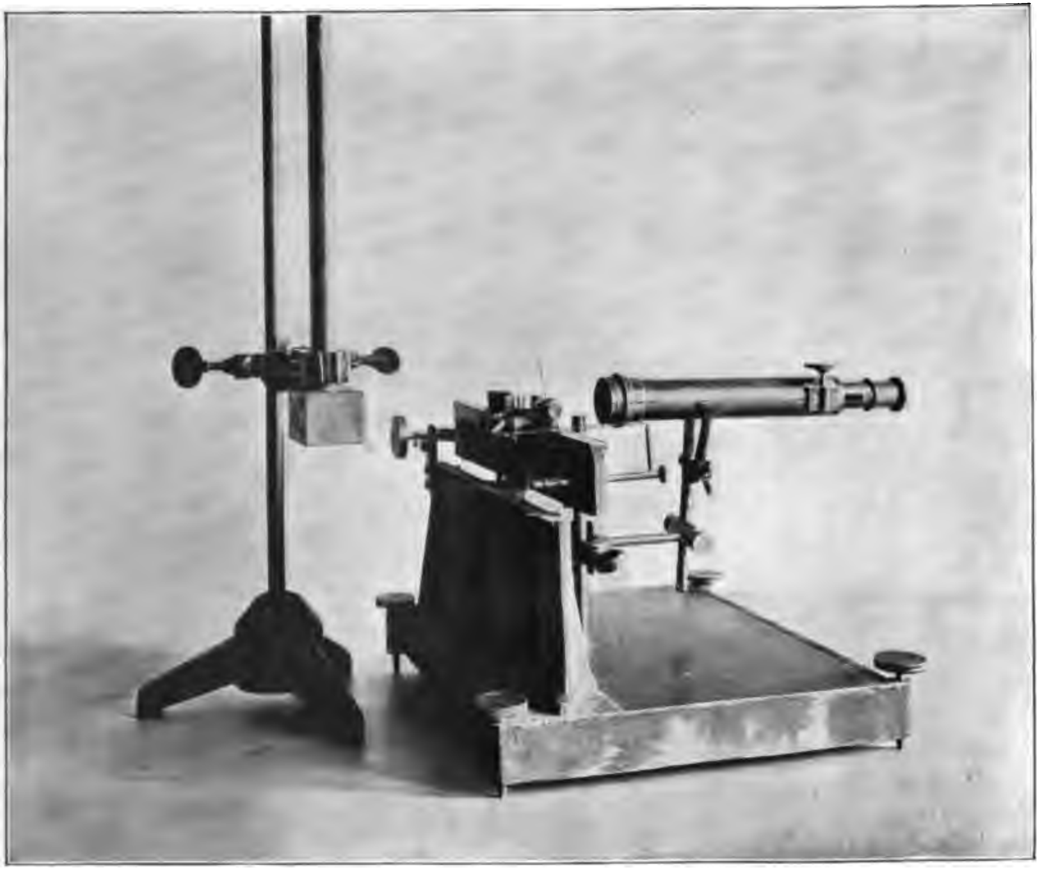

Fig. 12 a.

not on the carriage with *B'*, but just in front of it (between *B* and *B'* instead of between *B'* and *A* as in the diagrams in Figs. 2 and 12), on a heavy brass rod screwed into the base of the instrument. The spiral spring which twists this rod is attached, the one end to a light lever projecting from it, the other to the graduated sliding bar seen near the front. The strength of spring is so adjusted that a motion of this bar of from 15 to 20 cm. is necessary in order to twist the compensator glass sufficiently to introduce a difference of path of one wave length.

The suspended system consisted of the two small mirrors *B*, *D*, which were about 5 mm. square, mounted on a stiff though light arm made of glass fibers about 5 cm. long, and suspended at its center from a quartz fiber about 25 cm. long. It is of course impossible to adjust either of the suspended mirrors during the course of the experiment, and it was therefore necessary to be extremely careful in mounting them originally so that their surfaces lie very nearly in the same plane. This is done by laying them face down upon a piece of optical glass and examining the fringes between this surface and the faces of the mirrors.

The whole suspended system was inclosed in an air-tight chamber which consisted of a small cast-brass box with heavy walls, about 4 cm. square and 6 cm. long. The heavy brass tube[1] which carried the fiber was soldered into the top of the box and the front was closed by a thick plate of optical glass, cemented on with a mixture of ordinary glue and molasses. The top of the brass tube could also be closed by means of a brass cap which could be slipped over the head to which the upper end of the fiber was attached, and cemented in place after the latter had been adjusted. The tube and box were clamped in a heavy cast-iron stand, as shown in the plate, and the two parts of the instrument, the inclosed suspended system mounted as just described, and the optical "head," as it may be called, placed together on a very heavy stone slab, supported on rubber blocks. Blocks of stone, not shown in the plate, were also placed around the feet of the iron stand and under the brass box so as to steady the latter from below, as well as at the point where it was clamped above. It was an easy matter to obtain the fringes when the suspended system was held stationary, but it was only after many trials that they were obtained with a free system. The most convenient method of procedure was finally found to be as follows: The suspended system was first arrested in as nearly as possible its position of rest by means of a wire fork, mounted on a short brass rod passing through a hole in the side of the brass box, as shown in Fig. 13. After the fringes have been obtained in white light by the method already described, the system was gradually released by turning the rod by the long arm, *b*, outside the box, the bed being rotated by means of the tangent screw already spoken of, so as to follow the suspended system as it swings one way or the other, and keep the fringes in the field.[1] At first, as soon as the system was completely free, it began to swing and tremble so that it was very difficult to follow the fringes. The difficulties due to vibration were finally very considerably reduced, and at times completely overcome, by floating the stone carrying the whole apparatus in mercury, in a box very slightly larger than the stone itself, the stone being kept from touching the box by silk threads.[2] The observing telescope was also removed from the base of instrument and mounted on a separate stand,[3] so that after the preliminary adjustments had been made, nothing on the floating stone itself needed to be touched. The difficulties due to the small constant swinging of the system to and fro were largely overcome by proper damping. In the preliminary experimental work this was effected by attaching to the suspended system a long light bar magnet and placing strong bar magnets on the stone outside. Such an arrangement would not in general answer because of the directive force of the magnets and consequent loss of sensibility. Probably the best system of damping which would leave the sensibility unchanged would be that indicated by dotted lines in Fig. 13. It consists of two strips of copper, or better, pure silver foil, hung from the ends of the suspended system with the plane of the strips at right angles to the mirrors. These strips swing edgewise between the poles of two powerful horseshoe or electro magnets. The brass box containing the system was not large enough to allow this

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[1] A glass tube cemented into the box was first used, but afterward replaced by the metal as far more satisfactory.

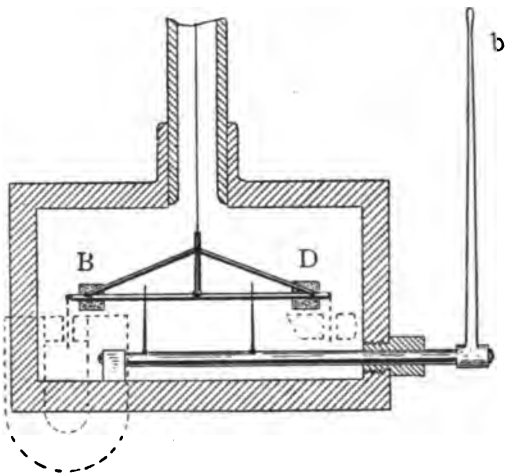

Fig. 13.

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[1] Any motion of translation, or of rotation about a horizontal axis, of the system is of course without effect on the fringes.

[2] What seems to be an even better method of support to prevent vibration has recently been described by Professor Brace. See Astrophysical Journal, Vol. V., p. 214, March, 1897.

[3] The telescope may of course be placed at any desired distance from the instrument itself.

arrangement to be tried, but I feel no doubt that it would prove effective in holding the system at rest, or so nearly so that there would be no difficulty in noting the position of the fringes as they slowly swing to and fro. A considerable movement of the fringes is no disadvantage, — is in fact even an advantage, provided only the swing is steady and not so large as to throw the central white fringe (which is the only one which can be identified) out of the field. The zero point or position of rest is in such a case determined by the method of oscillations, just as in the case of the ordinary telescope and scale method of reading. To avoid parallax and other errors, the excursions of the central fringe are best noted by means of a scale engraved directly on one of the mirrors D or B.

These experiments having demonstrated the practicability of this method of reading deflections, I next began the construction of a large D'Arsonval galvanometer, designed with special reference to this new method of observation. Before this was finished, however, I was called upon to devote my whole time and energy to the design and construction of the large Paris interferometer, and immediately after its completion and installation in the laboratory of the International Bureau at Breteuil, I went to Washington to accept a position in the Smithsonian Astrophysical Observatory, where I had no opportunity to continue these experiments. It is only within the last few months, since my transfer to the Yerkes Observatory, that I have had an opportunity to resume the long-deferred completion of the galvanometer, the unfinished parts of which I have fortunately preserved.

It is as yet too soon, perhaps, to predict anything in regard to the degree of sensitiveness which it will be possible to practically attain with this instrument. A preliminary estimate, based on the dimensions of field-magnets and proposed coil and mirror system, would indicate that it would be possible to attain a sensitiveness such that a deflection of 0.1 fringe corresponds to a current of between 10^{-16} and 10^{-17} amperes. If this degree of sensitiveness can be attained, particularly with the D'Arsonval type of instrument, it will permit of certain problems of great theoretical interest being attacked for the first time experimentally.

There is one point of considerable importance in the theory of galvanometers in this connection. It might be urged by some, that this method of reading considerably increased the mass and, more particularly, the moment of inertia of the optical parts of the suspended system. This is true, but the remedy is simply to increase the mass of the rest of the suspended system in the same ratio. This is exactly the reverse of the modern practice of using very light systems to secure great sensitiveness; but as I have elsewhere[1] shown, this increased sensitiveness results not at all from the lightness of the system, but from the higher degree of astaticism reached. Given the same degree of astaticism in the heavy system, and it would be the more sensitive of the two, although (and this is the practical difficulty generally attending the use of the heavy system) the time of swing will be much longer. With the interferometer method of reading, however, this slowness of swing is fortunately just what is wanted.

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[1] Description of a Very Sensitive Form of Thomson Galvanometer and some Methods of Galvanometer Construction, Phil. Mag., Vol. 38, p. 553, December, 1894.

In conclusion it may be of interest to point out certain changes and modifications in the apparatus which have suggested themselves as desirable. In the first place, in making another instrument of this kind I would have all the mirrors larger, particularly the moving ones. In the instrument just described the mirrors A, B', C, D', and D'' were each about 10 mm. × 15 mm. and B and D, as already stated, about 5 mm. square. It would be better to have the first-named mirrors 5 mm. larger each way and the two last of about the same width as before, but at least twice as long, so as to allow greater amplitude of swing without causing the central fringe to pass out of the field.[1] The tangent screw which moves the bed in azimuth should be finer and act on the end of a longer arm, in order to be able to follow the movement of the mirror more slowly and steadily. It would be better for the same reason to have the steel pin upon which the bed turns, at one end of the latter instead of at the center. Both the bed and the mountings of the mirrors should be heavier (following more nearly the proportions given in the first part of this article) and the movable carriages on which the latter are mounted longer, and arranged to clamp more rigidly to the bed.

In the preceding form of instrument the difference in path Δ, introduced by a given angular movement θ, of the system is $2\,b\beta$, b being the distance BD between the centers of the mirrors. If we consider that the smallest movement that can be measured with certainty in a freely suspended system is one twentieth of a fringe,[2] the smallest angle that can be measured with the interferometer is

$$\theta_0 = \frac{1}{40}\frac{\lambda}{b}.$$

The smallest movement that can be observed with a mirror as ordinarily used is probably not more than one fifteenth its angular resolving power.[3]

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[1] Or what amounts to the same thing, without allowing the point p, in Fig. 4, to get beyond the edge of the mirrors.

[2] With the type of instrument with fixed mirrors (linear measurement) a movement of one two-hundredth of a fringe may be detected under favorable conditions.

[3] See paper, "Manufacture of very Accurate Straight Edges," already referred to, p. 6, also review of Boys' determination of the Newtonian Constant of Gravitation, Astrophysical Journal, Vol. III., p. 303, April, 1896.

This, as is well known, is

$$\alpha = \frac{\lambda}{b},$$

and the smallest angular movement is therefore

$$\theta_m = \frac{1}{15}\frac{\lambda}{b}.$$

The interferometer system therefore is not so very much more accurate than a mirror of the same horizontal aperture used with the telescope and scale. The principal advantage of the former, as has been pointed out by Michelson, is that with it we need to use only two small pieces at the ends of a diameter instead of the whole mirror.[1] We could also do this in the case of the telescope without loss of resolving power provided we could make these two pieces form parts of one and the same optical surface, and also give sufficient light; but in general neither of these conditions could be fulfilled. When they can be, the interferometer does not offer sufficient increase in accuracy to compensate for the greater difficulties encountered in its use.[2] It occurred to me, however, during the course of these experiments, that we can further increase the attainable accuracy of the interferometer method by employing the principle of multiple reflection. One very simple and symmetrical arrangement which suggested itself is that shown in Fig. 14, in which, as before, *B* and *D* are the movable mirrors and *F*, *F'*, *F''* are three additional fixed mirrors. Each ray is reflected from the movable mirrors three times, and in such directions that the total difference in path produced by a given movement θ is now $6\,b\theta$, and the resolving power is therefore, on the same assumption as before,

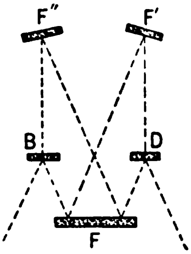

Fig. 14.

$$\theta_o = \frac{1}{120}\frac{\lambda}{b},$$

or eight times as great as for the mirror. But the greatest advantage of such a system is that we may, for a given resolving power, make the length of the arm supporting the two mirrors one third as great as in the more elementary form, and therefore reduce the amount of inertia to less than one tenth of what it was in the latter.

YERKES OBSERVATORY, UNIVERSITY OF CHICAGO,
March, 1897.

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[1] This is an advantage only when the distance *b* is very large; at least fifteen times the height of the mirrors *B* and *D*.

[2] A fuller discussion of the comparative advantages of the different methods of observing small angular movements will be found in a paper, "On the Conditions which Determine the Ultimate Optical Efficiency of Methods for Observing Small Rotations," which will soon be published in the Phil. Mag.

SOME DETERMINATIONS OF THE SLIDE MODULUS OF GLASS, AND THE SHORTENING OF GLASS FIBERS WITH AGE.

BY W. S. FRANKLIN AND L. B. SPINNEY.

I. DETERMINATIONS OF SLIDE MODULUS.

Preparation of Fibers.

THE glass to be tested was drawn into fibers. Great care was taken to secure circular section and approximately uniform size, and the tension in drawing was carefully relieved as the glass began to harden. Some of the fibers were annealed, as indicated, by heating for about ten minutes to a temperature of about 750° C. in a long brass tube surrounded by a German silver resistance coil. To fasten the fibers in the torsion pendulum, the ends were platinized and imbedded in solder.

Measurement of Fibers.

For purposes of calculation the fibers were considered to be portions of two cones, and the diameter of each fiber was measured at each end and in the middle by means of a micrometer microscope, using a Zeiss stage micrometer as a reference standard. The lengths of fibers were measured in terms of a Brown & Sharpe steel scale.

Observations for slide modulus were taken in each case with a torsion pendulum made by suspending a body of known moment of inertia by the fiber to be tested.

Reduction Formula.

The formula for calculating slide modulus from the observation data was derived from the solution of the problem of the twisting of an acute cone. The authors are not aware of previous publication of this simple problem, and the outline of it is here given as applied to the present case.

Let
$$r = ml \qquad \text{(i.)}$$

express the radius r of fiber from the near end to the middle; l is a coördinate parallel to the axis of the fiber and measured from the apex of the cone formed by this half of fiber, and m is a constant.

Similarly,
$$r = m'l + a \qquad \text{(ii.)}$$

expresses the radius throughout the remainder of the fiber, m' and a being constants. Let l', l'', and l''' be the l coördinates of the near end, middle, and far end of fiber, respectively, and r', r'', and r''' the radii of the fiber at these points. Then

$$\left.\begin{aligned} r' &= ml' \\ r'' &= ml'' \\ l'' - l' &= L \\ r'' &= m'l'' + a \\ r''' &= m'l''' + a \\ l''' - l'' &= L \end{aligned}\right\} \qquad \text{(iii.)}$$

in which L is the half length of the fiber. Each element of the fiber considered as a circular cylinder of radius r and length dl will be twisted through an angle

$$d\theta = \frac{2T}{\pi n} \cdot \frac{1}{r^4} \cdot dl,$$

by a torque T, n being the slide modulus. Therefore the angle of twist of the whole fiber is

$$\theta = \frac{2T}{\pi n} \int_{l'}^{l''} \frac{1}{m^4 l^4} dl + \frac{2T}{\pi n} \int_{l''}^{l'''} \frac{1}{(m'l + a)^4} \cdot dl. \qquad \text{(iv.)}$$

Substituting in this equation the values of m, m', a, l', l'', and l''' from (iii.), and remembering that the coefficient of T in (iv.) is equal to $\frac{t^2}{4\pi^2 K}$, in which t is the vibration period of a body of moment of inertia, K, suspended by the fiber, we have

$$n = \frac{2\pi KL}{3\, t^2 r'^3 r''^3 r'''^3} \left[r'''^3 (r'^2 + r'r'' + r''^2) + r'^2 (r''^2 + r''r''' + r'''^2) \right]. \qquad \text{(v.)}$$

This equation was used in the calculation of slide modulus from the observation data.

Results.

The results of tests of nine fibers are exhibited in the accompanying table. The wide variations in the value of the modulus cannot be ascribed to errors of observation, but it seems that the value of the modulus varies with the size of the fiber, and that it is changed by annealing the glass. German soft glass is not distinctly different from Jena normal thermometer glass.

II. SHORTENING OF FIBERS WITH AGE.

The primary object of these tests was to determine the variation in length of extremely fine glass fibers with the hygrometric state of the surrounding air. This effect was at first confused with the shortening of the fibers with age, which entirely masked the effect sought. The apparatus was then used in the study of the shortening with age of several fibers, and one of these, when twenty days old, was subjected to a rapid change of humidity up to saturation, without any distinct alteration in length. A change of one part in seven or eight millions could have been detected.

Fiber.	Glass.	Length $2L$.	Radii.	$n \times 10^{-11}$.
1	Jena ther. glass.	30.16	r' 0.003504 r'' 0.003920 r''' 0.004013	1.59
2	Jena ther. glass.	29.70	r' 0.006713 r'' 0.007378 r''' 0.007265	2.40
3	Jena ther. glass.	29.30	r' 0.002473 r'' 0.002389 r''' 0.002460	6.83
4	Jena ther. glass.	29.50	r' 0.003335 r'' 0.003435 r''' 0.004474	1.92
5	German soft glass.	29.30	r' 0.002777 r'' 0.002501 r''' 0.002496	2.08
6	German soft glass.	30.30	r' 0.002502 r'' 0.001865 r''' 0.002392	2.04
7	German soft glass.	28.60	r' 0.003860 r'' 0.003295 r''' 0.003323	1.77
9	German soft glass annealed.	29.50	r' 0.003555 r'' 0.003671 r''' 0.003852	3.63
10	German soft glass annealed.	29.20	r' 0.003433 r'' 0.003068 r''' 0.003740	2.93

Arrangement of Apparatus.

Two very fine glass fibers were stretched horizontally over a glass frame, and the ends of the fibers were bent and imbedded in solder. One fiber was tightly stretched and the other was loose, and was weighted at its middle with a bit of fine wire in the form of a hook. This glass frame was encased in an air-tight chamber, which was mounted, together with a micrometer microscope, upon a cast-iron plate. The micrometer microscope served to measure the vertical distance between the middle points of the two fibers. In this way any changes in length of the loose fiber could be determined with great accuracy.

Results.

The effects of humidity could not be measured, as explained above. The accompanying diagram (Fig. 1) shows graphically the alterations in length of a very fine fiber of German soft glass. The first observation was taken about twenty minutes after the fiber was drawn. Jena thermometer glass gave a similar curve of shortening, and showed about the same percentage shortening in seven or eight days as German soft glass. The fibers were very fine, and the tension in drawing was, of course, not relieved as the glass began to harden. The fiber and frame were repeatedly removed from the case and replaced during the course of the observations from which the curve (Fig. 1) was derived, and the roughness of the curve is no doubt due to this fact.

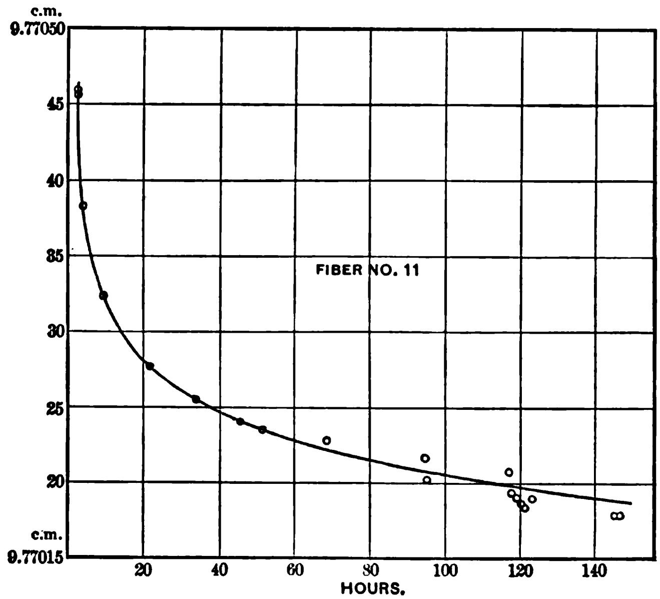

Fig. 1.

AMES, IOWA, October 7, 1896.

THE MOST SENSITIVE ARRANGEMENT OF A WHEATSTONE BRIDGE WITH SPECIAL REFERENCE TO THE BOLOMETER.

BY C. D. CHILD AND O. M. STEWART.

THE usual theory of the Wheatstone bridge indicates that the most sensitive arrangement is secured when all of the resistances, including those of the galvanometer and battery circuits, are equal.[1] However, this theory assumes that we have at our command an invariable electromotive force. In reality, one is usually limited not by his inability to secure a larger electromotive force, but by the thermal effects in the bridge circuits when too large a current is used. This is especially the case in using a bolometer. When the current in the bolometer exceeds a certain maximum, excessive heating effects occur, and irregular movements of the galvanometer are observed. This maximum current is fixed entirely by the properties of the bolometer strips.

It has been the usual practice to work out the theory of the bolometer on the assumption that the electromotive force is constant, and then, after connecting up the bolometer in the manner indicated by this theory, to proceed to vary the electromotive force until a certain maximum current in the bolometer strips is reached. As this theory assumed a constant electromotive force, while in reality the electromotive force is varied, one is not warranted in concluding that this is the most sensitive arrangement. In fact it seems probable that for this maximum current there might be some other arrangement which would be more sensitive.

Thus we see that in solving the equations of the bolometer connections for maximum sensibility, the current in the strips should be treated as a constant, and the electromotive force as a variable. From this standpoint Dr. H. F. Reid[2] has shown that there are two solutions to the problem. (1) If, in Fig. 1, a and b are the bolometer strips, the deflection of the galvanometer due to a small change in the resistance of the exposed strip is proportional to $\sqrt{\frac{1}{a+d}}$ and d and c should therefore be as small as possible. (2) If a and d are the bolometer strips, the deflection is proportional to $\sqrt{\frac{b}{a+b}}$, and, therefore, b and c should be as large as possible. As it has been found necessary to make the bolometer strips as nearly equal as possible, this equality was assumed in the derivation of the preceding results.

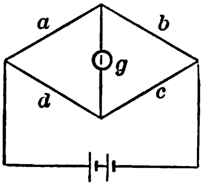

Fig. 1.

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[1] Gray's Absolute Measurements, Vol. I., p. 331, and elsewhere.

[2] American Journal of Science, 135, p. 160.

For the first case, however, Dr. Reid concludes that if the resistances *d* and *c* are made small, the current through them would be large, and the heating effects would cause much trouble. And, therefore, as the gain in diminishing *d* and *c* is not large, the best working results will be obtained when all the resistances are made equal. In the second case, when *a* and *d* are the bolometer strips, he says that if the resistances *b* and *c* are made large, heating effects would again follow. This, as before, is to be avoided, and the resistances should again be made equal. This last conclusion we regard as incorrect. There is a difference between the effect of heat developed in a small resistance by a large current, and that developed in a large resistance by a small current. In the first case the heat is concentrated in a short wire, while in the second case it may be distributed over a long one.

Our attention was called to this matter while one of the writers was engaged on the work described elsewhere in this number of the Physical Review, p. 433. In that work one of the bolometers used was first connected in the traditional way. Each of the arms, including the galvanometer, had a resistance of about one ohm. This arrangement was afterwards changed, and the connections made as in Fig. 1, where *a* and *d* represent the bolometer arms and *b* and *c* the other two arms, which were about 26 ohms each. The theory indicates that this change should produce a gain of about 30 per cent in the sensitiveness of the bolometer. The actual increase observed was in accordance with the predictions of this theory, and no increase in the unsteadiness of the galvanometer could be detected.

In order to verify the theory before reconstructing the bolometer connections, the following simple experiment was performed. The usual connections on a Wheatstone bridge were changed and others made as shown in Fig. 2. At *A* were two mercury cups which could be connected at will. The resistances *R* and *R'* were equal and were about 100 ohms each. The resistance in each of the arms *a, b, c, d* was one ohm. This arrangement fulfills the necessary condition of the theory, viz., that the

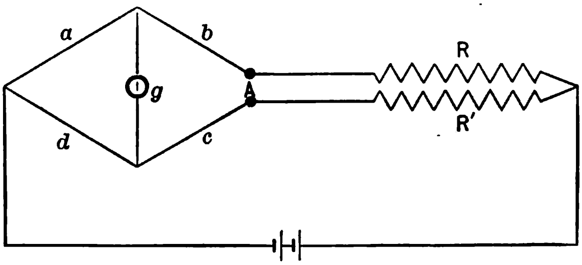

Fig. 2.

current in the arms *a* and *d*, which correspond to the bolometer strips, is constant. With connection made at *A* the usual arrangement of the bolometer connections is given, but when not made at that point we have the second case of Reid's theory. The resistances were carefully balanced, both when the connection at *A* was made and when it was broken. Then a slight increase was given to the resistance *d*, and the deflections of the galvanometer noted when connection was made at *A* and when it was broken. In accordance with the theory the deflection was greater in the latter case than in the former. In fact there seems to be no doubt but that a gain of about 30 per cent in the sensitiveness of a bolometer is secured by changing the resistance from the form commonly used to one where the resistance of the arms not used for bolometer strips is large and the corresponding increase made in the electromotive force.

Theory indicates that the galvanometer should have a resistance $\frac{2lR}{1+l}$ where R is the resistance of one of the bolometer strips and l the ratio between this resistance and that of one of the arms not a bolometer strip. Often it is not practicable to change the resistance of the galvanometer. But for this case also, theory shows that the arrangement of resistances suggested in this article is preferable.

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DEMONSTRATION OF THE DOPPLER EFFECT.

BY R. W. WOOD.

IN treating of the Doppler effect in the class room, it is usual to cite the example of the change in the pitch of a locomotive whistle in rapid motion. As a matter of fact scarcely one person in ten has any distinct recollection of ever having noticed the phenomenon, unless his attention has been directed to it, and it may be of interest to teachers to know that it can be shown by means of a very simple and inexpensive piece of apparatus.

A small pitch pipe such as is used by musicians is securely fastened to the end of a tapering stick six or seven feet long, such as a blackboard pointer. This can be blown with a piece of rubber tubing fastened along the rod, and by waving the device rapidly, it is possible to give the sounding pipe a to-and-fro velocity of twenty or thirty feet a second, which is sufficient to cause a marked rise and fall in the pitch. By waving the stick first in the direction of the "line-of-sight," and then at right angles to it, it can be shown that the effect occurs only in the former case. A bamboo fish pole with the partitions bored out, the pipe fastened into the tip by means of sealing wax, and a short piece of rubber hose inserted near the breach, would make a less clumsy piece of apparatus.

NOTE.

Galileo Ferraris. — Galileo Ferraris, physicist and engineer, died on the 7th of February, 1897. The city of Turin, where he met his sudden death from pneumonia, had been the central scene of his lifelong scientific activity. Although he was born at Livorno-Vercellese (1847), it was at Turin that he was educated; and that city has the honor of counting him among the illustrious of the graduates of her university (1867) and of her royal school of engineering.

In 1869, immediately upon the completion of his technical training, he was appointed assistant in applied physics in the *Museo Industriale* at Turin, and in 1879 he became professor of technical physics in that institution. The latter post he occupied until his death.

In the development of electrical engineering, which had its beginnings during this period, Ferraris was a pioneer. The laboratory, of which he was director, became one of the chief centers of activity in Italy in the domain of applied electricity. Ferraris was one of the first to investigate the currents generated in the telephone, and he was also one of the first to make practical experiments upon the transformer, and to establish the theory of that apparatus. In the development of the polyphase motor he was contemporaneous with Tesla, if indeed he did not anticipate the inventions of that electrician; and in this field, as in every subject upon which he wrote, his treatment was fundamental and exhaustive.

Ferraris' scientific activity, however, was by no means confined to the field of applied electricity. His contributions to optics, to mechanics, and to thermodynamics are scarcely less notable. His book on the *Cardinal Properties of Optical Instruments*, which appeared in 1878, is justly regarded as one of the most important treatises upon that subject. Every topic which he subjected to investigation he treated with that breadth and precision which characterizes the trained physicist, together with an insight into the purely practical and technical aspects of the subject which give his papers high rank from the standpoint of the engineer.

Ferraris' eminence as an electrician was fully recognized by his countrymen from the first. He was appointed by the government as an official delegate to the Electrical Congress in Paris in 1881, and he took part in the International Conference held in that city in 1882. At the Vienna exhibition, the following year, he again acted as delegate. Of the Turin exhibition of 1884 he was one of the chief organizers, and he was chosen president of the section devoted to electricity. In 1886 he organized the Electrotechnical School for Engineers at Turin. This was the first school for electrical engineering established in Italy.

In one sense Ferraris' technical career may be said to have had its culmination at the Frankfort exhibition of 1891, where the principles of polyphase transmission, which he had worked out in so admirable a manner, were applied to the transmission of power from Lauffen to the exhibition grounds in Frankfort, a distance of over a hundred miles. Although this occasion may be regarded as a sort of triumphant climax, it did not mark the beginning of any decadence in Ferraris's activity, which indeed continued unabated up to the time of his sudden death. In the work of the Chamber of Delegates of the World's Congress of Electricians, held in Chicago in 1893, he took an important part.

During the very last year of his life Ferraris represented Piedmont as senator in the upper house of the Italian legislature.

E. L. N.

NEW BOOKS.

Molecules and the Molecular Theory of Matter. By A. D. Risteen. 8vo, pp. 223. Boston, Ginn & Co., 1895.

The experimental and theoretical developments of the past thirty years in molecular physics and chemical dynamics have almost completely transformed our views regarding the constitution of matter and created a new branch of natural science.

Although we have in some of the larger physical and chemical compendia elaborate discussions of certain phases of the subject, no exhaustive treatment of both the theoretical and experimental matter has yet appeared. The 1500-page compendium of Lehmann, or the treatise of Ostwald, illustrates the amount of material which can be selected out of the chaotic mass of matter which has accumulated in a comparatively short time.

Much, however, has been done in classifying facts and developing theory. Thomson's interesting lectures in physical and chemical dynamics suggest great possibilities in this line of development; and Boltzmann's *Gastheorie*, embodying Maxwell's theory, shows the finished groundwork for a dynamical theory in the simplest case.

The attempt to deduce thermodynamic laws from dynamical principles, initiated by Clausius and proved possible by Kirchhoff, and later perfected by von Helmholtz in his paper on "Least Action," and further supplemented by Boltzmann, has been crowned with success. Further extension in the same line has been made by Duhem and others, and indicates the present possibility in molecular dynamics.

The application of thermodynamics to chemical phenomena and electrical processes has received unusual attention during the past twenty years, and the work of Massieu, Gibbs, von Helmholtz, van't Hoff, and others has firmly established the true principles involved. With this link between chemical processes and dynamical principles, it is not a visionary possibility to expect that some day molecular phenomena will be as well established on dynamics as have Clausius, Maxwell, and Boltzmann established those of monatomic gases on the principles of Lagrange and Hamilton.

While the molecular theory has been so largely extended in this field, it has been sadly neglected in other lines. Little has been done on a molecular theory of sound. Such a theory might complicate rather than simplify this already difficult subject. However, many phenomena will not receive their complete solution short of the use of such a theory. The propagation of very loud sounds of varying intensities may not be exactly determined with present methods, unless corrections depending on molecular considerations are introduced. In ordinary vibrations, thermodynamic considerations must enter, as instance the Laplacian correction to Newton's law as discussed and further corrected by Stokes.

In the subject of light, however, much has been done since the papers of Sellmeyer and von Helmholtz appeared. Kelvin's Baltimore lectures have done much to expand our views on the subject, and his gyrostatic molecule goes far toward a mechanical solution of magnetic rotary polarization, and serves also as a basis of an elastic solid theory.

Rayleigh has shown that our present laws in photometry are what should follow from the arbitrary motions in our kinetic theory, and also how the spectral lines are broadened from the same cause; and he has further extended the dynamics of diffraction of Stokes to the scattering of light by small particles. On the other hand, Kirchhoff has, by use of Green's theorem, extended Huyghens' principle, but he has, however, eliminated the idea of secondary or molecular disturbances assumed by Huyghens.

The complete agreement of von Helmholtz' conclusions on dispersion, both from the elastic solid and from the electromagnetic standpoint, with recent observations on both Röntgen rays and on the extreme ultra-red rays, strengthens the correctness of the assumptions; while the observed absorption by gratings of long heat waves indicates the true dynamical theory in doubly refracting substances.

The revival of Weber's theory of magnetism, with the adoption of a bipolar state of polarization in a dielectric by von Helmholtz, and the interesting application of Faraday tubes by Thomson, illustrate the activity in this branch of the subject. The experimental observations of the action of static charges in motion have done much to confirm, on the one hand, Weber's hypothesis of electrodynamic action in his famous controversy with von Helmholtz, and on the other, Maxwell's views as to dielectric polarization. This apparent anomaly in nature, of an action involving time as a variable, offers, perhaps, the only instance so far which is not reconcilable with the principle of the conservation of energy, as von Helmholtz has shown in his discussion.

The kinetic theory of gravitation has been much strengthened, and some of the inherent difficulties explained away.

Evidently a molecular theory for natural phenomena must mean much more than merely constitutions of the physicist's molecules or of the chemist's atoms in ponderable matter; we must premise other physical units differently endowed.

Any attempt to bring before the scientific student a brief résumé of some of the present theories, and the facts upon which they are based, must certainly be appreciated. We have in the small volume before us an expansion of a popular lecture, which will be gladly welcomed by the beginner and commended by the teacher. However, the reader must content himself with only a brief reference to some of the better known phenomena and discussions. The author, however, has made an excellent selection for the purpose in view, and has presented most of it clearly and concisely. Several of the topics are interesting as a matter of history only, and might have been omitted for greater expansion on other points which have a living value in the theories of to-day, and which the author hardly carries far enough for the beginner.

The first part of the book is devoted to a short historical introduction on molecular constitution and the discussion of the kinetic theory of a gas treated as monatomic. Boltzmann's theorem of energy distribution, the explanation of pressure, temperature, specific heat, viscosity, law of molecular force, van der Waals' equation, etc., follow with appropriate examples. "Free path" and its bearing on high vacua receive some attention, inadequate to the extent and interest of the subject. Less exact, as it must be, is the next part on liquids. But much suggestive matter is given in regard to evaporation, ebullition, critical temperature, compressibility, etc., which, however, is largely speculation. Some classic experiments on capillarity are given, and an example of the action of molecular forces is explained in vaporization.

In the case of solids, the kinetic theory is illustrated rather briefly; but solutions, diffusion, osmotic pressure, crystallization, etc., are discussed and might well be expanded, considering the interest in this branch of physical chemistry. The discussion of the various methods of determining molecular magnitudes is more satisfactory and certainly interesting. Several pages are devoted to Dulong and Petit's law, the compound nature of the elements, the periodic law and the harmonics in atomic vibrations and the ultimate nature of matter.

The elastic solid theory is compared with the electromagnetic theory and rejected. This is unjust to the exponents of the contractile ether form of this theory, which, as proposed and worked out by Kelvin and extended to double refraction, dispersion, etc., by Glazebrook, explains as well as the electromagnetic theory, which is not entirely satisfactory. Duhem found on using von Helmholtz' earlier equations that there is the same difficulty in reflection as on the elastic solid theory. The difficulty at the ultimate bounds of such a medium is of the same order as that at the limits of an electrical medium containing a charge within it.

The author does not extend the molecular theory to include other physical units, but limits the discussion to the case of gravitation, to which several pages are devoted in an interesting résumé of Kelvin and Maxwell's suggestions on the subject, omitting reference to more recent speculation "on the Continent."

Considering the scarcity of compiled material in English, the book is certainly a valuable addition to the popular literature of the subject.

D. B. Brace.

A Manual of Physics. By William Peddie. Second edition. 8vo, pp. xvii 573. New York, G. P. Putnam's Sons, 1896.

The first edition of this work has already been referred to in the pages of the Physical Review.[1] In the new edition the general plan of the book has not been altered, although several portions have been rewritten. One of the most noticeable changes is in the mathematical methods used, which are considerably simplified. New material is added to the extent of nearly a hundred pages.

In the words of the author "this book is intended to be an accompaniment to a course of lectures on physics, by the use of which a student may largely avoid the evils of note-taking, and give more attention to the words of his teacher." Dr. Peddie's familiarity with the methods and requirements of the Scotch Universities makes it probable that the book is well suited to the needs of the Scotch and English student. Whether it will be equally useful in our American Universities is quite another question, and one which I should be inclined to answer in the negative. In this country a lecture course on general physics is usually largely experimental, and is intended to give to the student as concrete a notion as possible of the phenomena with which the science deals. To supplement such a lecture course a text-book is required which shall give a clear and rigorous statement of the underlying principles and theories. If descriptive matter can also be included, without making the book too bulky, this is doubtless desirable. In my opinion Dr. Peddie's *Manual* does not fulfill these requirements. While possessing many excellent features, the book is distinctly superficial in its treatment. The discussion of the second law of thermodynamics (p. 382) may be cited in support of this statement. The student who reads this discussion, and believes that he has learned something about the subject, will be in a worse position than one who has never studied it; for before proceeding further he will be compelled to disabuse his mind of many notions that are, to say the least, vague, and in some cases actually unsound.

While the superficial character of the book renders it unsuitable, in my opinion, as a text-book, a large amount of valuable matter is contained in Dr. Peddie's *Manual.* Many subjects are touched upon which are usually not mentioned in text-books at all; take, for example, the brief discussion of the various theories that have been suggested to explain gravitational attraction (p. 84). It would be a great help to the reader if references were given in such cases to some work where the subject is discussed in more detail. The tendency of the book is unquestionably to make one think, and for one who is well grounded in the fundamental principles of physics, so as to be on his guard against plausible but inexact methods of proof, the *Manual* will furnish interesting and suggestive reading.

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[1] Vol. I., p. 79.

ERNEST MERRITT.

Practical Work in Physics for Use in Schools and Colleges. Part III., Light and Sound. By W. G. WOOLLCOMBE. pp. x + 94. Oxford, The Clarendon Press, 1896.

This volume follows Part I., Practical Work in Heat, and Part II., Practical Work in General Physics.[1] The experiments are simple and suitable for beginners; no elaborate apparatus is required. The thirty experiments in light are classed in the following groups: Photometry, Reflection at Plane Surfaces, Reflection at Spherical Surfaces, Refraction at Plane Surfaces, Refraction through Lenses, Simple Optical Instruments. The twenty experiments in sound are grouped as follows: Transverse Vibrations of Wires, Velocity through Gases, Velocity through Solids, Interference. The directions are clear and the selection of material good, when viewed from the standpoint of the needs of elementary classes. B.

The Magnetic Circuit. By H. DU BOIS. Translated by Dr. ATKINSON. 8vo, pp. xviii + 362. London, Longmans, Green, & Co., 1896.

The translation of Dr. du Bois' work on the magnetic circuit makes available to the English-speaking public a treatment of the subject of magnetism that possesses several distinctive features. As I have already mentioned in discussing the German edition,[2] the book is not to be looked upon as one which will take the place of the well-known English works on magnetism. It will rather serve to fill some of the gaps in the treatment of the subject which have resulted from the special character of previous treatises. It is this characteristic of the work that has especially impressed me in using it as a book of reference.

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[1] Reviewed in the PHYSICAL REVIEW, Vol. II., p. 317.

[2] PHYSICAL REVIEW, VOL. II., p. 311.

Even in those parts of the subject which must of necessity be common to all books on magnetism, the mode of presentation is sufficiently different from the usual one to offer valuable suggestions. In deriving the general laws of the magnetic field, for example, Dr. du Bois has discarded completely the conception of a magnetic pole. Strength of field is defined in terms of the electromotive force induced in a small test coil when the latter is rotated through 180°, and the whole system of equations in the theoretical part of the book is developed by reference to electro-dynamic actions and the phenomena of electro-magnetic induction. This method of approaching the subject possesses advantages that have already been recognized by several writers on the subject of magnetism. But I am not acquainted with any other work in which the method has been so systematically developed.

The work of translation is in general excellent. Numerous notes that have been added by Dr. du Bois, himself a master of the English language, make the translation more complete than the original.

Reference must be made to the review of the German edition for a more detailed discussion of the plan of the book.

ERNEST MERRITT.

A Primer of the Calculus. By E. SHERMAN GOULD. 12mo, pp. 91. Plates. New York, D. Van Nostrand Company, 1896.

This little book aims to give a working knowledge of the calculus where one independent variable and the first differential only are involved. The work is quite complete so far as it goes, and will prove useful in helping the reader to use the calculus as a tool in solving many of the problems in physics and mechanics.

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